Dynamics simulations of a rigid body lying on a vibrating table with the use of special approximations of the resulting contact forces (MAT203-15)

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Abstract: The work presents simulations and dynamics of a rigid body lying on a vibrating table. An attempt of shaping and control the body dynamical behavior, by the use of manipulation of parameters of the table oscillations, is presented. This work is also an implementation of the specially prepared mathematical models of friction between table and the moving body. Those models are based on the integral model assuming the fully developed sliding on the plane contact area with the foundation of any pressure distribution. In order to simplify the calculations and reduce their computational cost, special approximations of the integral models of friction force and moment are used. They are based on Padé approximants and their generalizations.

1. Introduction

The examination of systems with friction forces resulting between contacting bodies is significant part of theoretical and applied mechanics. It has been investigated by scientists for years forming the separate branch of mechanics called tribology. Theoretical foundations analyzed in tribology focus mostly on stationary or periodical parts of motion. However, in mechanics one can encounter many examples, including the motion of bodies with friction, which shows that the transient stages between rolling and sliding can be a crucial part of analysis. The other problem in describing the system consisting of two contacting bodies with the dry friction forces is the normal pressure distribution over the contact area. The above mentioned difficulties can occur in daily life examples like the dynamics of billiard ball moving on the flat table, curling rocks, bowling ball, Celtic stone, rolling bearing and many others issues related to robotics. Many theoretical studies on such dynamical processes have been developed. In most cases they are based on simple static models of the static laws of Coulomb dry friction. Systems with Coulomb dry friction can behave interestingly in the instances of transition from sliding to rolling. However, the shape and size of the contact area may be the most important factor, which influence the global dynamics of the body. With the assumption that the flat base is deformed by the contacting body using the Hooke's law, MacMillan [1] described the normal pressure distribution as a linear function. He used the elliptic integrals to describe the friction forces and torque in the case of the circular contact area. Contensou in 1962 [2] has presented the paper in which he assumes fully developed sliding and the classical Coulomb law of frictions forces with the plane circular contact area. Applying the Hertzian contact distribution, Contensou showed the integral model of the resultant contact forces. Basing on the Contensou results, Zhuravlev [3] developed his theory and showed exact analytical functions, which defines the frictions forces and torque and he proposed special approximations on the basis of Padé approximations. Those models are much more suitable for the cases in which the relation between the sliding direction and the resultant friction components is considered. The dynamics of the motion of disk on a flat table was analyzed by Borisov et. al [8]. Kireenkov in [9] has proposed the model of rolling resistance, based on Padé approximations of resultant friction force and moment. He used the special contact pressure distribution on a circular contact area. In [4,5] authors presented the generalization of the models based on Padé approximants as the family of approximant models of friction forces. Application of the proposed models was considered in [6,7].

This work is the implementation of the proposed models of friction force and moment in the case of a rigid body lying on the vibrating table. By manipulation of the table oscillations' parameters we shape and control the behavior of the moving body. Proposed models assume the fully developed sliding on the plane contact area with any pressure distribution. To reduce the computational cost of the equations we propose the simplifications of the model as well.

2. Modeling of contact forces

This part of the paper is thoroughly described in work [5] and for the purpose of this study we present the most important parts, which are necessary to understand the theoretical foundations of the problem. We consider the dimensionless form of plane, circular contact area F, with the Cartesian coordinate system Axyz., where x and y axes lie in the contact plane. We define the dimensionless length as the quotient of the actual length and the radius of real contact $\hat{a}$. Point situated on the contact area F has the dimensionless coordinates $x = \hat{x}/\hat{a}$ and $y = \hat{y}/\hat{a}$. The following form of the non-dimensional contact pressure distribution is further assumed:

$$\sigma(x,y) = \hat{\sigma}(x,y)\frac{\hat{a}^2}{\hat{N}} = \frac{3}{2\pi}\sqrt{1-x^2-y^2}\left(1+d_c x + d_s y\right) , \tag{1}$$

where:

$$d_c = d\cos\gamma = d\,\frac{v_{rx}}{\sqrt{v_{rx}^2+v_{ry}^2}} \text{ and } d_s = d\sin\gamma = d\,\frac{v_{ry}}{\sqrt{v_{rx}^2+v_{ry}^2}} . \tag{2}$$

In above expressions (Eq.1-2) $\hat{\sigma}(x,y)$ is the real contact pressure, $\hat{N}$ - normal component of the real resultant force, *d* – rolling resistance parameter, γ - angle describing direction of rolling. The variables v_{rx} and v_{ry} are the components of the non-dimensional "rolling velocity" $\mathbf{v}_r = \hat{\mathbf{v}}_r / \hat{a} =$ $= v_{rx}\mathbf{e}_x + v_{ry}\mathbf{e}_y$ ($\hat{\mathbf{v}}_r$ is the corresponding real vector, $\mathbf{e}_i$ is unit vector of i axis). Proposed model of contact pressure distribution (Eq. 1) is the modification of Hertzian stress distribution, with the assumption that the center of pressure distribution does not coincide with the geometrical center A. Due to this assumption, we can implement the non-dimensional rolling resistance:

$$\mathbf{M}_r = \mathbf{f} \times \mathbf{e}_z = y_S\mathbf{e}_x - x_S\mathbf{e}_y = M_{rx}\mathbf{e}_x + M_{ry}\mathbf{e}_y, \tag{3}$$

where:

$$M_{rx} = \frac{1}{5}d_s, \; M_{ry} = -\frac{1}{5}d_c. \tag{4}$$

In above expression (Eq. 3) $\mathbf{f} = \overrightarrow{AS} = x_S\mathbf{e}_x + y_S\mathbf{e}_y$ is a vector denoting the position of center of non-dimensional pressure distribution S. The real counterpart of rolling resistance can be calculated as $\hat{\mathbf{M}}_r = \hat{a}\hat{N}\,\mathbf{M}_r$. The assumption of fully developed sliding on the contact area F is made. We consider that the deformations of moving bodies are small enough to describe the relative motion as a plane motion of rigid bodies. As the result the motion is described by the following dimensionless linear sliding velocity in centre A: $\mathbf{v}_s = \hat{\mathbf{v}}_s / \hat{a} = v_{sx}\mathbf{e}_x + v_{sy}\mathbf{e}_y$, and the angular sliding velocity: $\mathbf{v}_s = \hat{\mathbf{v}}_s / \hat{a} = v_{sx}\mathbf{e}_x + v_{sy}\mathbf{e}_y$, where $\hat{\mathbf{v}}_s$ and $\hat{\boldsymbol{\omega}}_s$ denote the corresponding real counterparts and $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$ are the unit vectors of the corresponding axes. We apply the Coulomb friction law on each element *dF* of the area F:

$$d\mathbf{T}_s = d\hat{\mathbf{T}}_s / (\mu\hat{N}) = -\sigma(x,y)\,dF\,\mathbf{v}_P / \|\mathbf{v}_P\| \;, \tag{5}$$

where $d\mathbf{T}_s$ and $d\hat{\mathbf{T}}_s$ are elementary non-dimensional friction force and its real counterpart respectively. $\mathbf{v}_P$ is the local dimensionless velocity of sliding and μ - dry friction coefficient.

The moment of the friction force $d\mathbf{T}_s$ about the centre *A* of the contact is described as follows:

$$d\mathbf{M}_s = \boldsymbol{\rho} \times d\mathbf{T}_s = d\hat{\mathbf{M}}_s / (\hat{a}\,\mu\hat{N}) \;, \tag{6}$$

where $d\hat{\mathbf{M}}_s$ is the corresponding real moment. Summing up the elementary friction forces $d\mathbf{T}$ and moments $d\mathbf{M}$ we get the total friction force **T** acting in the point *A* and moment **M**. The resulting integral expressions may possess the singularities in the case of lack of the relative motion. Therefore, we use a small numerical parameter ε_t to the corresponding integral expressions to avoid above mentioned singularities. Finally we obtain:

$$T_{sx} = \iint_F \sigma(x,y) \frac{(v_{sx} - \omega_s y)}{\sqrt{(v_{sx} - \omega_s y)^2 + (v_{sy} + \omega_s x)^2 + \varepsilon_t^2}} dxdy \ ,$$

$$T_{sy} = \iint_F \sigma(x,y) \frac{(v_{sy} + \omega_s x)}{\sqrt{(v_{sx} - \omega_s y)^2 + (v_{sy} + \omega_s x)^2 + \varepsilon_t^2}} dxdy \ , \tag{7}$$

$$M_s = \iint_F \sigma(x,y) \frac{\omega_s \left(x^2 + y^2\right) + v_{sy} x - v_{sx} y}{\sqrt{(v_{sx} - \omega_s y)^2 + (v_{sy} + \omega_s x)^2 + \varepsilon_t^2}} dxdy \ .$$

The real counterparts of the friction force and moment can be found as $\hat{\mathbf{T}}_s = \mu \hat{N} \mathbf{T}_s$ and $\hat{\mathbf{M}}_s = \hat{a} \mu \hat{N} \mathbf{M}_s$ respectively. The above mentioned expressions (Eq. 7) have the integrals over the contact area and for the numerical simulations they are very time consuming and may be inconvenient. Therefore, we propose the corresponding components of the integral model, basing on special modifications of Padé approximants:

$$T_{sx}^{(I)} = \frac{v_{sx} - b_T c_{0,1,1}^{(x,y)} \omega_s}{\sqrt{\left(|v_s|^{m_T} + b_T^{m_T} |\omega_s|^{m_T}\right)^{2m_T^{-1}} + \varepsilon_t^2}} \ ,$$

$$T_{sy}^{(I)} = \frac{v_{sy} + b_T c_{1,0,1}^{(x,y)} \omega_s}{\sqrt{\left(|v_s|^{m_T} + b_T^{m_T} |\omega_s|^{m_T}\right)^{2m_T^{-1}} + \varepsilon_t^2}} \ , \tag{8}$$

$$M_s^{(I)} = \frac{b_M c_{0,0,-1}^{(x,y)} \omega_s - c_{0,1,0}^{(x,y)} v_{sx} + c_{1,0,0}^{(x,y)} v_{sy}}{\sqrt{\left(b_M^{m_M} |\omega_s|^{m_M} + |v_s|^{m_M}\right)^{2m_M^{-1}} + \varepsilon_t^2}} \ ,$$

where:

$$c_{i,j,k}^{(x,y)} = \iint_F x^i y^j \left(x^2 + y^2\right)^{-\frac{k}{2}} \sigma\left(x, y\right) dxdy\,. \tag{9}$$

Full set of relations and expressions can be found in the work [5].

Using the proposed model of contact pressure distribution (Eq. 1) we get:

$$c_{0,1,1}^{(x,y)} = \frac{3}{32}\pi d_s\,,\quad c_{1,0,1}^{(x,y)} = \frac{3}{32}\pi d_c\,,\quad c_{0,0,-1}^{(x,y)} = \frac{3}{16}\pi\,,\quad c_{0,1,0}^{(x,y)} = \frac{1}{5}d_s\,,\quad c_{1,0,0}^{(x,y)} = \frac{1}{5}d_c\,. \tag{10}$$

Model presented in (Eq. 8) has the constant parameters: b_T , m_T , b_M and m_M , which can be found by the process of optimization of the approximate model to the integral components. Those parameters may be identified experimentally as well. For this work we propose the following objective functions:

$$F_T\left(b_T, m_T\right) = \int_D \left(\left(T_{sx} - T_{sx}^{(I)}\right)^2 + \left(T_{sy} - T_{sy}^{(I)}\right)^2\right) dD\,,$$

$$F_M\left(b_M, m_M\right) = \int_D \left(M_s - M_s^{(I)}\right)^2 dD\,, \tag{11}$$

where D is the representative area of model's kinematic parameters. Above expressions results in the following set of optimal parameters: $b_T = 0.771$, $m_T = 2.655$, $b_M = 0.419$ and $m_M = 3.073$.

Finally, using expressions (Eq. 10) and the simplified model (Eq.8) adding a small parameter to avoid singularities we get the following form of total friction force and moment:

$$T_{sx\varepsilon}^{(I)} = \frac{v_{sx} - \frac{3}{32}\pi b_T d_{s\varepsilon}\omega_s}{\sqrt{\left(\left|v_s\right|^{m_T} + b_T^{m_T}\left|\omega_s\right|^{m_T}\right)^{2m_T^{-1}} + \varepsilon_t^2}}\,,$$

$$T_{sy\varepsilon}^{(I)} = \frac{v_{sy} + \frac{3}{32}\pi b_T d_{c\varepsilon}\omega_s}{\sqrt{\left(\left|v_s\right|^{m_T} + b_T^{m_T}\left|\omega_s\right|^{m_T}\right)^{2m_T^{-1}} + \varepsilon_t^2}}\,, \tag{12}$$

$$M_{s\varepsilon}^{(I)} = \frac{\frac{3}{16}\pi b_M \omega_s - \frac{1}{5}d_{s\varepsilon}v_{sx} + \frac{1}{5}d_{c\varepsilon}v_{sy}}{\sqrt{\left(b_M^{m_M}\left|\omega_s\right|^{m_M} + \left|v_s\right|^{m_M}\right)^{2m_M^{-1}} + \varepsilon_t^2}}\,.$$

3. Modeling the rigid body on the vibration table

In this paper we propose the application of the friction model presented in (Eq.12) as the rigid body lying on the vibration table. Rigid body is assumed to be a ball (for example the billiard ball), situated on the table, which is allowed to make vibrations in x and y direction. Those vibrations have influence on the movement of the ball. We assume that the table can be rotated around x or y axis with angle α, which result in the external force $\mathbf{F}_g$ acting on the body. Model is presented in figure 1, where the following notation is used: $\mathbf{v}$ – velocity of the ball center O; $\hat{\mathbf{v}}_{\mathbf{r}}$ – linear rolling velocity; $\hat{\mathbf{v}}_{\mathbf{s}}$ - linear sliding velocity at the point A; $\hat{\boldsymbol{\omega}}_s$ – angular sliding velocity; $\hat{\mathbf{T}}_{s\varepsilon}$ – the resultant friction force acting at the contact center A; $\hat{\mathbf{N}} = \hat{N}\mathbf{e}_z$ - the normal reaction acting on the ball; $\hat{\mathbf{M}}_{s\varepsilon}$ – moment of friction forces; $\hat{\mathbf{M}}_{r\varepsilon}$ – moment of rolling resistance. It is assumed that a rigid ball rolls and slides over the vibration table, then: $\hat{\mathbf{v}}_{\mathbf{r}} = \mathbf{v}$.

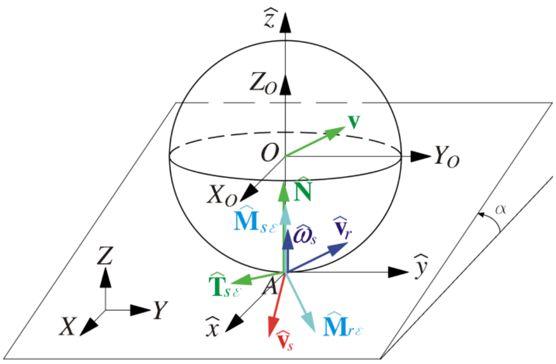

Figure 1. Rigid body and the vibration table contact area

The analyzed dynamical system has five degrees of freedom and is governed by the following set of differential equations:

$$m\frac{d\mathbf{v}}{dt} = \hat{\mathbf{T}}_{s\varepsilon} + \mathbf{F}_g, \quad \mathbf{B}\frac{d\boldsymbol{\omega}}{dt} = \mathbf{r}\times\hat{\mathbf{T}}_{s\varepsilon} + \hat{\mathbf{M}}_{s\varepsilon} + \hat{\mathbf{M}}_{r\varepsilon}, \tag{13}$$

where $\mathbf{r} = \overrightarrow{OA}$, m is the mass of the ball, $\mathbf{B}$ is a tensor of inertia in the mass center O. The matrix representation of the equation 13 can be presented as:

$$\mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}, \quad \boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} 0 \\ 0 \\ -r \end{bmatrix}, \quad \hat{\mathbf{T}}_{s\varepsilon} = \mu mg\cos(\alpha)\begin{bmatrix} T_{sx\varepsilon} \\ T_{sy\varepsilon} \\ 0 \end{bmatrix}, \quad \mathbf{F}_g = \begin{bmatrix} mg\sin(\alpha) \\ 0 \\ 0 \end{bmatrix},$$

$$\hat{\mathbf{M}}_{s\varepsilon} = \hat{a}\mu mg\cos(\alpha)\begin{bmatrix} 0 \\ 0 \\ M_{s\varepsilon} \end{bmatrix}, \quad \hat{\mathbf{M}}_{r\varepsilon} = \hat{a}mg\cos(\alpha)\begin{bmatrix} M_{rx\varepsilon} \\ M_{rx\varepsilon} \\ 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{bmatrix}, \tag{14}$$

with the assumption that ball is homogenous and $B=2/5mr^2$ is the central inertia moment of the body.

The dynamical system in the Cartesian coordinate system *Axyz* is represented by the two scalar differential equations for the linear motion:

$$\frac{dv_x}{dt} = -\mu g\cos(\alpha)T_{sx\varepsilon} + g\sin(\alpha), \ \frac{dv_y}{dt} = -\mu g\cos(\alpha)T_{sy\varepsilon}. \tag{15}$$

Since the permanent contact of the ball with the table is assumed, the third equation is $dv_z/dt = 0$. The angular motion of the billiard ball is governed by the following equations:

$$\frac{d\omega_x}{dt} = -\frac{5}{2}\frac{g\cos(\alpha)}{r}\left(\mu T_{sy\varepsilon} - \frac{1}{r}\hat{a}M_{rx\varepsilon}\right),$$

$$\frac{d\omega_y}{dt} = \frac{5}{2}\frac{g\cos(\alpha)}{r}\left(\mu T_{sx\varepsilon} + \frac{1}{r}\hat{a}M_{ry\varepsilon}\right), \ \frac{d\omega_z}{dt} = -\frac{5}{2}\frac{g\cos(\alpha)}{r^2}\hat{a}\mu M_{s\varepsilon}, \tag{16}$$

where: r – radius of the ball; v_x, v_y – the corresponding components of velocity of the ball center *O*; ω_x, ω_y, ω_z – components of angular velocity. In the global coordinate system XYZ presented in figure 2, X_O and Y_O denote the coordinates of the ball center O. To compute the absolute position of the ball center we use the following expressions:

$$\frac{dX_O}{dt} = v_x, \ \frac{dY_O}{dt} = v_y. \tag{17}$$

The following relations can be used for calculations of the arguments of the above presented models of friction, in the case of no vibrations of the table:

$$v_{sx} = \frac{v_x - r\omega_y}{\hat{a}}, \ v_{sy} = \frac{v_y + r\omega_x}{\hat{a}}, \ v_{rx} = \frac{v_x}{\hat{a}}, \ v_{ry} = \frac{v_y}{\hat{a}}. \tag{18}$$

Since, the rigid body is lying on the vibrating table we apply the vibrations to the dynamical system by changing the expressions (18). We add the vibration parameters to the sliding and rolling velocities of the ball as follows:

$$v_{sx} = \frac{v_x - r\omega_y + A_x\cos(\omega_a t)}{\hat{a}}, \quad v_{sy} = \frac{v_y + r\omega_x + \mathrm{A}_y\cos(\omega_b t + \varphi_y)}{\hat{a}},$$

$$v_{rx} = \frac{v_x + A_x \cos(\omega_a t)}{\hat{a}}, \quad v_{ry} = \frac{v_y + \mathrm{A}_y \cos(\omega_b t + \varphi_y)}{\hat{a}}. \tag{19}$$

where A_x, A_y are the amplitudes of the vibrations, ω_a, ω_b are the angular frequencies and φ_y is the phase of the oscillations in y direction.

The results of the simulations of the ball's motion are presented in figure 3. We use the model of friction forces (Eq. 12), changing the sliding velocity (Eq. 19) with the following parameters and initial conditions: $\hat{a} = 0.003\mathrm{m}$, $\mu = 0.2$, $d = 1$, $\varepsilon_t = 0.01$, $\varepsilon_r = 0.01$, $g = 9.81\mathrm{m/s}^2$, $r = 0.02m$, $m_T = 2.655$, $m_M = 3.073$, $b_T = 0.771,$, $b_M = 0.419$, $X_O(0) = 0$, $Y_O(0) = 0$, $\alpha_x(0) = 20$, $\alpha_y(0) = 0$, $\alpha_z(0) = 0$, $v_x(0) = 0$, $v_y(0) = 0.4$.For the simulations presented in figure 2 the following other parameters were used for the graph on the left: $A_x = 1$, $A_y = 0.5$, $\omega_a = 1000 rad/s$, $\omega_b = 500 rad/s$, $\varphi_y = 0$; and for the right graph: $A_x = 1000$, $A_y = 800$, $\omega_a = 1500 rad/s$, $\omega_b = 1200 rad/s$, $\varphi_y = 1$. Table was rotated around y axis resulting.

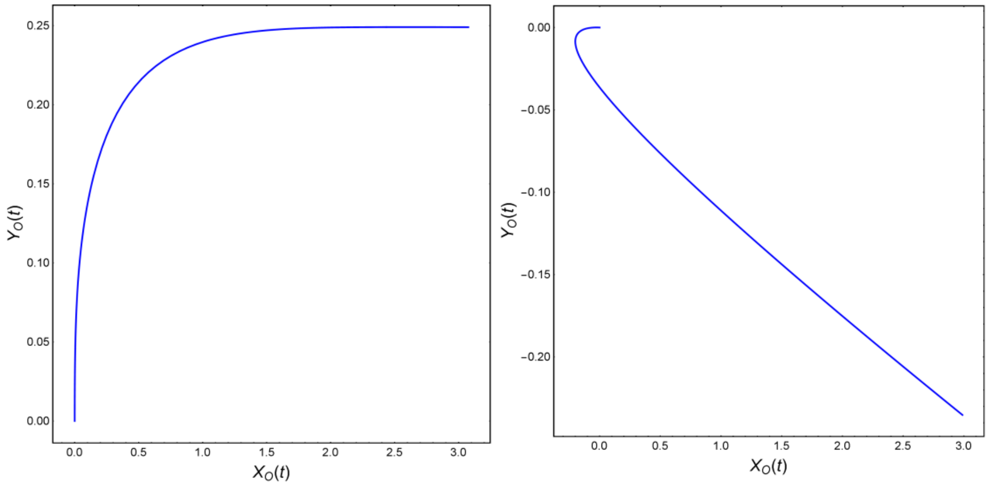

Figure 2. Simulations of rigid body's movement

For the simulations presented in figure 3 the following parameters were used for the top left graph: $A_x = 1$, $A_y = 1$, $\omega_a = 1000 rad/s$, $\omega_b = 500 rad/s$, $\varphi_y = 0$; for the top right graph: $A_x = 1$, $A_y = 3$, $\omega_a = 1200 rad/s$, $\omega_b = 1000 rad/s$, $\varphi_y = 1$; for the bottom left graph: $A_x = 1$, $A_y = 1$, $\omega_a = 2000 rad/s$, $\omega_b = 1000 rad/s$, $\varphi_y = 1$; finally for the bottom right graph: $A_x = 1$, $A_y = 1$, $\omega_a = 800 rad/s$, $\omega_b = 1500 rad/s$, $\varphi_y = 0.5$ and table was rotated around x axis.

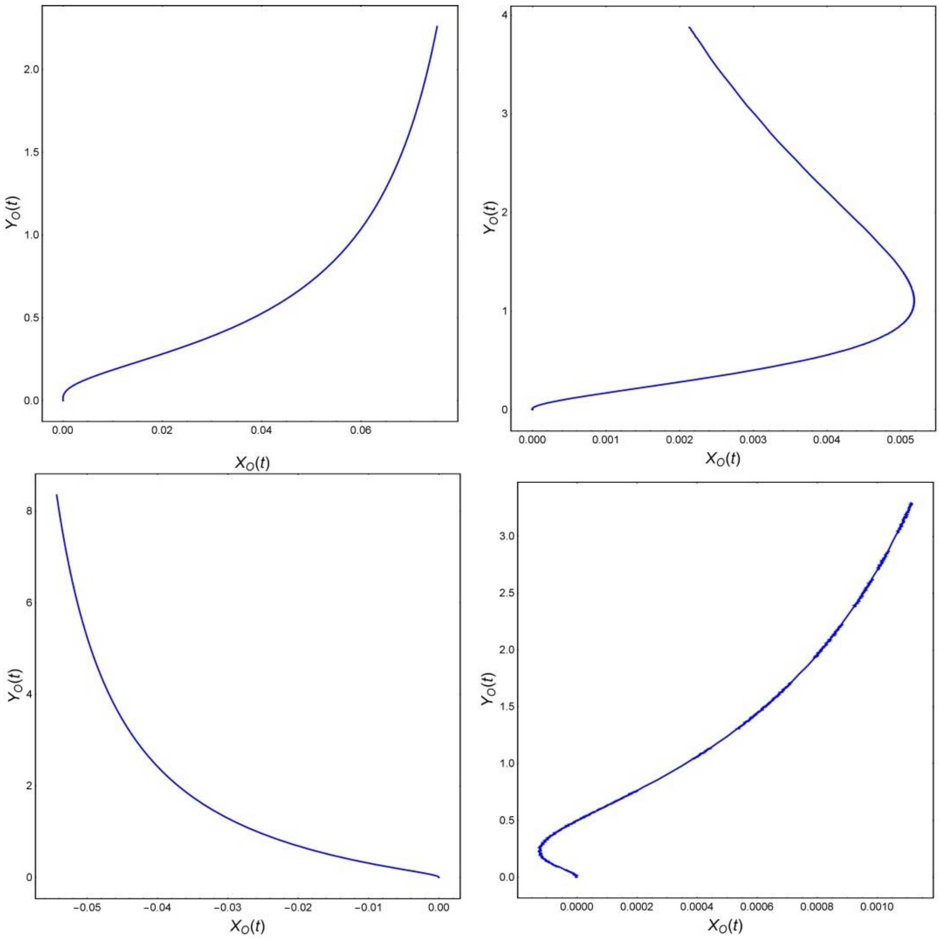

Figure 3. Simulations of rigid body's movement

4. Conclusions

This work presents the possible implementation of the model of the resultant contact forces. Due to the complexity of computation of the integrals over the contact area, the authors proposed a model for the numerical simulations, which is based on Padè approximant. Possible application of the model is a dynamical system consisting of a rigid body (billiard's ball) lying on a vibrating table. By manipulating the oscillations of the vibrating table, different ball's movements have been achieved. Results presented in this paper may be a good starting point for the verification of the presented model not only by numerical simulations, but with the real object as well. Developing such a dynamical system and its technical implementation may be the authors' object of interest in future.

Acknowledgments

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