STABILITY OF CROSS-PLY COMPOSITE PLATE WITH PIEZOELECTRIC ACTUATORS

P. WŁUKA, T. KUBIAK
Department of Strength of Materials, Lodz University of Technology,
Stefanowskiego 1/15, 90-924 Lodz, Poland

In this paper a model of a cross-ply composite plate with piezoelectric actuators has been presented. The model has been built up by making use of the finite element method (FEM). Analysis of piezoelectric composite plate has concentrated on the effects of forces that are formed as a result of activation of piezoelectric layer. The forces cause changes in displacement, stresses, strains and deflection as well. Numerical calculations have provided insight into static nonlinear responses of the investigated structure under compression, thus enabling the evaluation of critical force.

1. INTRODUCTION

Applications of piezoelectric materials have a great deal of attention these days. The advantages of using active materials in mechanical structures are mainly due to their structural functioning. The active material functions as an embedded actuator that responds to electric loads and generates strains, deformations and forces. Moreover, they also function as an integrated part of the structural skeleton and contribute to the mechanical load carrying mechanism. This advantage is even more significant in the design and construction of composites with the piezoelectric materials. The theory of piezoelectricity was discussed by Yang [25]. Not to mention the fact that there have been number of PhD thesis concerning smart materials [3, 11, 12, 13]. Although very few problems in piezoelectric structures can be directly analysed by the three-dimensional theory, in order to obtain results useful for device applications. Usually numerical methods have to be used. As a consequence plenty of works published by scientists are focused on numerical analysis. The investigation of deflection control of plates with piezoelectric actuators has received a lot of attention [2, 7, 10, 15, 17, 20, 22,23]. Mostly the dynamic responses of the structures with piezoelectric are considered. Many of works have focused on the influence of the electrical actuation on the buckling and post-buckling behaviour of the active plate and investigated the ability to control and enhance the buckling load of the active structure [5, 16, 18, 19, 20, 21].

The present work investigated the stability of composite plate with piezoelectric actuators. The proposed method is an extension of model derived by Her Shiuh-Chuan and Lin Chi-Sheng [6]. The model consists of two piezoelectric actuators symmetrically surface bonded on a cross-ply simply supported composite plate subjected to compression. The effects of voltage applied to the piezoelectric actuators, initial imperfections and different numerical conditions on the stability of the composite plate are presented. The feasibility of controlling the stability of the plate is illustrated by application of appropriate voltage to the piezoelectric actuators.

2. SOLUTION METHOD

Stability analysis of composite plates with piezoelectric actuators has been carried out using finite element method. Piezoelectric FEM equations can be written in term of nodal displacement {u} and nodal electrical potential {V} The mechanical efforts are expressed by {F} and the nodal electric loads {L}. The elemental equilibrium equation in matrix form for coupled mechanic and electric field is as follows [1]:

$$\begin{bmatrix} [\mathrm{K}] & [\mathrm{K}^{\mathrm{z}}] \\ [\mathrm{K}^{\mathrm{z}}]^{\mathrm{T}} & -[\mathrm{K}^{\mathrm{d}}] \end{bmatrix} \begin{Bmatrix} \{\mathrm{u}\} \\ \{\mathrm{V}\} \end{Bmatrix} = \begin{Bmatrix} \{\mathrm{F}\} \\ \{\mathrm{L}\} \end{Bmatrix}, \tag{1}$$

where:

$[\mathrm{K}] = \int_{\Omega} [\mathrm{B}]^{\mathrm{T}}[\mathrm{c}][\mathrm{B}]\mathrm{d}\Omega$ – mechanical stiffness matrix;

$[\mathrm{K}^{\mathrm{z}}] = \int_{\Omega} [\mathrm{B}]^{\mathrm{T}}[\mathrm{e}][\mathrm{B}]\mathrm{d}\Omega$ – piezoelectric coupling matrix;

$[\mathrm{K}^{\mathrm{d}}] = \int_{\Omega} [\mathrm{B}]^{\mathrm{T}}[\varepsilon][\mathrm{B}]\mathrm{d}\Omega$ – dielectric permittivity coefficient matrix;

{u} – vector of nodal displacements;
{V} – vector of nodal voltage;
{F} – vector of nodal forces;
{L} – vector of nodal, surface, and body charges;
[B] – strain displacement matrix (geometrical matrix);
[c] – elasticity matrix,

In linear piezoelectricity the equations of elasticity (constitutive equations of piezoelectricity) are coupled to the charge equation of electrostatics by means of piezoelectric constants:

$$\begin{Bmatrix} \{\sigma\} \\ \{D\} \end{Bmatrix} = \begin{bmatrix} [c] & [e] \\ [e]^T & -[\varepsilon] \end{bmatrix} \begin{Bmatrix} \{S\} \\ -\{E\} \end{Bmatrix}, \tag{2}$$

where:

$\{\sigma\} = \{\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{yz} \quad \sigma_{zx}\}^T$ – stress vector;

$\{\mathrm{D}\} = \{D_x \quad D_y \quad D_z\}^T$ – electric flux density vector;

$\{\mathrm{S}\} = \{\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \gamma_{xy} \quad \gamma_{yz} \quad \gamma_{zx}\}^T$ – strain vector;

$\{\mathrm{E}\} = \{E_x \quad E_y \quad E_z\}^T$ – electric field intensity vector.

The elasticity matrix for an orthotropic piezoelectric layer is:

$$[c] = \begin{bmatrix} \frac{(1-\nu_{23}\nu_{32})E_{11}}{\Delta} & \frac{(\nu_{12}+\nu_{13}\nu_{32})E_{22}}{\Delta} & \frac{(\nu_{13}+\nu_{12}\nu_{23})E_{33}}{\Delta} & 0 & 0 & 0 \\ \frac{(\nu_{12}+\nu_{13}\nu_{32})E_{22}}{\Delta} & \frac{(1-\nu_{31}\nu_{13})E_{22}}{\Delta} & \frac{(\nu_{23}+\nu_{21}\nu_{13})E_{33}}{\Delta} & 0 & 0 & 0 \\ \frac{(\nu_{13}+\nu_{12}\nu_{23})E_{33}}{\Delta} & \frac{(\nu_{23}+\nu_{21}\nu_{13})E_{33}}{\Delta} & \frac{(1-\nu_{12}\nu_{21})E_{33}}{\Delta} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix},$$

where: $\Delta = 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{12}\nu_{13}\nu_{32}$ and according to Betty-Maxwell the following relations are fulfilled:

$$\frac{\nu_{12}}{E_{11}} = \frac{\nu_{21}}{E_{22}}; \quad \frac{\nu_{23}}{E_{22}} = \frac{\nu_{32}}{E_{33}}; \quad \frac{\nu_{13}}{E_{11}} = \frac{\nu_{31}}{E_{33}};$$

$$[e] = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ - piezoelectric stress matrix;

$$[\varepsilon] = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$$ - dielectric matrix

3. FINITE ELEMENT MODEL OF THE PLATE

The analysed cross-ply composite plate is presented in Fig.1. Three dimensional solid elements SOLID 45 with eight nodes and three degree of freedom at each node are used for the composite part and solid elements SOLID 5 with 8 nodes and four DoF at each node for piezoelectric part of the structure. The composite material is carbon/epoxy with stacking sequence [0/90/90/0]. Material properties of the composite layer have been assumed as: E_{11}= 108 GPa, E_{22} =E_{33}= 10.3 GPa, G_{23}=4.02 GPa, G_{13}=G_{12}= 7.13 GPa, ν_{23}= ν_{13}= ν_{12}= 0.28. Material and electric properties of the piezoelectric actuators are: E_{11}= E_{22} =E_{33}= 63 GPa, G_{23}= G_{13}=G_{12}= 24.2 GPa, ν_{23}= ν_{13}= ν_{12}= 0.3, d_{13} = $1.9\cdot10^{-10}$ m/V; $\varepsilon_{11}/\varepsilon_0$ = $\varepsilon_{22}/\varepsilon_0$= $\varepsilon_{33}/\varepsilon_0$= 1780. Dimensions of the plate are: A=380 mm, B=300 mm, t_p=1.5876 mm. The piezoelectric actuators are bonded on the top and bottom surfaces of the composite plate, located in the centre of the plate with dimensions 6 x 4 mm, thickness of t_{pe}=0.15867 mm.

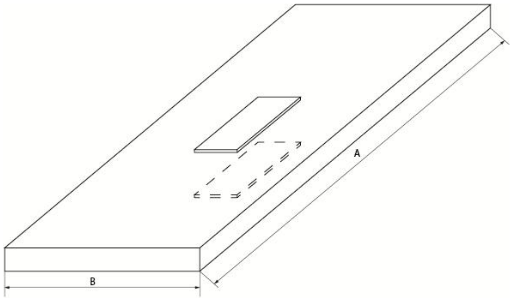

Fig. 1. Geometry of the plate

The plate is simply supported on each edge. Equal electric field is applied across thickness of the actuators, but with the opposite sign, resulting the deflections (upper layer is compressed and bottom layer is tensed) corresponding to bending.

4. RESULTS OF THE CALCULATIONS

The static nonlinear response of the analysed structure versus middle point deflection is presented in Fig. 2. The problem is solved using the Newton-Raphson method and initial imperfection of 0.1 of plate thickness (including piezoelectric actuators thickness) is applied. The force increment is 15 N in the first case of analysis. The results are compared with different actuations. These curves show the plate exhibits stiffening control behaviour with the variation of the applied voltage. For 15V the solution of the problem unconverged and further analysis was required. With the increase of number of subsets the solution converged and the results are depicted in Fig. 3.

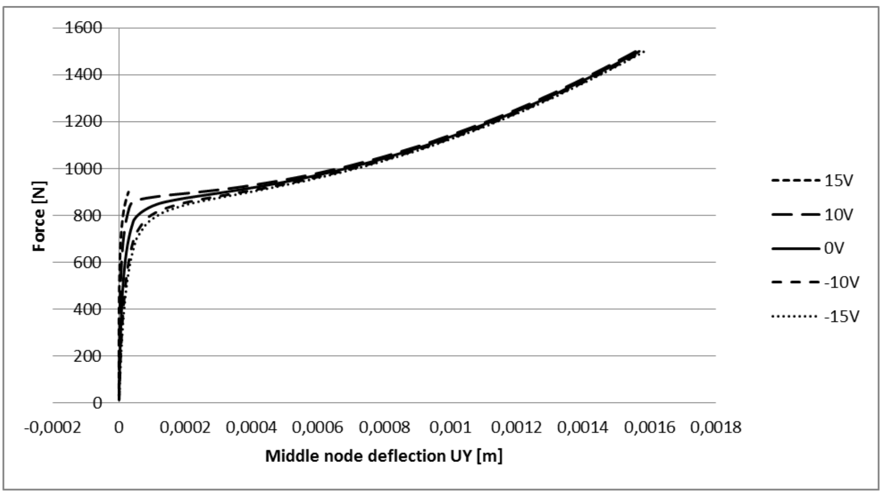

Fig. 2. Static response of plate with initial imperfection of 0.01 of thickness (number of subsets 100)

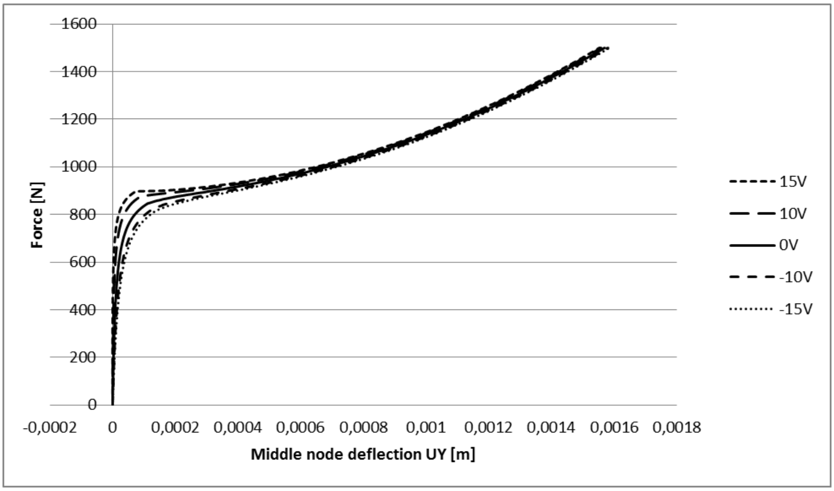

Fig. 3. Static response of plate with initial imperfection of 0.01 of thickness (number of subsets 500)

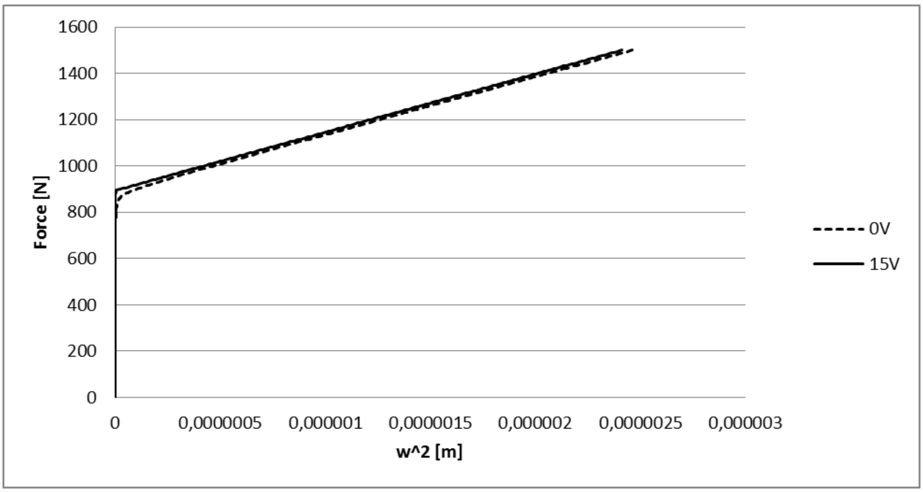

Fig.4. Identification of critical load with (Force-w^2) method

In order to identify the critical load load based on the results of experimental investigation the alternative (Force-w^2) [25] and inflection point (Force-w) [4] methods have been used. This alternative method is based on drawing a tangent to the imperfect curve at an arbitrary value of deflection beyond the small deflection range. The tangent line intersects Force-axis and corresponds to the critical load value. Identification of the inflection point is carried out by examination of the load-deflection curve. The results are depicted in Fig. 3 and Fig. 4. The aforementioned methods were used to evaluate critical loads and the results are listed in Table 1.

Table 1. Evaluation of critical load with different methods for plate with initial imperfection 0.01 of plate thickness

	Inflection method	Alternative method
Voltage [V]	Critical load [N]	Critical load [N]
15	919	894
10	829	872
0	817	865
-10	799	858
-15	790	855

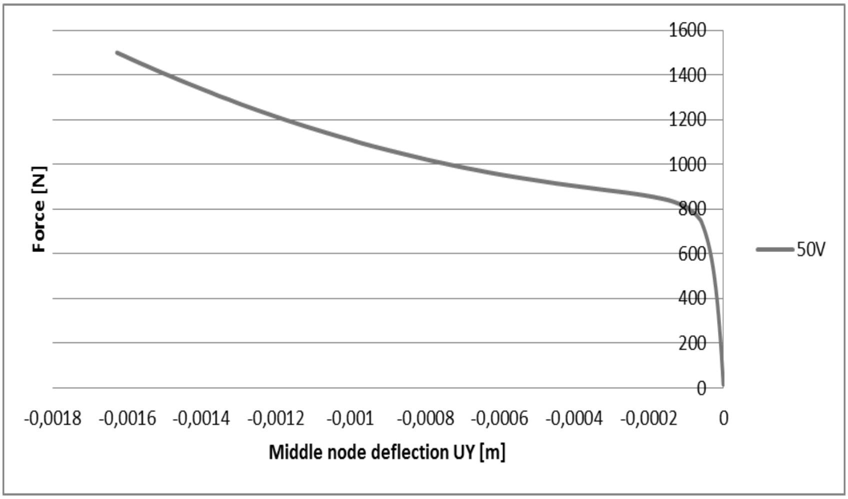

Fig. 5. Static response of plate for high value of voltage

Table 2. Comparison of critical loads for two different actuations

	Inflection method	Alternative method
Voltage [V]	Critical load [N]	Critical load [N]
50	903	864
15	919	894

Unwilling phenomenon has appeared for high value of voltage. Precisely speaking the position and geometry of the actuator at higher value of voltages changes plate initial imperfection and starts to lose its stability towards opposite direction (Fig.5). In such case actuation of the piezoelectric element firstly counteracts the effects of stress and tension but after the change of initial imperfection it advances the loss of stability. The results presented in Table 2 confirm this phenomenon.

In terms of numerical approach, the influence of initial imperfections has been analysed. Fig.6 and Fig. 7 show calculations for plate with initial imperfection of 0.1 of the plate thickness (excluding thickness of piezoelectric actuators). The analysis has been conducted for different voltage actuations.

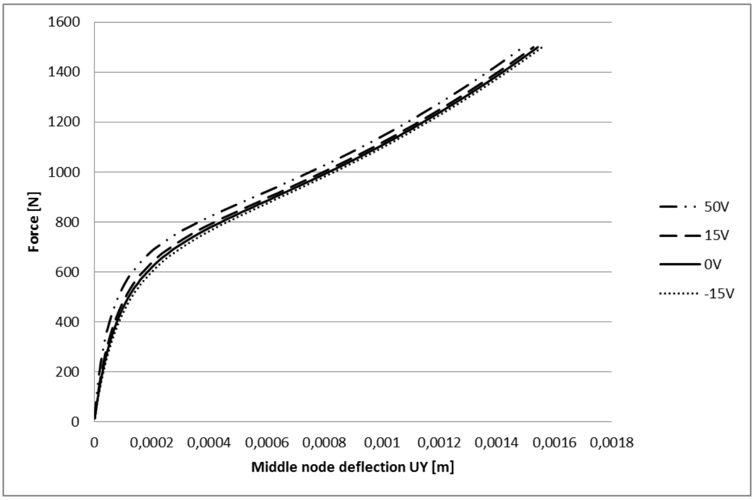

Fig. 6. Static response of plate with initial imperfection of 0.1 of plate thickness

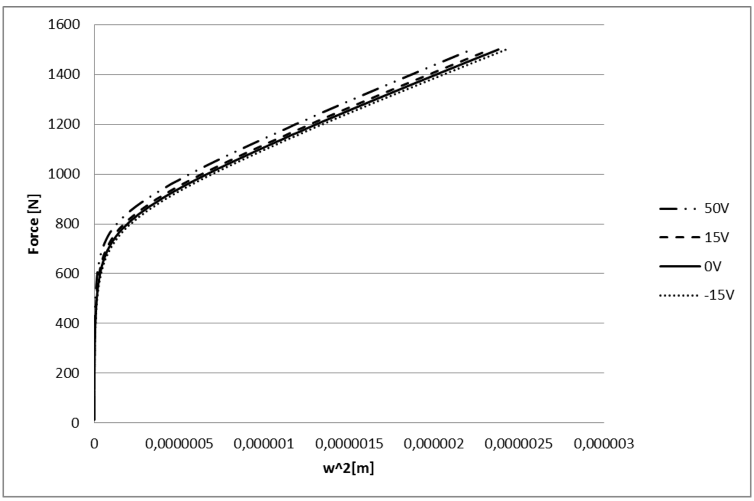

Fig.7. Identification of critical load with (Force-w^2) method

Buckling has been identified with the same alternative (Force-w^2) and inflection method (Force-w) as for the case where the plate had initial imperfection of 0.01 of its thickness and the results are shown in Table 3.

Table 3. Evaluation of critical load with different methods for plate with initial imperfection 0.1 of plate thickness

	Inflection method	Alternative method
Voltage [V]	Critical load [N]	Critical load [N]
50	873	824
15	839	807
0	826	800
-15	812	793

Additional results in terms of the static nonlinear response of the analysed structure versus middle point deflection are presented in Fig. 6. The curves reveal that different increment of the force applied in nonlinear analysis does not affect the results.

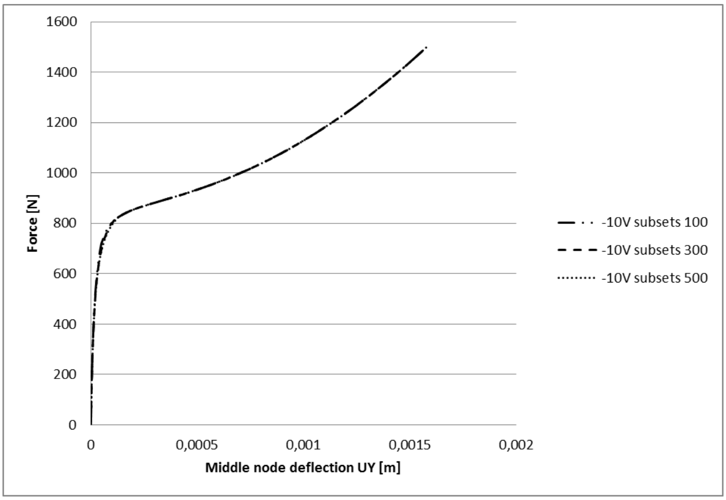

Fig. 7. Comparison of static responses of the plate for different number of subsets at constant value of voltage

5. CONCLUSIONS

In this work stability of cross-ply composite plate with piezoelectric actuators is studied. It is concluded that:

- The piezoelectric actuators induce bending moment acting on the composite plate by applying negative or positive voltages which may significantly enhance the buckling capabilities of structures.
- Appropriate voltage actuation increases the critical force in nonlinear buckling analysis.
- The choice of force increment and initial imperfections in nonlinear buckling analysis does not affect the results.

- The actuation of the piezoelectric elements may change the initial imperfection and the structure starts to buckle in an opposite direction

Further analysis should be carried out for checking the influence of distribution and size of piezoelectric actuators in controlled plate on the critical load.

ACKNOWLEDGEMENTS

This publication is a result of the research work carried out within the project subsidized over the years 2009-2012 from the state funds designated for scientific research (MNiSW - N N501 113636).

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