LIGHTWEIGHT STRUCTURES in CIVIL ENGINEERING
CONTEMPORARY PROBLEMS
Monograph from Scientific Seminar
Organized by Polish Chapters of
International Association for Shell and Spatial Structures
Łódź University of Technology
Faculty of Civil Engineering, Architecture
and Environmental Engineering
XXVII LSCE
Łódź, 2nd – 3rd December 2021

ANALYTICAL AND NUMERICAL RELIABILITY ANALYSIS OF THE PRATT STEEL TRUSS

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ABSTRACT: The main aim of this paper was to investigate a variation of the reliability index β for some steel truss girder depending with respect to its length and to check the differences in numerical results obtained with the use of numerical and analytical probabilistic calculations. Some small Pratt truss, which includes vertical members and diagonals that slope down towards its centre was selected to make such a comparison. The span of this truss has been adopted as the Gaussian input variable in this study, which is motivated by in situ observations and measurements statistics as well as by its nonlinear impact on the Serviceability Limit State. The basic probabilistic characteristics of the structural response have been determined using the 10^{th}-order Iterative Generalized Stochastic Perturbation Method and using Monte-Carlo Simulation also. The very important aspects in this work are (*i*) a contrast of analytical and numerical results and (*ii*) stochastic analysis of the load capacity in this truss joints, i.e. the stresses in the welds. Numerical results presented here have been obtained by the Finite Element Method system *Robot Structural Analysis Professional* 2021 and also via symbolic computer algebra environment *MAPLE*.

Keywords: reliability index, stochastic perturbation method, Monte-Carlo simulation, finite element method, truss girder, deflection.

1. INTRODUCTION

There is no doubt that stochastic computational mechanics based is very well and efficiently explored research area with many concurrent methods, existing however without wide computational implementations. The practising engineers must use some additional software or create some new small routines by themselves to determine at least the reliability indices mandatory for modern civil engineering structures. The very specific role is in distribution and usage of the computer algebra systems like MAPLE, MATHEMATICA or MATHCAD, where many stochastic methods may be implemented, but the Finite Element Method modeling still remains a separate part of the engineers job. It is well known right now that a huge portion of mathematical effort in development of analytical formulas for structural engineering analyses has been lost last years thanks to the common and fast methodology of „computing everything". This is not the very clever way as many simple structures may be quite efficiently designed using classical formulas extended relatively easy with mathematical software towards reliability assessment. The Pratt steel truss girder has been selected to demonstrate such an approach due to its popularity in academic analyses during Civil Engineering courses and a huge number of the existing old steel bridges all around the world (some example presented in Fig. 1).

Fig. 1. Old steel truss bridge over Brazos river (www.bridgehunter.com).

2. STRUCTURAL ANALYSIS

All the calculations have been made for Pratt truss given in Figure 2 below, whose height was adopted as h = 1,50 m, where the number of segments equal to 12 remains unchanged in further cases. The truss span l is treated as the input Gaussian random variable. It has been discretized by the discrete set of values increasing from 15,00 up to 21,00 m every 0,60 m. Each truss FEM model has been uploaded with the same uniform linear load q applied at the nodes as concentrated forces throughout the upper chord. Its characteristic value has been assumed as q_k = 15 kN/m, while the design load has been set as q_d = 20 kN/m. Let us underline that adoption and combination of safety indices has marginal influence on this study, so that has been postponed here. All the cross-sections of structural components have beem assumed as hot-finished square-hollow steel profiles made of structural steel S235, whose material parameters have been assumed according to the design code Eurocode 3 (EN 1990:2002). Structural parameters assigned to the individual elements have been presented in Table 1 below.

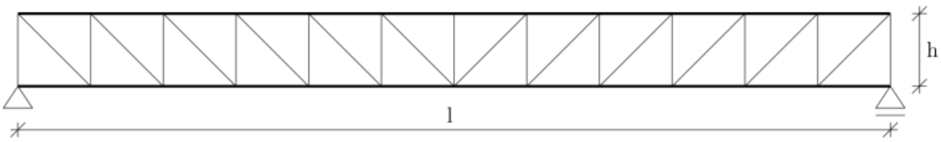

Fig. 2. Static scheme of the given Pratt truss.

Table 1. Structural parameters of steel elements

Parameter	Upper chord	Diagonals	Lower chord
Member length ly	0,90	1,00	0,90
Member length lz	1,00	1,00	1,00
Buckling length coefficient y	0,08	0,90	0,08
Buckling length coefficient z	0,16	1,00	0,50

Analytical calculations for expected values $E[u(x)]$ and the variances $Var(u(x))$ have been completed starting from the deterministic formula describing approximate deformation line $u(x)$ of such a truss as:

$$u(x)=\frac{ql^4}{24EJ}\left(\frac{x}{l}-2\left(\frac{x}{l}\right)^3+\left(\frac{x}{l}\right)^4\right), \tag{1}$$

$$E[u(x)] = \frac{qx}{24EJ}\left((E[l])^3 + 3\sigma^2(l)E[l] - 2E[l]x^2 + x^3\right), \tag{2}$$

$$Var(u(x)) = \frac{q^2x^2\sigma^2(l)}{576E^2J^2}\left(9(E[l])^4 + 36(E[l])^2\sigma^2(l) - 12(E[l])^2x^2 + 15\sigma^4(l) - 12\sigma^2(l)x^2 + 4x^4\right), \tag{3}$$

where q is a characteristic value of load, l is theoretical span of the truss, E is Young modulus, J denotes inertia moment of the upper & lower chords, while $m(l)$ and $\sigma(l)$ stand for the expectation and standard deviation of the truss girder length. Such a characterization of probabilistic structural response is definitely wider than that coming from numerical methods (Kamiński 2013, Kamiński 2015), because it is able to draw the diagrams of expectations and variances throughout the girder lenght as the continuous function of their parameters and, further, this is exact from probability theory point of view. The engineers do not need to contrast these results with Monte-Carlo simulation results and the only error comes from approximate deterministic model. The reliability index β have been determined for the numerical and analytical results of truss deflection (fourth order polynomial of the truss span l) and the numerical results of stresses in joints (third order polynomial of l appeared to be optimal here). The approximating functions have been obtained all on the basis of the Least Squares Method unweighted approximation (Bjorck 1996). Numerical effort analyzed with the FEM system ROBOT ranges from about 65% up to 95% for the structural joints (while l varies from 15.0 up to 21.0 meters), while the effort for lower and upper chord elements varies from 45% up to more than 85% at the same time. It enables to conclude that the ULS-based reliability index should be based upon the joints rational designing, while the SLS – traditionally – upon this truss midspan deflection.

3. NUMERICAL RESULTS

In this work the values obtained from the 10th-order iterative stochastic perturbation method and the Monte-Carlo Simulation were compared in Table 2 assuming the sample size equals to $n = 10^6$. This analysis has shown that the values of the reliability index obtained from numerical and analytical calculations for the stochastic perturbation method and Monte-Carlo simulation are very close to each other. The differences do not exceed 3%, so they are practically negligible.

Table 2. A contrast of the reliability indices by numerical and analytical methods based upon the stochastic perturbation technique and the Monte-Carlo simulation method

	Stochastic perturbation method		**Monte-Carlo simulation**	
α [-]	β (NC) [-]	β (AC) [-]	β (NC) [-]	β (AC) [-]
0,025	38,11	38,43	38,10	38,42
0,050	19,05	19,20	19,18	19,32
0,075	12,69	12,79	12,65	12,75
0,100	9,50	9,58	9,53	9,60
0,125	7,59	7,65	7,69	7,74
0,150	6,31	6,37	6,33	6,38
0,175	5,40	5,44	5,35	5,40
0,200	4,71	4,75	4,81	4,85
0,225	4,17	4,21	4,22	4,25
0,250	3,74	3,77	3,65	3,69

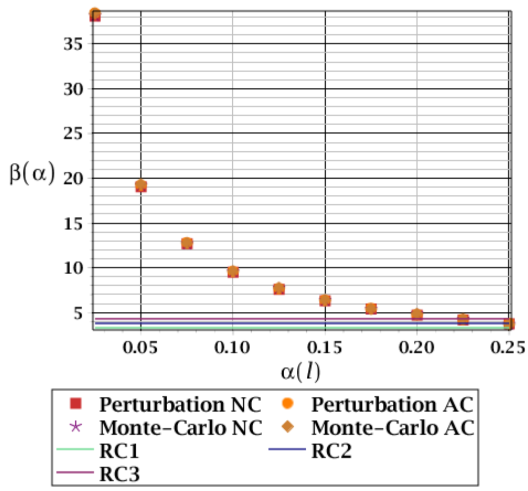

Figure 1. Reliability index β due in the SLS

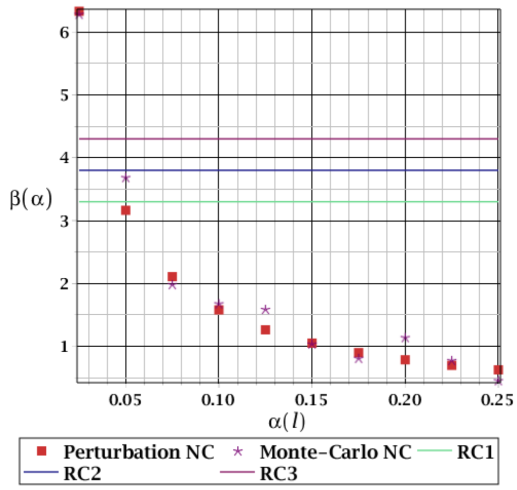

Figure 2. Reliability index β in the ULS for the joints

A comparison of the results shows that the values of the reliability index β calculated on the basis of the analytical formula are each time greater than those determined from numerical calculations. The difference in value is about 1%, so it can be concluded that the accuracy of the analytical method is sufficient for practical engineering calculations. Comparing further the reliability indices β with the values recommended by the EN 1990 standard it should be stated that the joints are the most sensitive elements of this truss. In the case of the analysis of the same truss, it can be seen that for nodes the value o reliability index β is insufficient for the coefficient of variance $\alpha > 0.025$ (Figure 3), while for the global analysis of truss such situation has been when $\alpha > 0.200$ (Figure 2).

4. CONCLUSIONS

Numerical analysis delivered in this work proves that structural reliability of the Pratt truss structures subjected to geometrical uncertainty can be efficiently modelled with the use of analytical formula describing its deflection line. This conclusion has been drawn on the basis of a contrast of the Stochastic perturbation-based FEM, Monte-Carlo-enriched FEM analysis and their analytical counterparts. This is very promising and important result as the deflection line of these trusses can be easily implemented in any computer algebra systems with probabilistic libraries. This would enable further applications of many correlated uncertainty sources having not necessarily Gaussian but any desired probability density function(s). Such a reliability analysis may replace in the future traditional deterministic design including a couple of the safety indices. A difference in-between numerical and analytical results for the reliability index increase rather slowly together with input statistical scattering, so that initial hypothesis is with quite good efficiency unbiased by the experimental statistics. It is also seen that this approach can be extended over many existing analytical models in civil engineering to develop stress formulas as well as deflections lines for some typical beams, plane frames and other girders to make analytical reliability analysis more popular.

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