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Vibration of the system with nonlinear springs connected in series

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Abstract: Solution of the problem and qualitative analysis of the forced vibration of the spring pendulum containing nonlinear springs connected in series is made in the paper. The method of multiple scales in time domain (MMS) has been employed in order to carry out the analytical computations. The MMS allows one, among others, to predict the resonances which can appear in the systems. The approximate solution of analytical form has been obtained for vibration at main resonance.

Keywords: asymptotic analysis, multiple scales, differential-algebraic system, springs in series

1. Introduction

Elastic elements arranged in various kinds of connections are widely applied in many mechanisms and mechatronic devices [1], [5]. The current analysis concerns a system with massless springs connected in series. The mathematical model of such a system includes differential and algebraic equations. Various kinds of similar problems were investigated by researchers mostly numerically. Asymptotic research of oscillators with two nonlinear springs connected in series is presented in papers [2-4].

2. Mathematical Model

The pendulum with two serially connected springs (Fig. 1), moves in the vertical plane. Z_1 and Z_2 stand for the total elongations of the springs whereas L_{0i} denotes the length of the ith non-stretched spring. The springs nonlinearity is of the cubic type, i.e. $F_i = k_i(Z_i + \Lambda_i Z_i^3)$ for $i = 1,2$, and the nonlinear contributions to the whole elastic force are assumed to be small. Moreover, there are two purely viscous dampers in the system. The system is loaded by the torque of magnitude $M(t) = M_0 \cos(\Omega_2\, t)$ and by the force **F** whose magnitude changes also harmonically i.e. $F(t) = F_0 \cos(\Omega_1\, t)$.

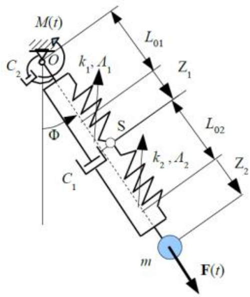

Fig. 1. Forced and damped spring pendulum with two nonlinear springs

Although the system has two degrees of freedom, its state is unambiguously determined by three time functions: the elongations Z_1 and Z_2 and the angle Φ.

Two equations of motion, obtained using the Lagrange formalism, and the equilibrium equation of the joint *S* govern the dynamic behaviour of the pendulum. They are as follows:

$$m\left(-g\cos(\Phi(t)) - L_0\dot{\Phi}(t)^2 + \ddot{Z}_2(t)\right) + C_1\dot{Z}_2(t) + Z_2(t)\left(k_2 - m\dot{\Phi}(t)^2\right) \\ - k_2\Lambda_2(Z_1(t) - Z_2(t))^3 - k_2 Z_1(t) = F_0\sin(\Omega_1 t), \tag{1}$$

$$\ddot{\Phi}(t)\left(mZ_2(t)\left(2L_0 + Z_2(t)\right) + L_0^2 m\right) + C_2\dot{\Phi}(t) + gm\left(L_0 + Z_2(t)\right)\sin(\Phi(t)) \\ + 2m\left(L_0 + Z_2(t)\right)\dot{\Phi}(t)\dot{Z}_2(t) = M_0\sin(\Omega_2 t), \tag{2}$$

$$k_1 Z_1(t)(\Lambda_1 Z_1(t)^2 + 1) = k_2\left(Z_2(t) - Z_1(t)\right)\left(\Lambda_2\left(Z_2(t) - Z_1(t)\right)^2 + 1\right), \tag{3}$$

where: $L_0 = L_{01} + L_{02}$.

Equations (1) – (3) are supplemented by the initial conditions of the following form

$$Z_2(0) = Z_0,\ \ \dot{Z}_2(0) = v_0,\ \ \ \Phi(0) = \Phi_0,\ \ \ \dot{\Phi}(0) = \omega_0, \tag{4}$$

where $Z_0, v_0, \Phi_0, \omega_0$ are known quantities.

The method of multiple scales in time domain with three time variables is applied to solve the considered problem. The method is appropriately modified due to the algebraic-differential character of the equations of motion. The approximate solution allows for the prediction of the resonance conditions. This allowed the equations of motion to be modified appropriately to describe the principal resonance. The solution to the resonant vibration problem has a semi-analytical form because the equations of modulation of the amplitudes and phases are solved numerically.

3. Concluding Remarks

The approximate solution to the governing equations, up to the third order, has been obtained using MMS with three time scales. The forced vibration of the pendulum has been analysed for two cases: far from resonance and in the resonance conditions. The analytical or semi-analytical form of the solution is the main advantage of the applied approach giving the possibility of the qualitative and quantitative study of the pendulum dynamics in a wide spectrum.

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