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PROBABILISTIC ANALYSIS OF THE 6061 ALUMINIUM ALLOY TENSILE TEST WITH RANDOM INITIAL MICRODEFECTS

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ABSTRACT: The main aim of this paper was to investigate an application of the generalized higher order iterative generalized stochastic perturbation technique in numerical simulation of 6061 aluminum alloy plastic behavior with structural initial microdefects. Gurson-Tvergaard-Needleman constitutive porous material model implemented in ABAQUS system has been employed for this purpose with its 2D triangular and quadrilateral elements. A volume fraction f_0 of the initial microdefects naturally appearing in the material has been adopted as the input random variable having symmetric but not necessary Gaussian distribution. A verification of its uncertainty influence on statistical scattering of material specimen deformations, stresses and void volume fractions has been carried out thanks to the common usage of the FEM system ABAQUS and computer algebra system MAPLE 2019. The 10th order Iterative Stochastic Perturbation Technique has been compared with Monte Carlo Simulations and Semi-Analytical Method to analyze uncertainty in extreme stresses and deformations within the tensioned material coupon.

Keywords: porosity, GTN-material model, stochastic finite element method, aluminium alloy.

1. INTRODUCION AND CONSTITUTIVE MODEL DESCRIPTION

Microstructural defects may play remarkable role during deformation of both homogeneous, heterogeneous and especially composite materials. According to both experimentation and engineering intuition they need to have uncertain nature concerning formation, nucleation, coalescence and growth, where these phenomena should have correlated character and be initiated at some uncertain level. Various constitutive theories were proposed to include such defects (Dormieux and Kondo, 2010) and one of the most popular was the Gurson model, whose improvement available in the literature as Gurson-Tvergaard-Needleman (GTN) model (Tvergaard and Needleman, 1984) is frequently applied in modern computer analysis. The yield condition of the modified Gurson model, which takes into account Tvergaard coefficients, has been given in the following way:

$$\left(\frac{\sigma_e}{\sigma_0}\right)^2 + 2q_1 f^* \cosh\left(\frac{3q_2\sigma_m}{2\sigma_0}\right) - \left(1 + q_3 f^{*2}\right) = 0 \tag{1}$$

where σ_e means the reduced stress according to Huber-Mises-Hencky hypothesis, σ_0 is yield stress, σ_m denotes hydrostatic stress here, q_i are constants called Tvergaard coefficients and f^* represents current voids volume fraction in the material calculated according to formula (Kossakowski 2012):

$$f^* = \begin{cases} f & \text{for} \quad f \le f_c \\ f_c + \dfrac{1/q_1 - f_c}{f_F - f_c}\left(f - f_c\right) & \text{for} \quad f_c < f \text{,} \end{cases} \tag{2}$$

where f_c means the critical voids volume fraction corresponding to onset of their coalescence and f_F means critical void volume fraction corresponding to the material failure. Influence of the number of voids existing in the al alloy material in the beginning of the research on resulting displacements and stresses is the man aim of this paper.

2. NUMERICAL MODEL

A numerical model for this tensile test has been created in ABAQUS 2017 software. Axisymmetric elements have been used. Total length of the specimen is 40 mm, radius of the top and bottom part is 5 *mm* and radius of the middle range is 3 *mm*. Two types FEM elements have been used for discretization. Aluminium specimen has been divided into two regions with qualitatively different meshing. Main part, where necking occurs, has been meshed by the 3-node linear axisymmetric triangle marked as CAX3 (2878 elements). The edge of each triangle is 0.05 *mm*. The remaining part has been discretized with the 4-node bilinear axisymmetric quadrilaterals with an application of the reduced integration (12360 hourglass control CAX4R finite elements). Size of the rectangular FEM elements is 0.1x0.5 *mm*. Structured meshing technique has been used. Extension of this material has been defined using kinematic forced displacement of the top and bottom side. A symmetry of the specimen is modelled using kinematic boundary conditions $u_1 = 0$ on the left vertical edge. To ensure the expected necking in the middle of the specimen a small cut-out has been modelled.

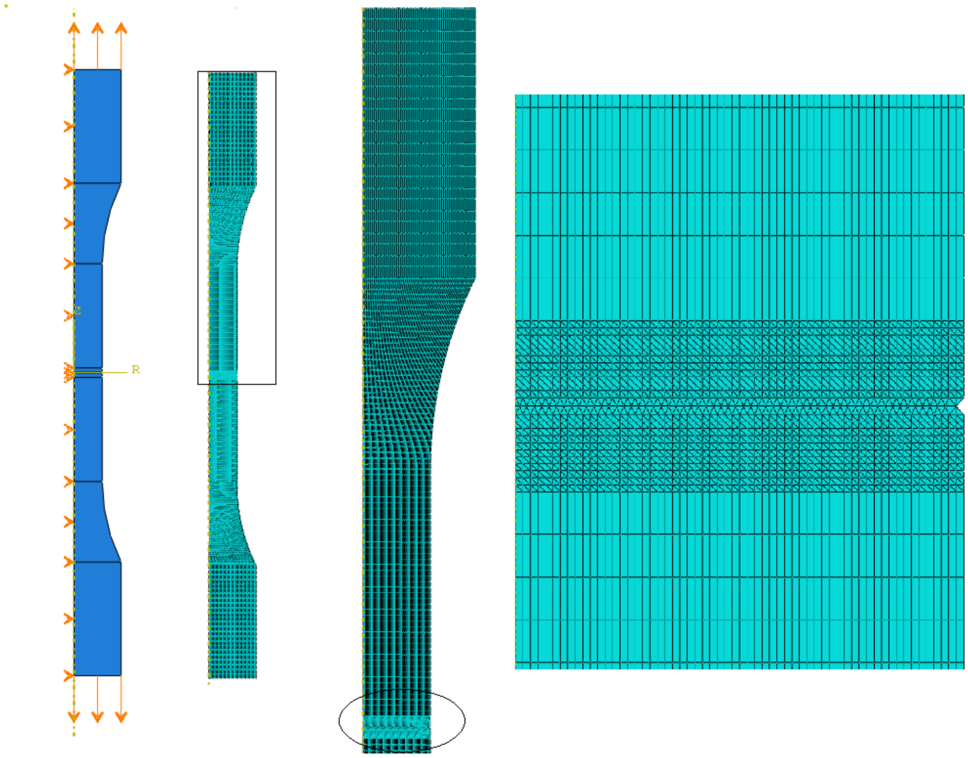

Fig. 1. Boundary conditions and discretization of the aluminum specimen.

Aluminium alloy Al 6061 has been used in this research. Its Young Modulus equals to 65.66 *GPa*, yield strength has been assumed as 296.9 *MPa* and Poisson ratio was taken as equal to 0.33. As it is known, the Hubert-Mises-Hencky hypothesis is insufficient for numerical simulation of the damage problems, so that more advanced constitutive model with additional parameters appeared to be necessary. Gurson-Tvergaard-Needleman material model has been implemented in ABAQUS system by assuming parameters, which describe its porosity – these have been displayed in Table 1 below. In this research material and geometrical nonlinearity has been assumed. Quasi-Newton incremental solution technique with large displacements allowed has been used. Initial increment has been assumed as 0.001, minimum one - as 0.00001 and maximum increment was set as equal to 1.0.

Table 4. GTN material model parameters

f_0	q_1	q_2	q_3	s_N	ε_N	f_N	f_c	f_F
0.00025	1.5	1	2.25	0.07	0.165	0.1285	0.03493	0.06294

3. UNCERTAINTY ANALYSIS

Initial void volume fractions f_0 existing in the material before tensile test has undoubtedly an uncertain nature. The volume fraction of voids existing in the material f_0 which is one of the most important parameters in this study has been selected as an input random variable having symmetric but not necessarily Gaussian distribution. Its dispersion has been predefined as 0.000÷0.005 and then discretized with the constant step equal to 0.0005. Based on (Yildiz & Yilmaz 2020) its mean value has been predefined as equal to f_0=0.00025. Three different probability distributions have been examined and contrasted in this study – uniform, triangular as well as Gaussian to see how PDF choice and input statistical scattering affects the final structural responses probabilistic characteristics. To investigate an influence of the initial porosity on the state functions like horizontal displacements, longitudinal stresses as well as Void Volume Fractions VVF eleven numerical FEM tests have been performed. The resulting values of this state functions have been received after each 5% progress up to its end. Further computational experiments concern stochastic analysis and they start with determination of polynomial bases represent local response functions and this is done for the extreme horizontal displacements and for longitudinal stresses localized in the necking area. These bases have been determined using statistically optimized unweighted Least Squares Method (Bjorck 1996) at each 5% of the incrementation process. Computer analysis has been conducted here using three concurrent probabilistic methods – semi-analytical approach, iterative stochastic perturbation method as well as the Monte-Carlo simulation method adopted as the reference technique here. The most interesting were the results obtained with the iterative perturbation scheme as this is the technique extremally reducing simulation time and effort. It is well known that the main idea of this approach is to expand all input random variables and all the resulting state functions in the given boundary initial problem via Taylor series about their expectations using some perturbation parameter ε. Higher order probabilistic moments and characteristics (coefficient of variation, skewness & kurtosis) are determined here analytically using the iterative scheme, where Taylor series expansions have been derived separately for uniform, triangular and Gaussian distributions (Kamiński 2015). Contrary to the linearized version of higher order perturbation methods (Kamiński 2013), this new methodology results in some negative contributions to the Taylor series expansions, which avoids any overestimation in the final higher order statistics.

4. FUNDAMENTAL RESULTS

Spatial distribution of the longitudinal stresses in the middle of the extended specimen is shown in Figure 2. The necking effect and stress concentration are traditionally expected at the half of height of the tensioned specimen and they have been compared for aluminum and steel structural elements.

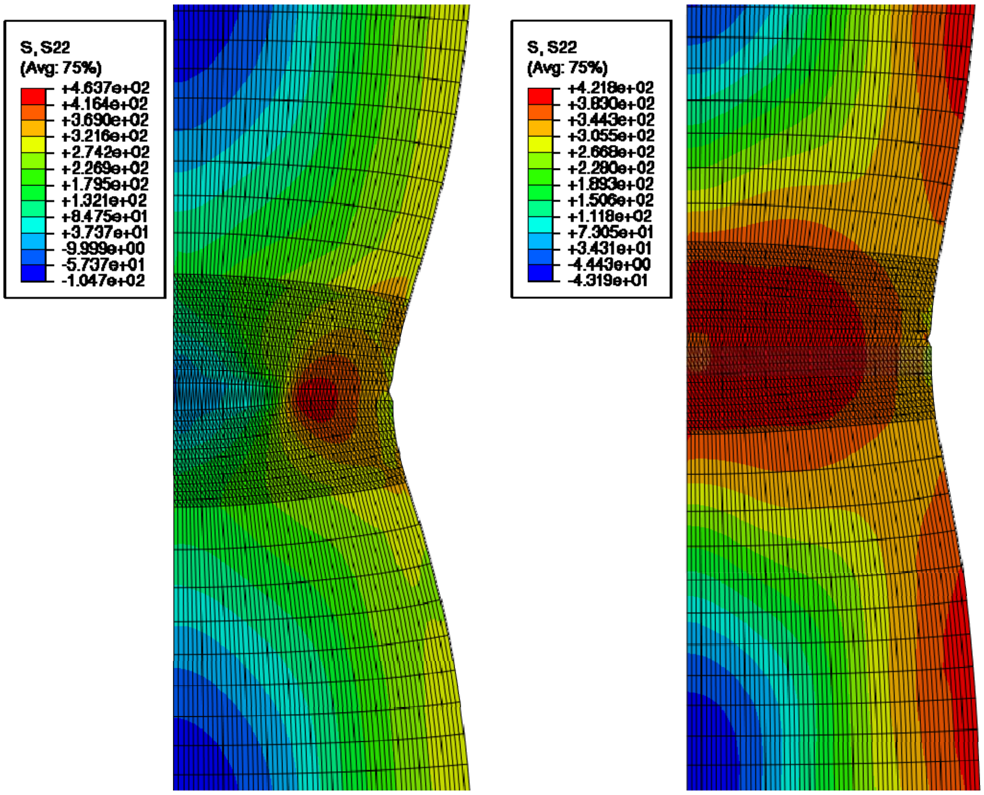

Fig. 2. Distribution of S_{22} stresses for f_0=0.001 - for the aluminum (left) and for the steel (right) coupons.

As one may noticed, the necking effect is deeper and larger in case of the aluminum alloy element, extreme stresses occur in the necking area only and they are localized into a very small region, which is in clear contradiction to the results obtained for structural steels. It is seen that the complex mesh composed with CAX3-CAX4R finite elements guarantees the very efficient and reliable FEM solution.

5. CONCLUSIONS

As it was expected from the series of results obtained for metal porosity analyses, the initial porosity plays remarkable role during the tensile tests for the aluminum alloys, while the extreme stresses localization is restricted to the necking area only. This conclusion seems to be very important concerning further stochastic analysis, where extreme structural responses traditionally are taken into account.

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