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Parametric Vibrations of a System of Oscillators Connected with Periodically Variable Stiffness

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Abstract: The work is a part of a larger project concerning investigations of different configurations of connected oscillators with direct and parametric forcing sources. Among others, we present preliminary numerical studies of dynamics of a system composed of two oscillators connected with periodically variable stiffness. Each of the oscillators is connected to the fixed support with the use of a nonlinear magnetic spring. Based on the derived mathematical equations of the parametric system, a bifurcation plot was calculated for different shaft speeds. Bifurcation analysis shows both the ranges of periodic and chaotic motion. As an additional part, we will realize this kind of mechanical system from an electrical point of view, and we shall observe the phenomena experimentally.

Keywords: parametric oscillations, rectangular cross-section shaft, impact, bifurcation

1. Introduction

The dynamics of parametric systems are characterized by the existence of many stable and unstable regions. Examples of such systems where parametric vibrations occur are, for example, shafts with non-circular cross-section or with asymmetrical stiffness in different directions of vibration of such a shaft. A few nonlinear behaviors of these systems have been studied, and the theory is still under process to reach some maturity [1-4]. Our recent work presents a preliminary numerical investigation of a system composed of two oscillators connected with periodically variable stiffness (see Fig. 1a). Each of the oscillators is connected to the fixed support with the use of a nonlinear magnetic spring.

2. Results and Discussion

The system is described by the following differential equations: $m_1\ddot{x}_1 + F_{R1}(\dot{x}_1) + F_{S1}(x_1) + k_c(t)(x_1 - x_2) = 0$ and $m_2\ddot{x}_2 + F_{R2}(\dot{x}_2) + F_{S2}(x_2) + k_c(t)(x_2 - x_1) = 0$, where, $F_{Ri}(\dot{x}_i)=c_i\dot{x}_i + T_i\dot{x}_i(\dot{x}_i^2 + \varepsilon^2)^{-1/2}$ is model of resistance in the bearings and the term $(\dot{x}_i^2 + \varepsilon^2)^{-1/2}$ is smooth approximation of function $\operatorname{sign}(\dot{x}_i)$, $F_{Si}(x_i) = F_M(\delta - x_i) - F_M(x_i + \delta)$ is magnetic spring force, where $F_M(z) = F_{M0}(d_1 z + 1)^{-4}$, and $k_c(t) = \frac{k_\xi + k_\eta}{2} + \frac{k_\xi - k_\eta}{2}\cos(2\omega t)$ is the stiffness coupling of the oscillators that varies periodically. We are going to build an experimental rig which is a special configuration of the stand used in the work [5]. For the initial simulation tests, we have assumed the parameter values as: masses of the carts $m_1 = 8.0985$ kg and $m_2 = 6.7838$ kg, linear damping coefficients $c_1 = c_2 = 8$ Ns/m, dry friction forces $T_1 = T_2 = 1.9$ N, parameters of the magnetic spring model $F_{M0} = 360$ N and $d_1 = 50\ \mathrm{m}^{-1}$, shaft stiffness coefficients $k_\xi = 262.5$ N/m and $k_\eta =$

2916.67 N/m, the distance between the magnets in equilibrium position $\delta = 0.03$ m. Moreover, it is assumed that, $\varepsilon = 0.001$ s/m.
In Fig. 1b, we have shown the numerically obtained bifurcation diagram of the considered system with the angular frequency of parametric forcing ω, playing the role of a control parameter, keeping the rest parameters fixed. There, we can see that there is an interplay between the periodic and chaotic attractors just after the bifurcation point. The periodic attractors with different periodicities are the windows in between chaos.

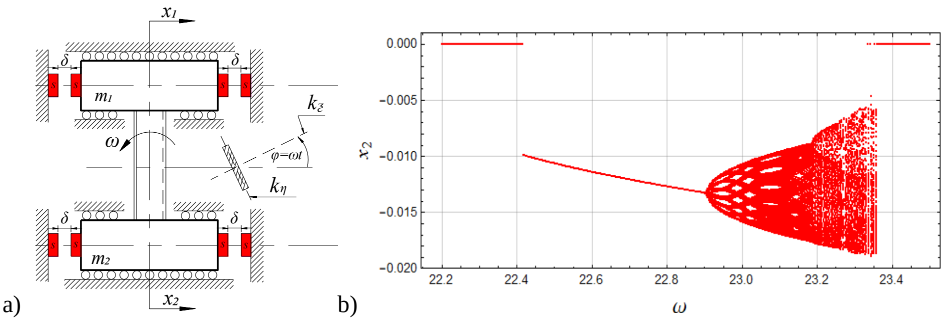

Fig. 1. Scheme of the investigated system (a) and bifurcation diagram (b)

3. Concluding Remarks

The presented investigations are a part of the larger project concerning parametric vibrations of the systems of connected mechanical oscillators with different kinds of excitations. Preliminary studies show that even the dynamics of the system composed of two oscillators connected to the fixed support with the nonlinear magnetic springs and the use of periodically variable linear stiffness exhibit complex bifurcation and chaotic dynamics. The hardening stiffness nonlinearity prevents the system from large oscillations in the vicinity of resonances and, instead of this, leads to complex bifurcation dynamics.
Additionally, we are going to construct the electrical analog of this mechanical system. The motivation is that such experiments are really inexpensive, and it is easy to vary the parameters in an electronic circuit. We shall simplify the equations of motion of the system and implement them in an electronic circuit.

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