JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
48, 2, pp. 517-524, Warsaw 2010

SYNCHRONISATION AND PERIODISATION OF DUFFING OSCILLATORS COUPLED BY ELASTIC BEAM: FINITE ELEMENT METHOD APPROACH

Agnieszka Chudzik

Technical University of Lodz, Division of Dynamics, Łódź, Poland

e-mail: agnieszka.chudzik@p.lodz.pl

Results of numerical analysis of a structure witch consists of two identical chaotic oscillators suspended on an elastic element are presented. The numerical calculations have been carried out with the use of the professional ANSYS software (User's Gude ANSYS 10). The findings show that for given conditions of the excitation, the initially uncollerated chaotic oscillations of the structure become periodic and synchronous.

Key words: oscillators, elastic structure, frequency

1. Introduction

Typical attractors of a dynamical system are fixed points, limit cycles, quasiperiodic trajectory and strange attractors (chaotic behaviour). One of the characteristic features of a non-linear system is the presence of co-existing attractors. This feature is called multistability. Multistability has been observed by many researchers dealing with dynamical systems (Feudel *et al.*, 1996; Kraut and Feudel, 2002; Pecora and Carroll, 1990). This phenomenon was examined with respect to the possibility of synchronous occurrence of oscillators (Pecora and Carroll, 1990; Kapitaniak, 1996; Chen, 1999; Boccaletti *et al.*, 2002).

The subject under consideration is dynamics of two Duffing chaotic oscillators suspended on an elastic beam. The Oscillators are excited by a periodic signal with frequency ω. In the previous study (Czołczyński *et al.*, 2009), the initially uncorrelated chaotic oscillator became periodic and synchronous as result of interaction with the elastic beam. Another interesting observation concerns the response of the elastic beam and the oscillators to the excitation with no accompanying synchronisation of their oscillations. In the work

Czołczyński *et al.* (2009), a simple discrete model of the beam was considered, whereas the aim of this papers is to find if the previously observed dynamical phenomena exist when the beam is discretised using the finite element method. The model of the analysed structure is shown in Fig. 1.

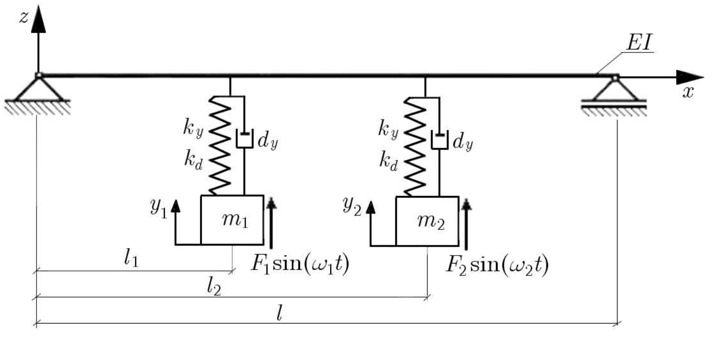

Fig. 1.

All Duffing oscillators used in the analysis are described by the following formula

$$m_i\frac{d^2y_i}{dt^2}+d_y\frac{dy_i}{dt}-k_yy_i+k_dy_i^3=f\sin(\omega_i t) \tag{1.1}$$

where d_y, k_y, k_d, f, ω are constants, $i=1,2$.

The above differential equation possesses a solution y(t) for an open set of initial conditions. The presence of damping in Eq. (1.1) means that this system is dissipative and its solution $y(t)$ tends to the attractor, i.e. the minimum subset $A\subset R^n$, where R^n is an n-dimensional phase space of Eq. (1.1), during the time evolution $(y(t)\to A$ as $t\to\infty)$. The possession of the attractor indicates global stability of the system. If, additionally, the system is locally stable (i.e. the solution $y(t)$ is insensitive on the initial condition $y(t=0)$), then the typical attractors of system (1.1) are fixed points (stable equilibrium position – only in the case of lack of excitation, e.g., $f=0$), limit cycles (periodic behaviour), tori (quasiperiodic behaviour). On the other hand, in the case of local instability, the sensitivity on initial condition appears, and we can observe the so-called strange attractor (chaotic behaviour). One of the characteristic features of a nonlinear system is the coexistence of attractors in the phase space, i.e. for given parameter values depending on the initial conditions, the system trajectory can go to a different attractor.

The beam is considered as a continuous homogenous linear elastic structure of the length l characterised by the modulus of elasticity E and the inertial moment of the cross section J. Details of the numerical discretization of the beam are given in Section 2. The results of the numerical computations are presented in Section 3. Finally, the results are summarised in Section 4.

2. Numerical model

The model has been developed for numerical calculations whose schematic view is presented in Fig. 2. The subject taken into consideration consists of two Duffing oscillators suspended on an elastic beam. For the numerical calculation, professional ANSYS software packages were incorporated (User's Guide ANSYS 10).

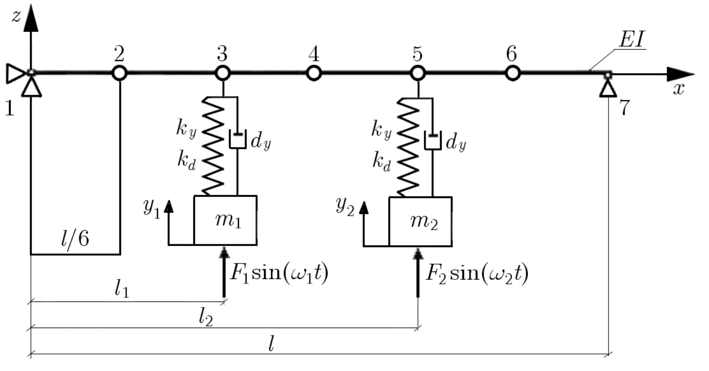

Fig. 2.

The structure has been built from the numerical elements:

- COMBIN 39 which describes non-linear springs,
- COMBIN 14 which describes viscotic damping.

For the numerical computation, transient dynamic analysis has been employed. The transient dynamic equation presented below has the following linear structure

$$\mathbf{M}\ddot{\boldsymbol{u}} + \mathbf{C}\dot{\boldsymbol{u}} + \mathbf{K}\boldsymbol{u} = \boldsymbol{F}^a \tag{2.1}$$

where: $\mathbf{M}$ is the structural mass, $\mathbf{C}$ – structural damping, $\mathbf{K}$ – structural stiffness, $\ddot{\boldsymbol{u}}$ – nodal acceleration, $\dot{\boldsymbol{u}}$ – nodal velocity, $\boldsymbol{u}$ – nodal displacement, $\boldsymbol{F}^a$ – applied load.

The parameters of the oscillators are: $d_y = 0.168\,\mathrm{Ns/m}$, $k_y = 0.5\,\mathrm{N/m}$, $k_d = 0.5\,\mathrm{N/m^3}$, $f = 1\,\mathrm{Hz}$, $\omega = 1\,\mathrm{s}^{-1}$, $g = 1$.

The beam is supported on both ends. Other structural parameters are as follows:

modulus of elasticity [N/mm^2]	$E = 2 \cdot 10^5$
Poisson's ratio	$\nu = 0.3$
density [kg/mm^3]	$\rho = 7.65 \cdot 10^{-6}$
sectional moment of inertia [mm^4]	$J = h^4/12 = 52.08$
damping coefficient	$g = 1$
mass [kg]	$m_1 = m_2 = 1$

The non-linear part of equation (1.1) $(k_d y_i^3 - k_y y_i)$ has been introduced as a discrete value of displacement forces described by this equation. The results of calculations for the range of spring dislocations are presented by a graph in Fig. 3.

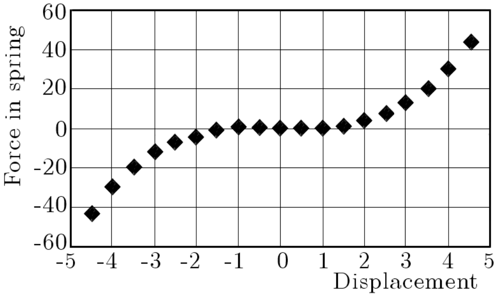

Fig. 3.

The load $[f \sin(\omega t)]$ was introduced to calculations of discrete values dependent on time. On the basis of the following data $(\omega = 1,\ f = 0.21)$ the vibration period $T = 2\pi$ s was accepted. The value of load is shown in Fig. 4.

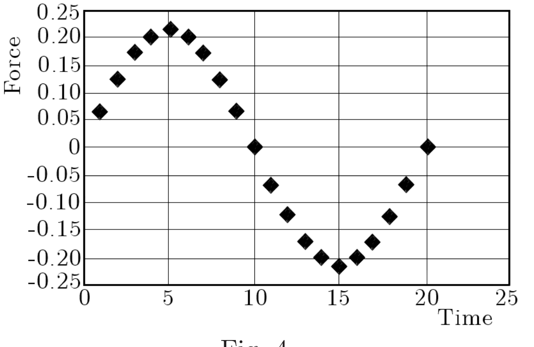

Fig. 4.

The numerical analysis was made for two different initial conditions. Figure 5 shows solutions for two combinations of the oscillators.

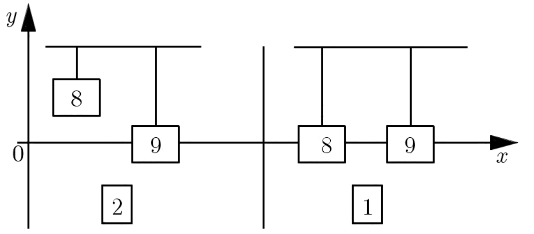

Fig. 5.

The responses of masses suspended on the oscillators to load changes specified above are shown in the graphs in Fig. 6.

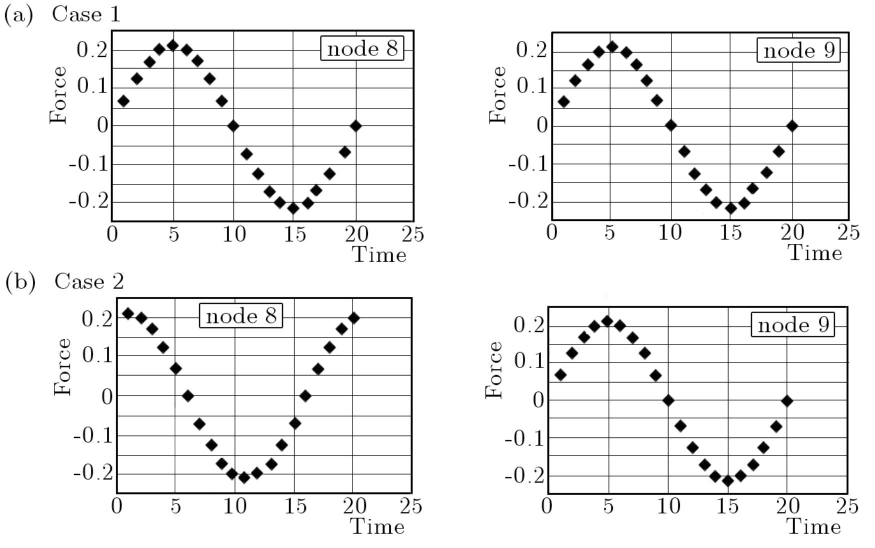

Fig. 6.

3. Results of numerical simulation

Results of numerical calculations are presented in graphs showing the node displacement. In both analysed cases, the displacement of node 8 in relation to node 9 and of node 3 in relation to node 5 is shown below, see Fig. 7a,b.

Figure 8a shows iterations of oscillations of nodes 8 and 9. Figure 8b presents oscillations of nodes 3 and 5. The displacement of nodes 3, 5, 8, and 9 is given in Fig. 8c.

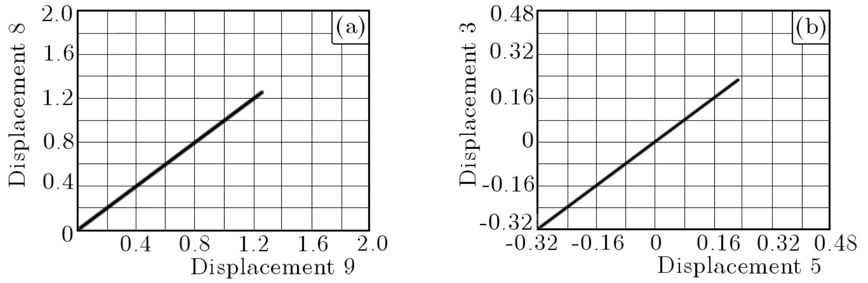

Fig. 7.

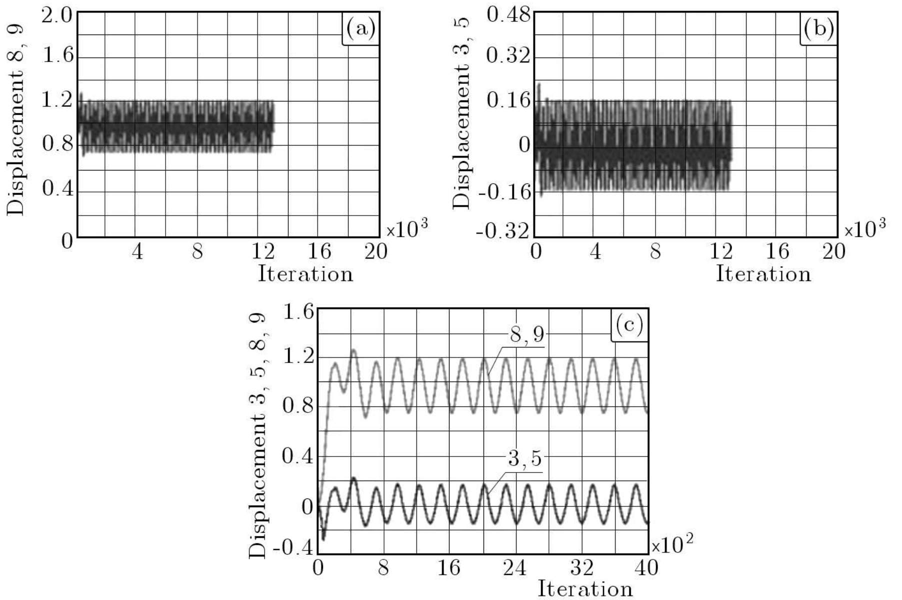

Fig. 8.

While analysing graphs of the displacement, it can be seen that the initially chaotic behaviour of the oscillators becomes periodic and synchronous in clusters. Some exemplary results of numerical calculations for case 2 are presented in the graph showing the displacement of:

- node 8 versus displacement of node 9 – Fig. 9a,
- node 3 versus displacement of node 5 – Fig. 9b.

The displacement of nodes 3, 5, 8 and 9 is given in Fig. 10.

The analysis of the obtained results proves that the synchronisation of the elastic beam and the Duffing oscillators does not occur. The oscillations

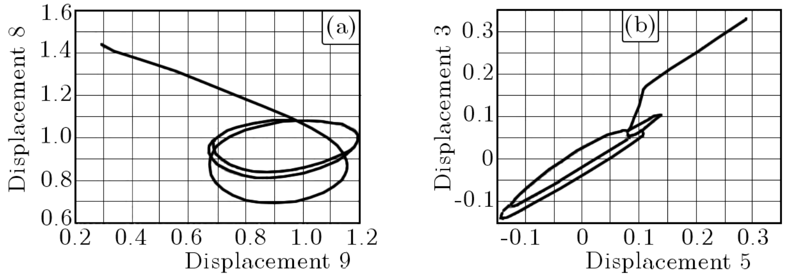

Fig. 9.

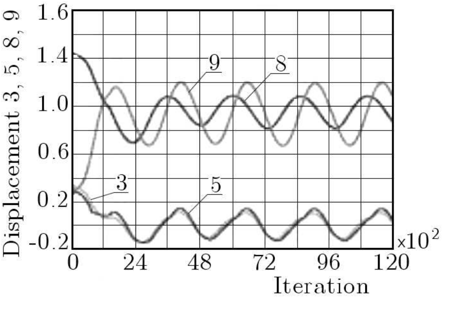

Fig. 10.

of the beam are periodical but they are not harmonic, on the other hand the displacement of Duffing's oscillators shows lack of synchronisation. In this case, many equilibrium points of the beam UY_3=UY_5 and oscillators UY_8=UY_9 hove been observed (Fig. 10). These points occur in different places along the course of experiment. They testify to the lack of synchronisation between the beam displacement and synchronisation of the oscillators.

4. Conclusions

Summarising, the use of the ANSYS software allowed one to analyse the structure consisting of an elastic beam to which two non-linear chaotic Duffing oscillators have been added. The numerical study enabled identification of the phenomenon in which the oscillators co-operating with the elastic beam behave:

- periodically and synchronically (case 1),
- periodically and not synchronically (case 2).

The numerical analysis confirmed the existence of relation between dynamics of the oscillators and the beam response.

The numerical computation obtained with the use of ANSYS supported the conclusions and observations formulated by Czołczyński *et al.* (2009).

References

1. Boccaletti S., Kurths J., Osipov G., Valladares D.L., Zhou C.S., 2002, The synchronization of chaotic systems, *Physics Reports*, **366**, I
2. Chen G., 1999, *Controlling Chaos and Bifurcations*, CRC Press, Boca Raton
3. Czołczyński K., Stefański A., Perlikowski P., Kapitaniak T., 2009, Mulitistability and chaotic beating of Duffing oscillators suspended on an elastic structure, *Journal of Sound and Vibration*, **322**, 3, 513-523
4. Feudel U., Grebogi C., Hunt B.R., Yorke J.A., 1996, A map with more than coexisting law-period periodic attractors, *Physical Review E*, **54**, 71-81
5. Kaneko K., 1997, Dominance of Milnor attractors and noise-induced selection in a multiattractor system, *Physical Review Letters*, **78**, 2736-2739
6. Kapitaniak T., 1996, *Controlling Chaos*, Academic Press, London
7. Kraut S., Feudel U., 2002, Multistability, noise and attractor hopping: the crucial role of chaotic saddles, *Physical Review E*, **66**, doi:015207(1-4)
8. Pecora L., Carroll T.S., 1990, *Physical Review Letters*, **64**, 821-824
9. User's Gude ANSYS 10

Synchronizacja mechanicznych oscylatorów Duffinga zamocowanych na elastycznej belce

Streszczenie

W artykule przedstawiono wyniki analizy numerycznej struktury składającej się z dwóch identycznych chaotycznych oscylatorów zawieszonych na sprężystej belce. Obliczenia numeryczne przeprowadzono stosując profesjonalny pakiet programu ANSYS. Wykazano, że dla danych warunków wzbudzenia, początkowo nie są skorelowane, chaotyczne oscylacje struktury stają się okresowe i synchroniczne. W warunkach, kiedy częstości wzbudzenia różnią się, zjawiska powyższe nie występują.

Manuscript received June 4, 2009; accepted for print November 25, 2009