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Comparision of Numerical Results with Known Solutions of Buckling Problem of Pressured Shallow Spherical Shells

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Buckling problem of pressured shallow spherical shells is studied numerically for two types of finite elements: axisymmetric and non–axisymmetric. A very good correspondence of obtained results and known solutions is revealed in the case of clamed and hinged spherical segments for both types of finite elements. The comparison of results also shows that using non–axisymmetric finite element lets one get full pre– and post–buckling equilibrium path in the range of relative deflections w/h.

Keywords: buckling, spherical shells, external pressure, numerical simulation, axisymmetric and non-axisymmetric finite elements.

1. Introduction

The most effective bearing elements used in many fields of technology are thin–walled spatial structures, in particular, thin–walled shells. The ability to cover large spans without intermediate supports, strength at relative lightweight, architectural expressiveness cause a wide application of shells in the practice of both industrial and civil constructions.

The comparison of different shapes of shells shows a certain tendency towards design of ever more shallow shells. From the viewpoint of material saving and uniform distribution of stresses, the most favorable shape of shells is spherical one.

The buckling of real spherical shells under external pressure is a very difficult process that can appear in axisymmetric or non–axisymmetric deformations. Former investigations of this problem were mostly realized numerically on the base of own program codes performed by different authors [2–4]. Thus, a complete solution of nonlinear axisymmetric problem was found by Mescall [1, 2] for clamped spherical panels. Equilibrium paths were obtained with numerous limit and bifurcation points. Known solutions of Weinitschke [3] and Thurston [4] also corroborated the first peak of Mescall's solution. There are also some new recent results of non–axisymmetric nonlinear deformation of spherical segments found by Noite and Makowski and by Marcinowski [2] for clamped shells of medium depths.

At the same time, there is almost no numerical buckling study of shallow spherical shells formulated within universal FE codes. That is why the aim of this work is to compare numerical solutions of ANSYS software with previously mentioned solutions for the buckling process of spherical segments.

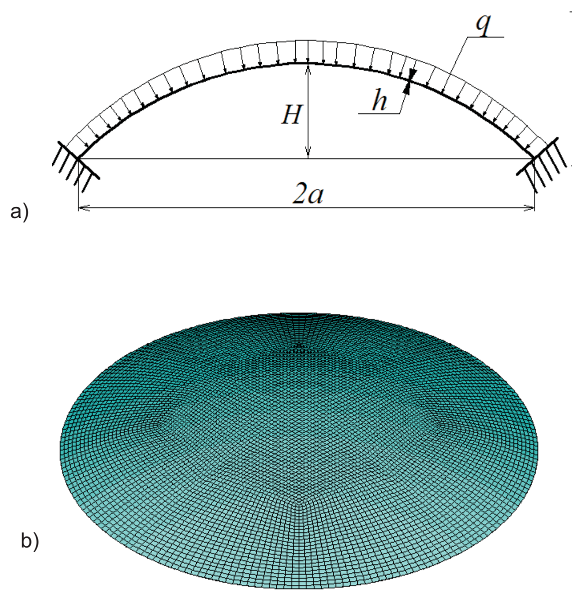


Figure 1 Loading scheme of a spherical shell (a) and its FE model for SHELL 281 (b)

2. Methodology of numerical research

Numerical analyses of buckling problem of elastic shallow spherical segments are carried out in a wide range of relative shallowness parameter $b = 2.5 \div 15$ [1]. The shells models have following geometrical characteristics: radius $R = 0.8\,m$, thickness $h = 4$ mm ($R/h = 200$). Material of the shells is highly elastic steel (modulus of elasticity $E = 2{\cdot}10^5$ MPa, Poisson's ratio $\nu = 0.3$). Spherical shells are constructed ideal by means of rotation of a generatrix line around the vertical axis. The loading (Fig. 1a) is applied as an external transverse pressure uniformly distributed over the entire surface of spherical segments. FE models consider two different boundary conditions: clamped support and fixed hinged support of bottom edges.

The numerical simulation of shells buckling is realized by means finite element method implemented in ANSYS software using two different standard structural FE from ANSYS library of shell elements. The first one is SHELL 208 which has two nodes with three degrees of freedom at each node: two displacements and one rotation. This FE is suitable for modeling thin and moderately thick axisymmetric shell structures with axisymmetric large strains. The second FE is a quadrilateral 8–node FE SHELL 281. It has six degrees of freedom (three displacements and three rotations) and is applicable for buckling problems of shells with large non–axisymmetric rotations and strains.

Despite of FE type, geometrically nonlinear buckling problems are performed for the definition of limit pressure (q) and corresponding buckling modes. Mention that nonlinear calculations are realized by means of arc length method.

3. Numerical results and discussion

Fig. 2 represents the comparison of ANSYS results with solutions of Weinitschke and Budiansky [1] as dependences of dimensionless pressure q/q_0 ($q_0 = 1.21E\,(h/R)^2$ – classical value of external pressure) on relative shallowness parameter b ($b = \sqrt[4]{12\,(1-\nu^2)}a/\sqrt{Rh}$ where a – base radius, see Fig. 1a).

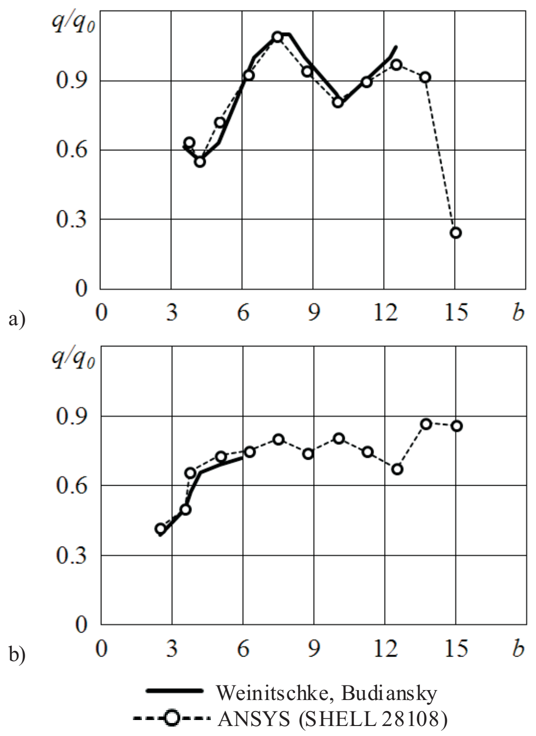
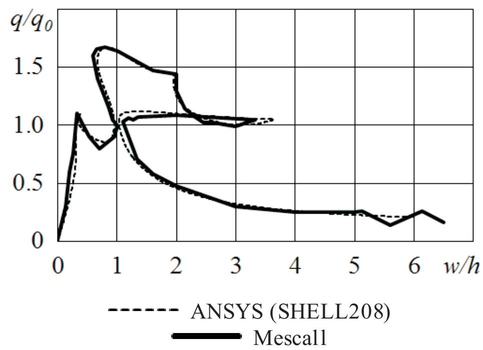


Figure 2 Dependences "$q/q_0 - b$" for the shell with $R/h = 200$ ($h = 4$ mm): a) clamped support, b) hinged support

A very good correspondence of ANSYS results (dotted lines) to the known solutions of Weinitschke and Budiansky (thick lines) is observed for both boundary conditions: clamped (Fig. 2a) and hinged (Fig. 2b) supports. Mention that present numerical solutions are accomplished for a large range of the shallowness parameter $b = 3.75 \div 15$. Here, values of critical pressure of hinged supported shells are lower than values of critical pressure found for clamped shells.

According to the previous research data, typical curves of spherical segments behavior are plotted as dependences of relative deflections w/h of pole points on the level of relative critical pressure q/q_0. Fig. 3 shows these curves of equilibrium paths for the spherical segment with $b = 8$ in the case of axisymmetric solutions (Fig. 3a) and non–axisymmetric ones (Fig. 3b).

a)


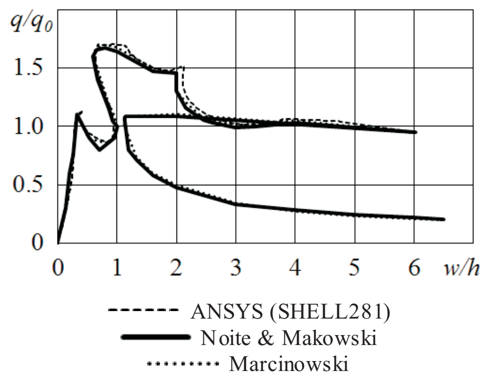
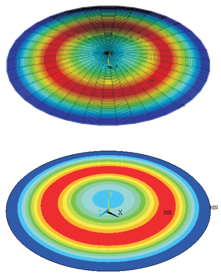
b)


Figure 3 Dependences "$q/q_0 - b$" for a clamped shell with $R/h = 200$ ($h = 4$ mm) and $b = 8$: a) axisymmetric FE, b) non–axisymmetric FE

The detailed analysis of presented graphs confirms that using the axisymmetric FE leads to the total coincidence of ANSYS results with Mescall's solution (Fig. 3a), and using non-axisymmetric FE also provides identical dependences "$q/q_0 - b$" for ANSYS solution and for the solutions of Marcinowski, Noite and Makowski (Fig. 3b). Note that buckling loads, as well as buckling modes obtained for different types of FE in the case of axisymmetric loading are identical (Fig. 4) despite of a different nature of possible deformations.



a)

b)

Figure 4 Buckling modes of shallow spherical shells with b = 8 for different FE: a) SHELL 208, b) SHELL 281

4. Conclusions

The buckling problem of spherical segments under external pressure is discussed for two different standard structural shell FE designed in ANSYS software.

Comparisons of numerical results obtained in ANSYS software with known solutions for the buckling of spherical shells are performed for two types of boundary conditions.

The full correspondence of ANSYS numerical solutions and known solutions shows that using non-axisymmetric element SHELL 281 gives a complete picture of the shell behavior in all the range of the parameter w/h.

Therefore ANSYS software can be successfully used for the investigation of buckling process of spherical shells in a wide range of geometrical parameters, boundary conditions, as well as in the cases of axisymmetric and non-axisymmetric loadings.

5. Acknowledgements

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