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ELECTROOPTIC MODULATION OF LIGHT BEAMS DEVIATED FAR FROM NORMAL INCIDENCE

It is shown that the electrooptic modulation of the phase difference for the light beams propagating oblique through the crystalline plane-parallel plate appears not only as the direct result of the field-induced changes in the refractive indices of refracted waves but also the modulation is due to the changes in the directions and paths lengths of the waves in the crystal. This indirect modulation may become important for light beams deviated significantly from the normal to the entrance and emergence crystal faces. Configurations with the light beams passing oblique through the plane normal to the optic axis are interesting in measurements of the linear electrooptic coefficients r_{41} and r_{42} in uniaxial crystals. Although in some crystals symmetries the coefficients r_{11} or r_{22} may contribute to phase difference, it is shown that the results obtained for different signs of the incidence angle allow to separate such contribution from that caused by r_{41} or r_{42}.

Keywords: linear electrooptic effect, double refraction, linear birefringence.

1. INTRODUCTION

Traditionally electrooptic measurements are performed for the light beams propagating normal to the planes of a crystalline plate cut in the form of right parallelepiped. The normal incidence is very convenient to perform measurements of electrooptic coefficients and allows to simplify mathematical relationships between the electrooptic coefficients and phase difference in the crystal. However, for some particular electrooptic coefficients it is worth to consider another strategy of measurements. For example, the linear electrooptic coefficients r_{41} and r_{42} and corresponding r_{52} and r_{51} coefficients are nonzero in some uniaxial crystals of 3, 4, $\bar{4}$, 6 symmetry [1]. Moreover for some other symmetries, i.e. 32, 3*m*, 422, 4*mm*, $\bar{4}2m$, 622, and 6*mm*, r_{41} or r_{42} is nonzero. When the light propagates along the optic axis or in any direction perpendicular to the axis these coefficients give no contribution to the phase modulation of the fast and slow waves in the crystal. Thus, if we need to measure these coefficients for traditional normal light incidence the crystal must be cut in a certain (σ_X, 0, σ_Z) or (0, σ_Y, σ_Z) direction. Such crystal samples are inconvenient to determine electrooptic coefficients other than r_{41} and r_{42}. In this paper we show that the coefficients r_{41} and r_{42} may be also measured in crystal samples with the pair of faces normal to the optic axis. However, the incident light beam must be deviated far from the normal to the entrance face. For such configurations the electrooptic modulation of the phase difference appears not only as the direct result of the field-induced changes in the refractive indices of the refracted waves (as it is in the case of normal incidence), but also the changes in the directions and the geometrical paths lengths of the waves in the crystal should be taken into account. So far, the contribution of this indirect mechanism to the total phase modulation was hard to estimate. Although equations describing the double refraction are known for any homogeneous birefringent medium, the analytical solutions in uninvolved form has been reported only for uniaxial media. As the field-free uniaxial crystal typically becomes biaxial in the external electric field [2] (except for some special field directions) the necessary calculations was difficult. Recently, we proposed a new approximated solution of the double refraction problem, in which the solution found for field-free uniaxial crystal is treated as the first approximation, and the perturbations due to the linear electrooptic effect are found analytically in the second step [3]. In this paper we applied the formulas derived in Ref. [3] to found the phase difference appearing in the plane-parallel crystalline plate for the directions far from the normal to the entrance face.

2. ELECTROOPTIC PHASE MODULATION IN THE UNIAXIAL CRYSTAL FOR THE OBLIQUE LIGHT INCIDENCE

The matrix **r** of the linear electrooptic effect for the 3 symmetry has the most complicated form, therefore we concentrate on this particular symmetry [1]

$$\mathbf{r} = \begin{bmatrix} r_{11} & -r_{22} & r_{13} \\ -r_{11} & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ -r_{42} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{bmatrix}. \tag{1}$$

The results obtained for this symmetry may be, easily adapted for other crystals belonging to tetragonal, trigonal, and hexagonal systems.

We consider the crystal sample cut in the form of right parallelepiped with the entrance and exit faces normal to the optic axis +*Z*. In our approach we assume that the optical activity is small enough to be neglected. Firstly we consider the configuration

$$\mathbf{E} = (E, 0, 0); \quad \boldsymbol{\sigma} = (0, \sigma_Y, \sigma_Z), \tag{2}$$

where the unit vector $\boldsymbol{\sigma}$ relates to the incident light direction, while the two refracted waves are described by theirs velocity vectors $\mathbf{u_o}$ and $\mathbf{u_e}$. The angle of incidence relative to the normal to the crystal face are denoted here as φ and its sign is determined by right-handed screw rule (in ref. [3] the incidence angle is always non-negative $\alpha = |\varphi|$ but the formulas depend on the signs of the σ_i components). For this configuration the formulas derived in paper [3] and transformed to the crystallographic coordinates *XYZ* lead to the following components of the velocity vector of the ordinary wave in the crystal perturbed by the electric field

$$u_{oX} = 0, \tag{3}$$

$$u_{oY} = -\frac{c^2 \sin\varphi}{n_o^2 v_{iso}} - \frac{c^2 r_{11} \sin\varphi}{v_{iso}} E, \tag{4}$$

$$u_{oZ} = \frac{c\sqrt{n_o^2 v_{iso}^2 - c^2 \sin^2\varphi}}{n_o^2 v_{iso}} + \frac{c\left(n_o^2 v_{iso}^2 - 2c^2 \sin^2\varphi\right)}{2 v_{iso}\sqrt{n_o^2 v_{iso}^2 - c^2 \sin^2\varphi}} r_{11} E. \tag{5}$$

Analogously, for the perturbed extraordinary wave

$$u_{eX} = 0, \tag{6}$$

$$u_{eY} = -\frac{n_e^2 c^2 v_{iso} \sin\varphi}{n_o^2 n_e^2 v_{iso}^2 - \left(n_o^2 - n_e^2\right)c^2 \sin^2\varphi} + \\ + \frac{n_o^4 n_e^2 c^2 v_{iso}\left(n_e^2 v_{iso}^2 - c^2 \sin^2\varphi\right)\sin(\varphi)}{\left[n_o^2 n_e^2 v_{iso}^2 - \left(n_o^2 - n_e^2\right)c^2 \sin^2\varphi\right]^2} r_{11} E +$$

$$-\frac{2n_{\rm o}^3 n_{\rm e}^3 c^3 v_{\rm iso}\sqrt{n_{\rm e}^2 v_{\rm iso}^2 - c^2\sin^2\varphi}\,\sin^2(\varphi)}{\left[n_{\rm o}^2 n_{\rm e}^2 v_{\rm iso}^2 - \left(n_{\rm o}^2 - n_{\rm e}^2\right)c^2\sin^2\varphi\right]^2} r_{41}E\,, \tag{7}$$

$$\begin{aligned} u_{\rm eZ} = {} & \frac{n_{\rm o} n_{\rm e} c v_{\rm iso}\sqrt{n_{\rm e}^2 v_{\rm iso}^2 - c^2\sin^2\varphi}}{n_{\rm o}^2 n_{\rm e}^2 v_{\rm iso}^2 - \left(n_{\rm o}^2 - n_{\rm e}^2\right)c^2\sin^2\varphi} + \\ & + \frac{\left[n_{\rm o}^2 n_{\rm e}^2 v_{\rm iso}^2 - \left(n_{\rm o}^2 + n_{\rm e}^2\right)c^2\sin^2\varphi\right]v_{\rm iso}}{\left[n_{\rm o}^2 n_{\rm e}^2 v_{\rm iso}^2 - \left(n_{\rm o}^2 - n_{\rm e}^2\right)c^2\sin^2\varphi\right]^2} \times \\ & \times\left[-\frac{1}{2} n_{\rm o}^3 n_{\rm e} c\sqrt{n_{\rm e}^2 v_{\rm iso}^2 - c^2\sin^2\varphi}\, r_{11} + n_{\rm o}^2 n_{\rm e}^2 c^2 \sin(\varphi)\, r_{41}\right]E\,. \end{aligned} \tag{8}$$

Here c and $v_{\rm iso}$ are the velocities of light in vacuum and isotropic medium around the crystal, respectively, and $n_{\rm o}$ and $n_{\rm e}$ are the field-free ordinary and extraordinary principal refractive indices, respectively. Generally the $u_{{\rm o}i}$ and $u_{{\rm e}i}$ components are nonlinear functions of the optical permeability tensor ε_{ij} and the forms of their expansions into power series differ essentially depending on the fact if the natural birefringence exists or not. The above results are valid only for the refracted waves deviated sufficiently far from the optic axis, that the natural birefringence becomes high in comparison to that field-induced. Thus, substitution of $\varphi = 0$ or a certain very small value leads to a wrong result.

The results in Eqs. (3)-(8) show, that beside the modules of the vectors $\mathbf{u_o}$ and $\mathbf{u_e}$, the directions of waves are also modulated by the applied field and the two directions are modulated in a different way. The phase difference Γ between the slow and fast waves is given by

$$\Gamma = \frac{2\pi}{\lambda}\left(n_{\rm s} l_{\rm s} - n_{\rm f} l_{\rm f}\right) = {\rm sgn}(n_{\rm o}^2 - n_{\rm e}^2)\frac{2\pi}{\lambda}\left(N_{\rm o} l_{\rm o} - N_{\rm e} l_{\rm e}\right), \tag{9}$$

where λ is the wavelength of the light; $l_{\rm s}$, $l_{\rm f}$, $l_{\rm o}$ and $l_{\rm e}$ are the geometrical paths lengths in the crystal traveled by the slow, fast, ordinary, and extraordinary waves, respectively; $n_{\rm s}$ and $n_{\rm f}$ are the refractive indices of the slow and fast waves, respectively; and $N_{\rm o}$ and $N_{\rm e}$ are the refractive indices of the field-perturbed ordinary and extraordinary waves propagating in the directions of the $\mathbf{u_o}$ and $\mathbf{u_e}$ vectors, respectively. To replace the slow and fast waves in Eq. (9) by the ordinary and extraordinary waves we noticed that for $n_{\rm o}^2 < n_{\rm e}^2$ the slow and fast waves are the ordinary and extraordinary waves, respectively, while for $n_{\rm o}^2 > n_{\rm e}^2$ this relationship is swapped. Utilizing the definition of the refractive index $N = c/u$ and some elementary geometrical dependencies, Eq. (9) can be transformed to the form

$$\Gamma = \mathrm{sgn}(n_{\mathrm{o}}^2 - n_{\mathrm{e}}^2)\frac{2\pi L}{\lambda}\left(\frac{c}{u_{\mathrm{o}Z}} - \frac{c}{u_{\mathrm{e}Z}}\right), \tag{10}$$

where L is the crystal length. Substitution of Eqs. (5) and (8) into (10) leads to the phase difference for the one selected configuration. Very similar formulas can be found for a few more configurations, thus we generalize our results in the form

$$\begin{aligned}
\Gamma \approx \frac{2\pi L}{\lambda}&\mathrm{sign}(n_{\mathrm{o}}^2 - n_{\mathrm{e}}^2)\times \\
&\times\Bigg[\frac{n_{\mathrm{o}}^2 v_{\mathrm{iso}}}{\sqrt{n_{\mathrm{o}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}} - \frac{n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - \left(n_{\mathrm{o}}^2 - n_{\mathrm{e}}^2\right)c^2\sin^2\varphi}{n_{\mathrm{o}} n_{\mathrm{e}} v_{\mathrm{iso}}\sqrt{n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}} + \\
&\quad + \frac{n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - \left(n_{\mathrm{o}}^2 + n_{\mathrm{e}}^2\right)c^2\sin^2\varphi}{n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 v_{\mathrm{iso}}\left(n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi\right)}\times \\
&\qquad\times\left(-\frac{1}{2}n_{\mathrm{o}}^3 n_{\mathrm{e}}\sqrt{n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}\, r_{\mathrm{A}} + n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 c\sin(\varphi) r_{\mathrm{B}}\right)E + \\
&\quad - \frac{n_{\mathrm{o}}^4 v_{\mathrm{iso}}\left(n_{\mathrm{o}}^2 v_{\mathrm{iso}}^2 - 2c^2\sin^2\varphi\right)}{2\left(n_{\mathrm{o}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi\right)^{3/2}} r_{\mathrm{A}} E\Bigg].
\end{aligned} \tag{11}$$

Here the meaning of r_{A} and r_{B} depends on the configuration of the applied field and incident light direction, as shown in Table 1.

Table 1

The meaning of the r_{A} and r_{B} symbols for some selected configurations of the electric field and incident light direction in the crystal of the point symmetry 3

field direction	light incidence	r_{A}	r_{B}
$(E, 0, 0)$	$(0, \sigma_Y, \sigma_Z)$	r_{11}	r_{41}
$(0, E, 0)$	$(0, \sigma_Y, \sigma_Z)$	$-r_{22}$	r_{42}
$(0, E, 0)$	$(\sigma_X, 0, \sigma_Z)$	r_{22}	r_{41}
$(E, 0, 0)$	$(\sigma_X, 0, \sigma_Z)$	$-r_{11}$	r_{42}

Let consider what error appears when the field-induced changes in the directions of refracted waves are neglected in calculations of the phase difference. To do this we calculate now the paths l'_{o} and l'_{e} taking only the constant terms appearing in Eqs. (3)-(8)

$$l'_{\mathrm{o}} = L\sqrt{u_{\mathrm{o}X}^{(0)2} + u_{\mathrm{o}Y}^{(0)2} + u_{\mathrm{o}Z}^{(0)2}}\Big/u_{\mathrm{o}Z}\ , \quad l'_{\mathrm{e}} = L\sqrt{u_{\mathrm{e}X}^{(0)2} + u_{\mathrm{e}Y}^{(0)2} + u_{\mathrm{e}Z}^{(0)2}}\Big/u_{\mathrm{e}Z}\ . \quad (12)$$

The indices N'_{o} and N'_{e} of the waves perturbed by an external field are calculated now from the optical indicatrix. To do this the field-free directions $\mathbf{u}_{\mathbf{o}}^{(0)}$ and $\mathbf{u}_{\mathbf{e}}^{(0)}$ of refracted waves are assumed instead of $\mathbf{u}_{\mathbf{o}}$ and $\mathbf{u}_{\mathbf{e}}$. The substitution of such intermediate results into Eq. (9) leads to the following phase difference

$$\begin{aligned}
\Gamma^{\sigma=\mathrm{const}} &= \frac{2\pi L}{\lambda}\,\mathrm{sgn}(n_{\mathrm{o}}^2 - n_{\mathrm{e}}^2)\times \\
&\times\Bigg[\frac{n_{\mathrm{o}}^2 v_{\mathrm{iso}}}{\sqrt{n_{\mathrm{o}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}} - \frac{n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - \left(n_{\mathrm{o}}^2 - n_{\mathrm{e}}^2\right)c^2\sin^2\varphi}{n_{\mathrm{o}} n_{\mathrm{e}} v_{\mathrm{iso}}\sqrt{n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}} + \\
&\quad - \frac{n_{\mathrm{o}}^2 n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - \left(n_{\mathrm{o}}^2 - n_{\mathrm{e}}^2\right)c^2\sin^2\varphi}{2 n_{\mathrm{e}}^3 v_{\mathrm{iso}}^3}\times \\
&\quad\quad \times\left(n_{\mathrm{o}}\sqrt{n_{\mathrm{e}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}\, r_{\mathrm{A}} - 2n_{\mathrm{e}} c\sin(\varphi) r_{\mathrm{B}}\right)E + \\
&\quad - \frac{n_{\mathrm{o}}^4 v_{\mathrm{iso}} r_{\mathrm{A}} E}{2\sqrt{n_{\mathrm{o}}^2 v_{\mathrm{iso}}^2 - c^2\sin^2\varphi}}\Bigg]. \qquad (13)
\end{aligned}$$

Taking as an example the PGO ($Pb_5Ge_3O_{11}$) crystal the discrepancies between the phase differences calculated from Eqs. (11) and (13) are shown in Figure 1.

3. DISCUSSION AND CONCLUSIONS

It is seen from Eq. (11), that the electric-field-induced phase difference depends on electrooptic coefficients r_{A} and r_{B} in a somewhat different way. While the contribution of the terms with r_{A} possess fixed sign for a given crystal and a configuration, the sign of the contribution of r_{B} depends on the sign of the incidence angle α. Therefore, the results obtained indicate that two measurements performed for any α and $-\alpha$ angles allow to separate the contributions related to r_{A} and r_{B}. The results may be also useful for crystals of symmetries 4, $\bar{4}$, 422, 4*mm*, $\bar{4}2m$, 6, and 622, where the electrooptic coefficients r_{11} and r_{22} are equal to zero, while at least one from the coefficients r_{41} and r_{42} does not vanish and may be found on the basis of measurements

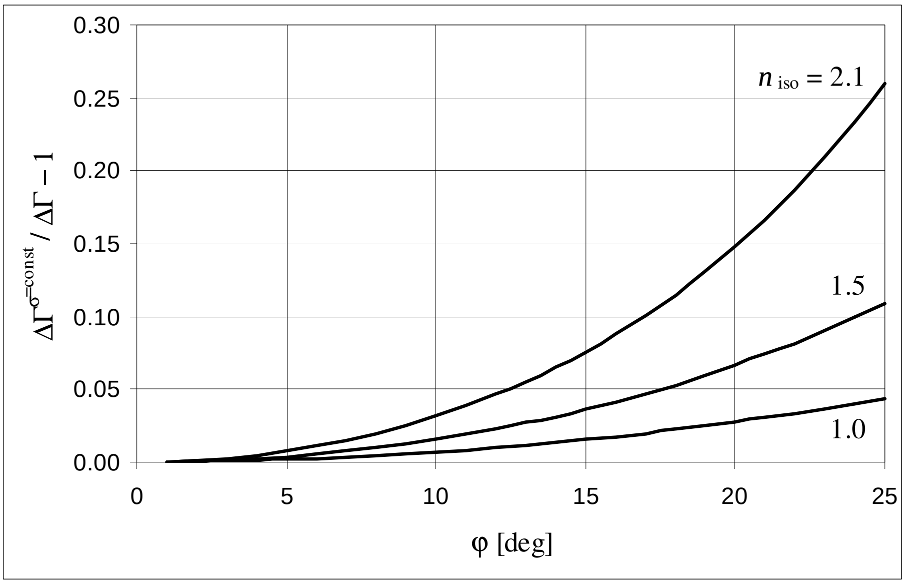

Fig. 1. The relative error ($\Delta\Gamma^{\sigma=\text{const}}/\Delta\Gamma - 1$) of the field-induced change in the phase difference $\Delta\Gamma = \Gamma(E) - \Gamma(0)$ obtained employing simplified Eq. (13) as compared to more exact Eq. (11). The plots are given for the PGO crystal ($n_o = 2.116$ and $n_e = 2.151$ for $\lambda = 0.6328$ [4], $r_A = r_{11} = 0.3\cdot10^{-12}$ m/V and $r_B = r_{41} = 0.6\cdot10^{-12}$ m/V [5]) in the configuration $\mathbf{E} = (E, 0, 0)$; $\boldsymbol{\sigma} = (0, \sigma_Y, \sigma_Z)$. The parameter of the series $n_{\text{iso}} = c/v_{\text{iso}}$ is the refractive index of the isotropic medium around the crystal

performed at a certain angle φ. Such configurations may replace the configuration $\mathbf{E} = (0, E, 0)$, $\boldsymbol{\sigma} = (1, 0, 1)/\sqrt{2}$, which was previously used, for example, to determine the $r_{41} = r_{52}$ coefficient in the KDP (KH_2PO_4) crystal [6].

In the configurations under consideration the directions of the refracted rays differ from the directions of the waves, furthermore, the directions are also modulated by the applied field. Thus, lateral displacement of two parallel light beams emerging from the crystalline plate is modulated. Such rays may interfere only partially in the plane of analyzer and their effective overlap S_{rel} is also modulated by the field strength. In Ref. [7] we have shown, that effective light intensity emerging from such optical system is given by

$$I = I_f + I_s + (I_m - I_f - I_s)\, S_{\text{rel}}\,. \tag{14}$$

Here I_m is the emerging light intensity for the full interference ($S_{\text{rel}} = 1$). I_f and I_s are the light intensities corresponding to the cases when only the fast or slow ray, respectively, propagates through the crystal while the other one is fully absorbed. In the absence of the interference the electric field cannot affect the emerging light intensity, so I_f and I_s intensities are constant, while I_m contains both the constant and electric field dependent components. Employing the Jones matrix calculus for the modulator working on the linear part of its transmission characteristic $I(\Gamma)$ one obtains

$$(I_m - I_f - I_s)|_{E=0} = 0. \tag{15}$$

Thus, if the measurements are made on the linear part of the transmission characteristic the simultaneous changes in S_{rel} and I_m due to the applied modulating sinusoidal field $E = E_0 \sin(\omega t)$ does not give any response of the system on the fundamental harmonic. The work on the linear part of the transmission characteristic may be provided by developed previously technique, which takes advantage of the temperature dependence of the natural birefringence in the crystal (see, e.g. Ref. [8]). Following the constant component of the emerging light intensity during changes of the temperature it is possible to find current working point and the relative overlap S_{rel}. Therefore, if the interference pattern is clear enough an additional optical system is not necessary.

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ELEKTROOPTYCZNA MODULACJA WIĄZEK ŚWIATŁA ODCHYLONYCH ZNACZNIE OD NORMALNEJ DO PŁASZCZYZNY PADANIA

Streszczenie

Pokazano, że elektrooptyczna modulacja różnicy faz dla wiązek światła przechodzących ukośnie przez płasko równoległą płytkę krystaliczną jest nie tylko bezpośrednim rezultatem zmian współczynników załamania fal w funkcji przyłożonego pola, lecz wynika również ze zmian kierunków i długości dróg fal w krysztale. Ten pośredni mechanizm modulacji może być znaczący dla wiązek światła odchylonych znacznie od normalnej do wejściowej i wyjściowej ściany kryształu. Konfiguracje z wiązką światła padającą ukośnie na ścianę normalną do osi optycznej są interesujące w pomiarach liniowych współczynników elektrooptycznych r_{41} i r_{42} w kryształach jednoosiowych. W przypadku niektórych symetrii kryształów mierzony efektywny współczynnik elektrooptyczny może zawierać wkład współczynnika r_{11} albo r_{22}. Pokazano, że wkład ten można oddzielić od wkładu współczynnika r_{41} lub r_{42} na podstawie pomiarów wykonanych przy różnych znakach kąta padania.