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# The Strength of a Cut–off Wheel Model Subjected to a Three Point Lateral Load Assessed by Different Methods

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In the paper the results of an assessment of a cut–off wheel model subjected to three–point loading with lateral forces in the form reduced stresses obtained by three methods: analytical, FEM and strain gauge ones have been presented. Recommendations on the applicability of the particular methods have been indicated.

*Keywords*: Cut–off wheel model, three point lateral load, strength, different methods

## 1. Introduction

Cut–off wheels are among the most frequently used tools. They allow very efficient cutting of metallic and non–metallic materials. Their high efficiency is due to high – compared with traditional grinding – grinding speeds, now exceeding even 100 m/s. In addition, abrasive cutting is often performed in objects or their elements positioned slantwise with respect to the plane of the cut–off wheel, which results in the load of the wheel with lateral forces. Owing to these difficult operating conditions, cut–off wheels are required to have an adequately high strength.

One of many investigation methods [1] is a method consisting in the static application of a three–point lateral load in the vicinity of the cut–off wheel circumference (Fig. 1). During this test the cut–off wheel does not rotate. This method is kinematically simple and the cost of a test–stand built for testing the method is low. Such test–stands can even be used by small or medium–sized cut–off wheel manufacturing plants. In a three–point loaded cut–off wheel stresses are formed, mainly of a tensile or compressive character, which can be determined by the following three methods: analytical: FEM and strain gauge ones.

## 2. A cut-off model and the diagram of its load

A cut–off wheel model was a thin circular plate subjected three–point lateral loads along the circumference: an applied force $F$ and two reactive forces $F/2$. It is presented schematically in Fig. 1, while the load of the section of the cut–off load is shown in Fig. 2.

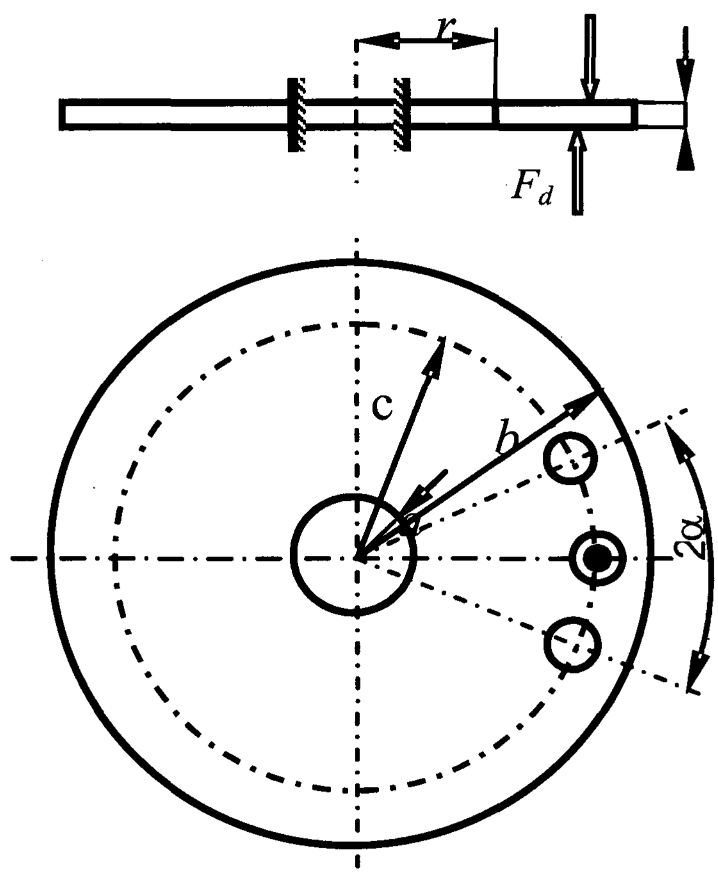


**Figure 1** Schematic diagram of the support and load of the cut-off wheel: $a$ – the radius of the cut–off wheel mounting hole, $b$ – the cut-off wheel outer radius, $c$ the radius of loading the system by the forces: $c = b - 18$ mm , $F$ – the lateral force applied to the cut–off wheel, $F_d$ – the reactive forces ($F_d = 0,5F$), $\alpha$ – the angle of mounting of the reactive forces $F_d(\alpha = 30^o)$

## 3. The analytical method of determination of stresses in the cut–off wheel model assumed

In the previous work [7], stresses caused by a single–point force in the cut–off wheel were analytically determined. To this end, the following assumptions were used: a differential equation of the deformed surface of the loaded circular plate, presented in [5], the proposition of a solution to these equations, given in [4] and the boundary conditions which also take into consideration the cut–off wheel fastening.

In the present paper, the solution from [7] for each of the three forces ($F$, $F/2$, $F/2$) and the principle of superposition with new additional boundary conditions

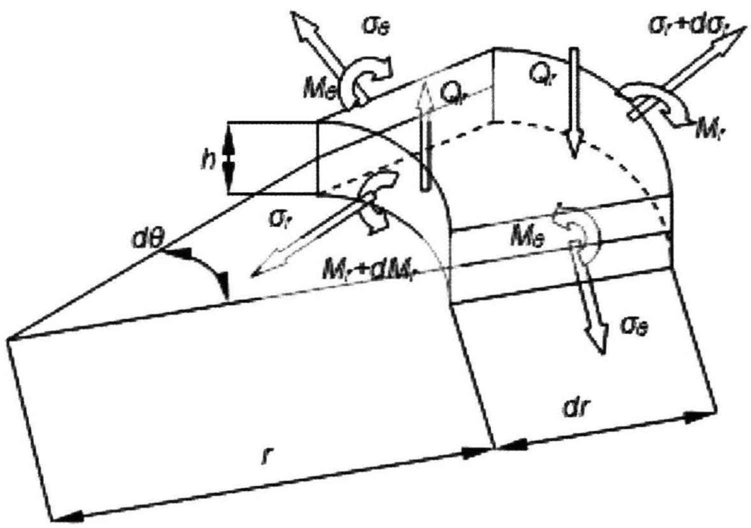


**Figure 2** Schematic diagram of the load of a section of the cut–off wheel for a single–point test

referring to the location of forces $F/2$ along the lateral direction have been used. The obtained mathematical relationships determining the stresses in the cut–off wheel are of a complex character and hence they are not presented herein. The software *Mathematica 7.0* was used for calculating the values of stresses. For previously determined [2] value of material properties of exemplary cut–off wheel *41-300x3,0x32 95A24RBF-100:* $\nu = 0,202$, $E = 29783$ MPa and force value $F = 100$ N, the curves of the stresses calculated are presented in Fig. 3. The calculation program, however, was not capable of calculating a continuous graph of the curve $\sigma_\theta(\theta)$ (Fig. 3b), in particular, at the place of occurrence of compressive stresses of the maximum absolute value, which were formed at the place of application of the force $F$. The program, however, allowed calculating their value: $\sigma_\theta(\theta = 0) = -44,5$ MPa. After the value of stresses $\sigma_r(\rho = 1) = -4,8$ MPa at this point were taken into account, the value of the maximum reduced stresses amounted to: $\sigma_{max} = 44,7$ MPa.

## 4. The determination of stresses in the cut–off wheel model by means of the FEM method

To determine stresses by the FEM method, the software *Ansys 12.1* with a typical division – for thin-walled circular plates – into fractional elements of the 2nd order of 5 degrees of freedom in each node. The stress curves obtained are presented in Fig. 4, while the visualization of stress distribution in the whole cut-off wheel is shown in Fig. 5. The value of the reduced stresses in this method is $\sigma_{max} = 42,3$ MPa.

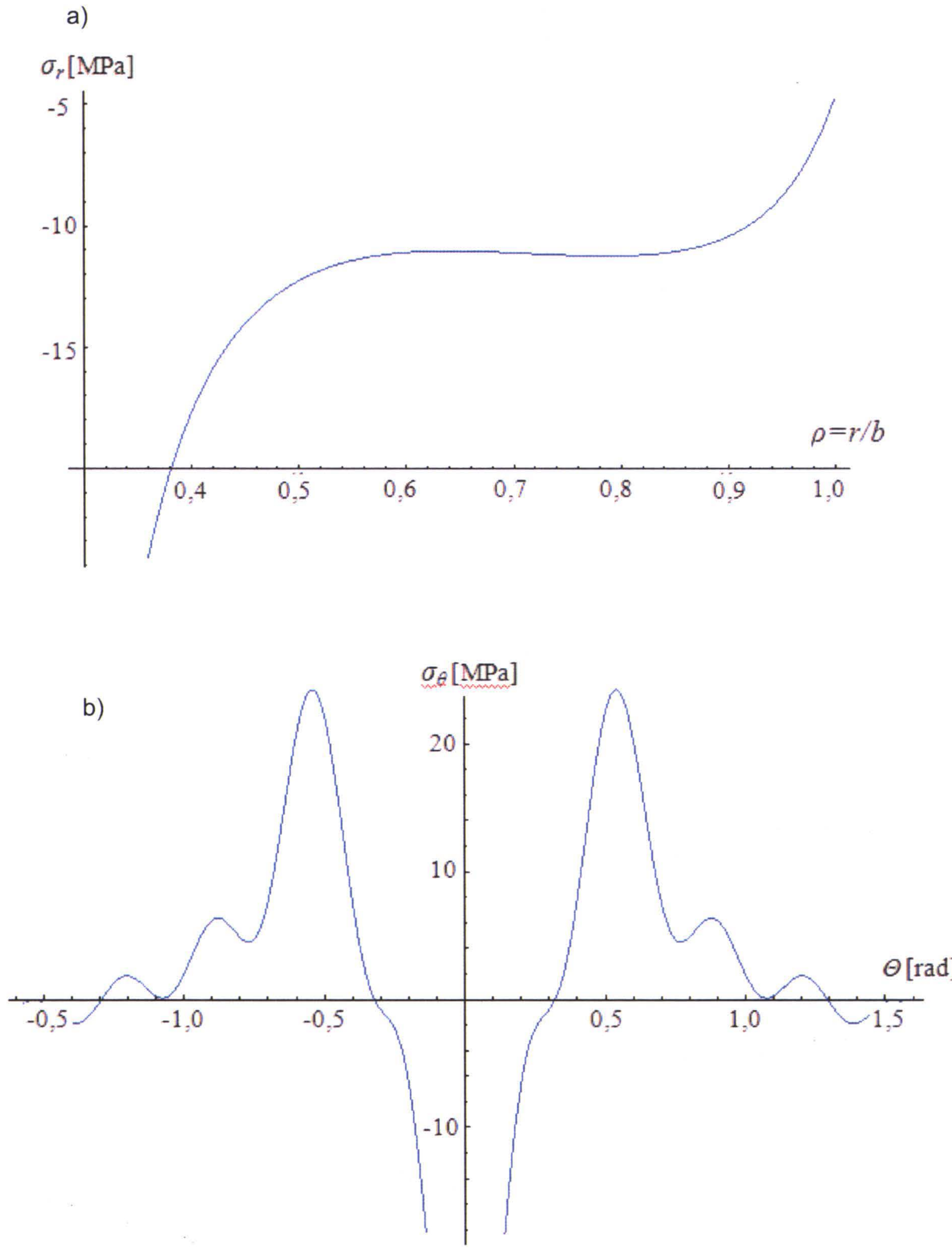


**Figure 3** Stress distribution in the cut–off wheel model determined by means of the software *Mathematica 7.0*: a) along the radius in the plane of operation of the force $F$ (radial component of stresses), b) along the circumference (circumferential component of stresses)

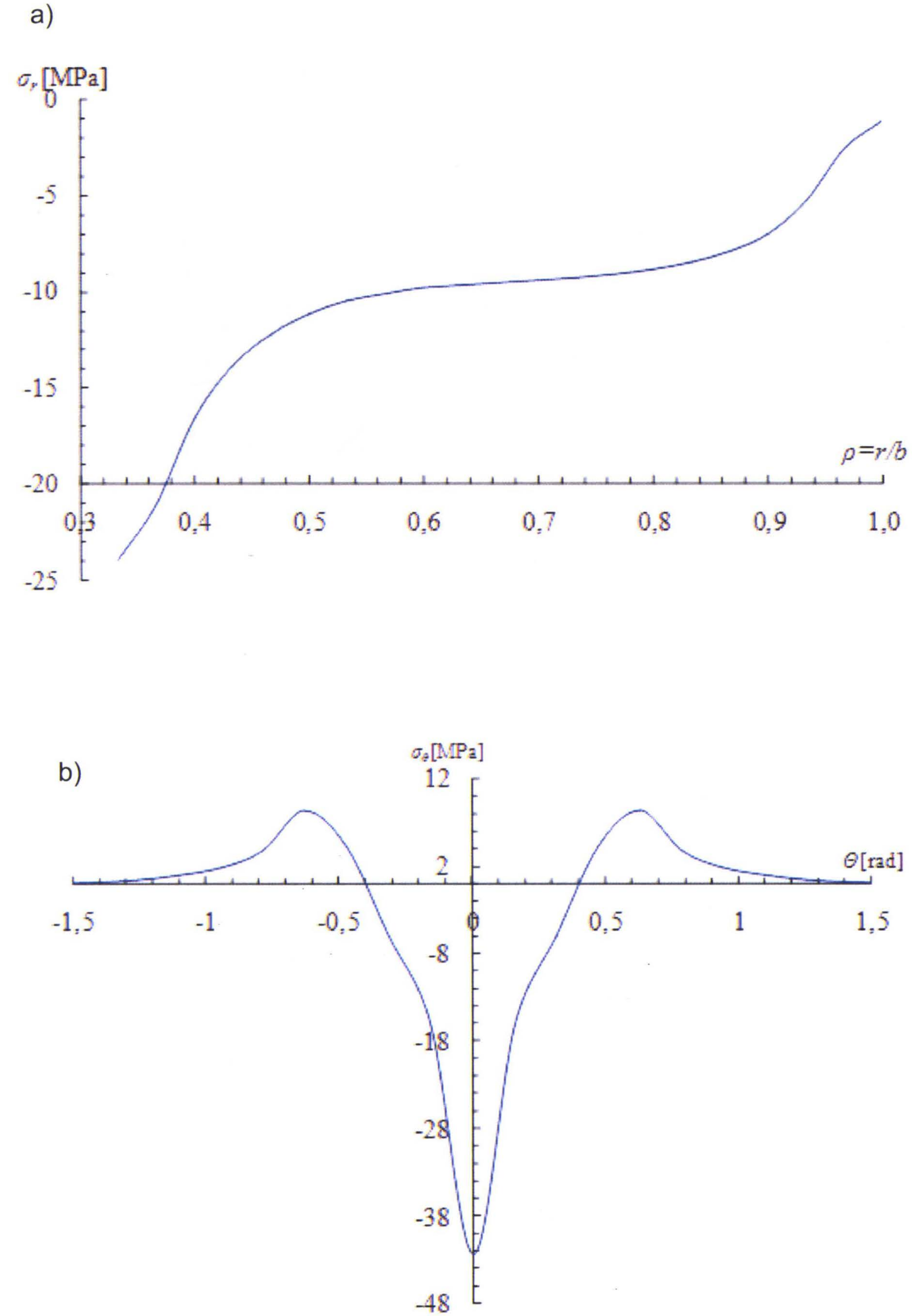


**Figure 4** Stress distribution in the cut–off wheel determined by means of the FEM method: a) along the radius in the plane of operation of the force $F$ (radial component of stresses), b) along half of the circumference (circumferential component of stresses)

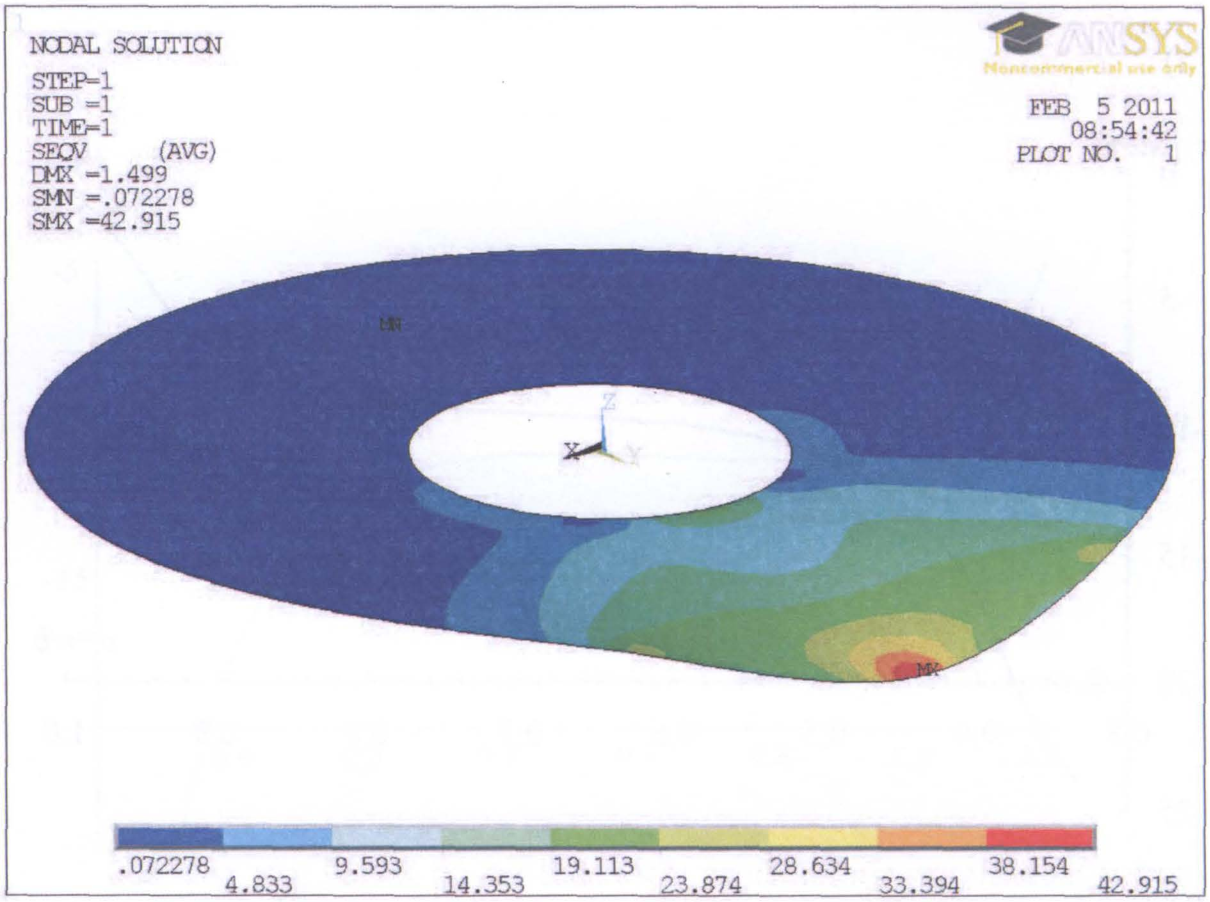


**Figure 5** Stress distribution in the cut–off wheel determined by means of the FEM method

## 5. The determination of stresses in the cut–off wheel by the strain gauge method

An experimental assessment of the strength of cut–off wheels was carried out by determining stress values by the strain gauge method on the test–stand presented above [2]. The measuring system consisted of: bi–axial strain gauge of the *TX-5-2x* type, an amplifier *Spider 8* controlled by the program *Catman Easy.* The strain gauge were situated at the place of occurrence of the maximum stresses. Three cut–off wheels of the same technical characteristics manufactured in the same production lot were investigated. The same lot was used to prepare specimens for determining Poisson's number and Young's modulus, necessary for determining stresses in all the methods considered. Each cut–off wheel was loaded in the vicinity of its circumference by the lateral force of a value $F = 100$ N. Using the reduced stress values read out from the measurement chain, mean values were calculated, which were: $\sigma_{max} = 45,4$ MPa.

## 6. The results obtained in the methods of assessment of the cut–off wheel model

The values of reduced stresses occurring in the cut–off wheel model obtained in the three assessment methods have been juxtaposed in Tab. 1.

**Table 1** Values of stresses $\sigma_{max}$ [MPa] in the cut–off wheel model and in the cut–off wheel obtained in the assessment methods under consideration

| Methods of assessment of strength of cut-off wheels | | |
|---|---|---|
| analytical | FEM | strain gauge |
| 44.7 | 42.4 | 43.4 |

## 7. Summary

The results obtained in the methods – considered in the paper – for the determination of stresses in circular plates treated as cut-off wheel models, are diversified only to a slight extent (the differences are below 7 %).

In the analytical method, it is questionable whether the trigonometric series discussed in [4] can be regarded as the solution to the equation of deformation of a circular–symmetric plate whose the task is to reflect the cut–off wheel model. The occurrence of a fragment of the stress curve $\sigma_\theta(\theta)$, similar to a sinusoid (the left and right part of the diagram in Fig. 3b) testifies to it, which is not confirmed by the stress curve obtained using the FEM method (Fig. 4b). This suggests that the results obtained by the analytical method can be burdened with a certain (difficult to determine) error. In addition, the analytical method is very labour-consuming despite the use of computer–aided calculation techniques.

The strain gauge method is equally labour–consuming since it requires special preparation (leveling) of the cut–off wheel before strain gauges can be glued to it, which can distort the stress values obtained.

The FEM method, making use of the program *ANSYS 12.1*, is the least labour–consuming of the three methods considered. This method can be regarded as the most reliable. In addition, this method has won recognition and is widely used for the determination of stresses in numerous constructions of different degrees of complexity.

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