Model of the gas journal bearing dynamics with a flexibly supported foil
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Eliza Tkacz, Zbigniew Kozanecki, Dorota Kozanecka, Jakub Łagodziski

Abstract: The work is devoted to an analysis of the journal bearing dynamics employing a numerical model which takes into account factors related to motion and friction of non–rotating elements of the sleeve. This model yields the basis for simulations carried out in order to determine correctly dynamic characteristics of an oil–free machine rotating system at the early stage of its design and during its operation. On the basis of those simulations, the thickness of gas film was determined as well as the pressure distribution in the bearing and its lift capacity. Moreover, the numerical program led to an analysis of proper vibrations and forced vibrations of the system under consideration. It was possible to obtain attenuation of the force and to visualize a journal trajectory for the dynamic harmonic input function. The FFT (Fast Fourier Transform) analysis of vibrations was implemented. In the spectrum, only a frequency of the input function was observed, whereas and a lack of subsynchronous frequencies pointed to the stable operation of the bearing.

1. Introduction

The development of modern turbomachinery of special reliability requirements resulted in search for new bearing systems solutions. Especially, nominal speeds of 10000–100000 rpm are impossible to reach with the use of conventional rolling element bearings or common oil–lubricated bearings, since their operation in these conditions may lead to instability of the machine rotating system. Furthermore, in ORC turbomachines propelled by a low–viscosity, organic working gas or liquid, an application of bearings lubricated with the working medium makes it possible to maintain the purity of the cycle.

In the present paper the coupling of a gas hydrodynamic model to a model of structure deformation is introduced. Accurate methods for the gas hydrodynamic behavior and the foil structure elastic behaviour are described. Finally, unsteady equations are formulated for the hydrodynamic model in order to predict the dynamic properties of a foil bearing. The equations are solved by means of numerical calculations.

As a result of the work, a parametric model has been created, which allows its universal use for calculations of the journal foil bearings dynamics . It was found that computational methods offer the possibility of deeper understanding of the phenomena occurring in the foil bearing.

2. Foil bearing technology

Two types of gas bearings can be distinguished: self–acting, so called aerodynamic bearings and gas powered or aerostatic bearings. The foil bearing is an aerodynamic type support, which is made up of compliant surfaces (a bump and a top foil). As shown in figure 1, during the foil bearing operation the top foil is clenched on the rotating journal by means of the elastic bump foil. The aerodynamic film of a very low thickness, theoretically close to the cylindrical one, is generated by viscosity effects. Since the viscosity of gas is much lower than for liquids, gas bearings have limited load capacity in comparison to oil–lubricated bearings.

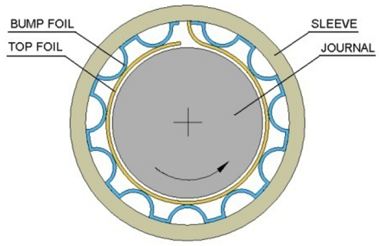


Figure 1. Compliant foil bearing

Miazga [7] has shown, that an important problem of the aerodynamic gas bearings application is related to the start–up and the shut–down in contact with the shaft surface. He has noticed a high drive moment during the start-up (disputable from the viewpoint of turbine mechanical characteristics) and so a limited number of start-up/shut-down cycles (wear). From this point, the heat generation was observed due to the friction phenomena, therefore the right choice of coating materials is crucial form the viewpoint of machine reliability.

Furthermore, the analysis of gas foil bearings is difficult due to interactions between the gas film pressure and the complicated deflection of the top foil and the underlying bump strip support structure. On the other hand, Garcia cited numerous benefits from aerodynamic bearing technology [3]:

- high rotational speeds,
- little maintenance,
- compliance thanks to the bump foil structure,
- compactness,
- no impact on the machinery environment thanks to the absence of external lubrication system.

From the application point of view, a great advantage of foil bearings is such that they require no external pressurization system for the working fluid and so the surrounding gas can be used as the lubricant.

3. Numerical model

In order to define a numerical model of the foil bearing, at first, three systems have to be isolated: rotor, gas film and elastic structure, [1]. In table 1 physical hypothesis are presented for each of the systems.

Table 1. Hypothesis adopted for foil bearing analysis

System	Hypothesis
Rotor	non-deformable in equilibrium position for given rotational speed
Gas film	thin–film compressible, Newtonian fluid isothermal, continue, laminar flow general curvature of the film is negligible non–slip boundary conditions inertial forces and mass forces are negligible viscosity and density do not vary along film thickness
Elastic structure	elastic deformation quasi-static

The mathematical analysis will be therefore applied to each of the systems separately, then, their mutual interactions will be determined. Main parameters values adopted for calculation are listed in tab. 2.

3.1. Gas film

The cylindrical geometry of the gas film in the bearing can be found from equation 1, [2].

$$H = 1 - \varepsilon \cos(\theta - \eta_L) \tag{1}$$

The Reynolds Equation formulated for a cylindrical coordinate system (equation 2) is used to describe the pressure distribution in nearly any type of fluid film bearing under hypothesis cited in table 1.

$$-\frac{\partial}{\partial\theta}\left(\mathbf{PH}^3\frac{\partial\mathbf{P}}{\partial\theta}\right) - \frac{\partial}{\partial\xi}\left(\mathbf{PH}^3\frac{\partial\mathbf{P}}{\partial\xi}\right) + \Lambda\frac{\partial}{\partial\theta}(\mathbf{PH}) + \frac{\partial}{\partial\tau}(\mathbf{PH}) = 0 \tag{2}$$

Table 2. Parameters values adopted for numerical simulations

Parameter	value
Bearing	
Bearing diameter, D_{B} [mm]	34.0
Bearing length, L [mm]	40.0
Radial clearance, C_{B} [mm]	0.035
Lubricant	
Ambient pressure, P_{a} [bar]	1.01325
Vapour dynamic viscosity, μ [Pa·s]	$0.185 \cdot 10^{-4}$
Meshing	
Circumferential number of nodes (gas), N	96
Axial number of nodes (gas), M	34
Foils	
Friction coefficient, ν	0.5

where $\Lambda = \frac{6\mu\omega R^2}{p_{\mathrm{a}}C^2}$ is the compressibility number, $\mathbf{P}$ – the dimensionless pressure, τ – time, θ and ξ are circumferential and axial coordinates respectively.

$$
\begin{aligned}
2P\frac{\partial H}{\partial \tau} + \frac{H}{P}\frac{\partial Q}{\partial \tau} + 2\Lambda P\frac{\partial H}{\partial \theta} + \Lambda\frac{H}{P}\frac{\partial Q}{\partial \theta} - H^3\left(\frac{\partial^2 Q}{\partial \xi^2} + \frac{\partial^2 Q}{\partial \theta^2}\right) + \\
-3H^2\left(\frac{\partial H}{\partial \theta}\frac{\partial Q}{\partial \theta} + \frac{\partial H}{\partial \xi}\frac{\partial Q}{\partial \xi}\right) = 0
\end{aligned}
\tag{3}
$$

By replacing $P^2 = Q$, the Reynolds equation can be rewritten (equation 3). According to the finite difference method, derivatives in partial differential equations can be replaced by central differences, [6]. In order to solve the Reynolds equation, the Alternating Direction Implicit (ADI) method was used.

3.2. Elastic structure

In order to calculate the deformation of the structure, four hypothesis have been formulated:

H1: The structure deflection in a point depends only on pressure acting in this point.

H2: The rigidity of the structure is linear and the elastic force is given by relation $F_{\mathrm{S}} = k_{\mathrm{f}}\Delta h$, where k_{f} is the elasticity coefficient and Δh – length change of the spring.

H3: There is no axial deformation of the structure.

H4: For given θ and different ξ the deformation has the same value.

$$fb_i^{t+\mathrm{d}t} - fb_i^t = \frac{h_{ij}^{t+\mathrm{d}t} - h_{ij}^t}{s} + c_\mathrm{f}\left(\left.\frac{\mathrm{d}h}{\mathrm{d}t}\right|_{t\to t+\mathrm{d}t} - \left.\frac{\mathrm{d}h}{\mathrm{d}t}\right|_{t-\mathrm{d}t\to t}\right) \tag{4}$$

Based on these hypothesis the spring–damper model has been programmed and the relationship between the change in punctual force, fb_i and foil deflection, h, can be expressed by equation 4, [4], where $s = 1/k_\mathrm{f}$ and c_f is a damping coefficient.

3.3. Rotor

The rotor of mass $2M$ is treated as a rigid structure and it is supported symmetrically in two identical finite lengths bearings (without gyroscopic motion) (see fig. 2).

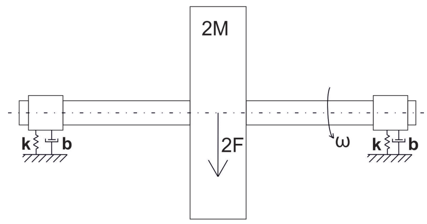


Figure 2. Rotating system model

Forces acting on the rotor result from its own weight, the unbalance, any dynamic (changing in time) loads and the pressure field in the gas film, [1]. They depend on the shaft position and the rotational speed. This can be expressed by Newton's second law equations in dimensionless form:

$$\begin{cases} \overline{M}\ddot{\overline{x}} = \overline{W}_0 + \overline{W}_{\mathrm{d}_x}(T) + \overline{M}e_\mathrm{b}\omega^2\cos(T) + \overline{F}_x(T) \\ \overline{M}\ddot{\overline{y}} = \overline{W}_{\mathrm{d}_y}(T) + \overline{M}e_\mathrm{b}\omega^2\sin(T) + \overline{F}_y(T) \end{cases} \tag{5}$$

where (x, y) are rotor centre coordinates. Dimensionless variables are:

– forces $\overline{F} = \frac{F}{P_\mathrm{a}R^2}$,
– displacements $X = \frac{x}{C}$, $Y = \frac{y}{C}$,
– accelerations $\ddot{X} = \frac{\ddot{x}}{C\omega^2}$, $\ddot{Y} = \frac{\ddot{y}}{C\omega^2}$.
– time $T = \omega t$
– mass $\overline{M} = \frac{MC\omega^2}{P_\mathrm{a}R^2}$

with P_a–ambient pressure, and $C = R - R_\mathrm{a}$ – a difference between the bush radius and the shaft radius.

3.4. Program for static calculations

In order to find the dynamic properties of the foil bearing, first the equilibrium position needs to be calculated. The equilibrium position is reached when the generated lift force, $\boldsymbol{F}$ balances the applied static load. This position is defined by the eccentricity, ε and the centres angle, φ. The algorithm for the equilibrium position calculation is shown in fig. 7.

3.5. Program for dynamic calculations

The aim of the nonlinear method is to find the journal trajectory within the bearing caused by external, time varying load. This problem requires to solve equations for elastic deformation of nonrotating parts, motion equations and unsteady equations for pressure distribution in the gas film. The algorithm for dynamic calculation is shown in fig. 3.

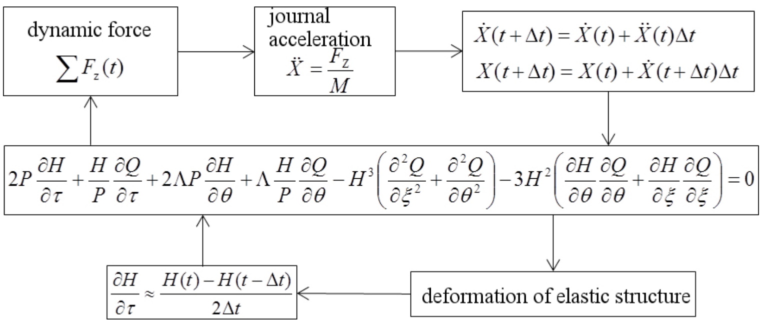


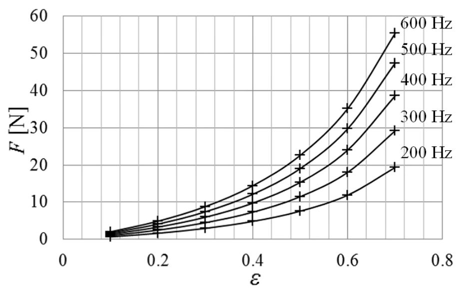
Figure 3. Dynamic calculation algorithm

4. Numerical results

The purpose of this chapter is to evaluate numerical procedures and verify bearing characteristics. The pressure distribution in the gas film is found so that the calculation of structure deformation is possible. It is also important to evaluate the structure model. The gas film is much more rigid than the foil structure, so it is the structure characteristics, which govern the static behaviour of the bearing.

4.1. Static characteristics

The static parameters are defined, when for given static force, the corresponding equilibrium position (ε, φ) is found. In figure 4 static characteristics curves are traced.



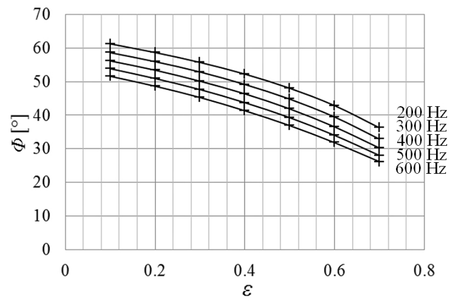


Figure 4. Flexible structure

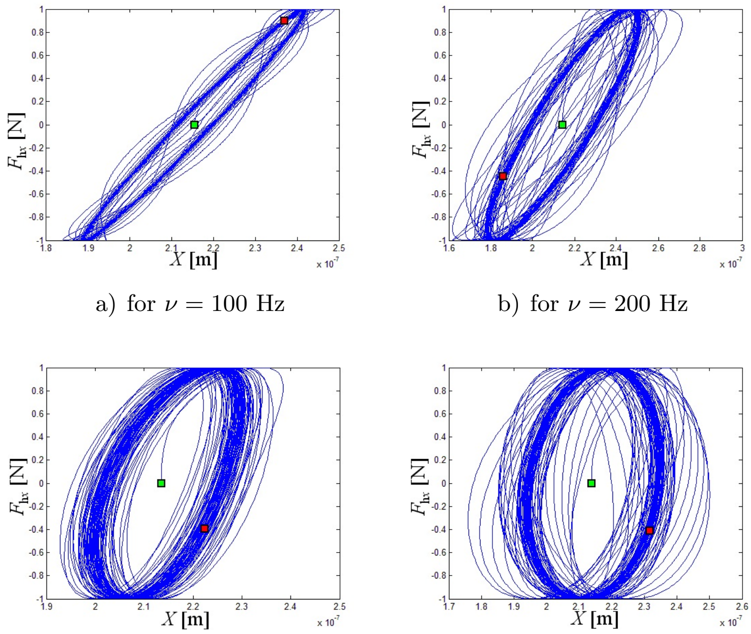
4.2. Dynamic characteristics according to the non–linear method

First, for given static force, F_0, the equilibrium position has been found (see fig. 4). This position is determined by an eccentricity and a centres angle,$(\varepsilon_0, \varphi_0)$. Then the harmonic perturbation has been added. In fig. 5 the rotor trajectory is shown in response to different frequencies of excitation, ν. Comparing those to experimental results, [5], the qualitative resemblance was found. The test rig consisted of a fixed journal, frictionlessly supported sleeve and a modal shaker. The shaker excited the sleeve with a sinusoidal waveform force. It simulated real synchronous excitation caused by rotor unbalance. During the experiment, the excitation force and the sleeve displacement were measured. A few obtained hysteresis loops allowed for identification of the experimental stiffness and damping values of the compliant foil bearings. The conclusion was made that the bearing stiffness increases with the excitation frequency.

Back to numerical simulation results, for the excitation frequency of $\nu = 400$ Hz, the Fast Fourier Transform (FFT) of vibrational response was calculated in order to examine the frequencies spectrum. The dominant frequency of response is of 400 Hz (see fig. 6), so it is equal to the frequency of excitation. This points to the stable work of the bearing.

5. Conclusions

In the present paper main algorithms for foil bearing static and dynamic calculations have been described. The analytical equations were programmed by means of numerical modelization and the programme in Fortran has been developed. The ADI numerical method for solving the Reynolds equation has given a good convergence. A choice of the spring–dumper model for the elastic structure deformation is an important simplification, since it requires less computational resources in comparison to finite element method based codes. Rotor tra-



a) for $\nu = 100$ Hz

b) for $\nu = 200$ Hz

c) for $\nu = 400$ Hz

d) for $\nu = 600$ Hz

Figure 5. Rotor trajectory

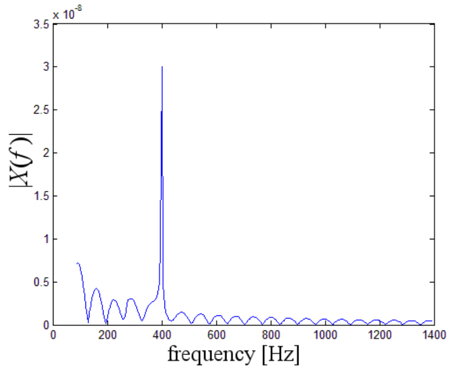


Figure 6. FFT of X vibrations for excitation frequency $\nu = 400$ Hz

jectories for harmonic excitations have been shown for different excitation frequencies. The results pointed to the good programme performance and the simulation behaviour shown the qualitative accordance to experimental tests. The vibrational numerical analysis has revealed a good stability of the rotating system.

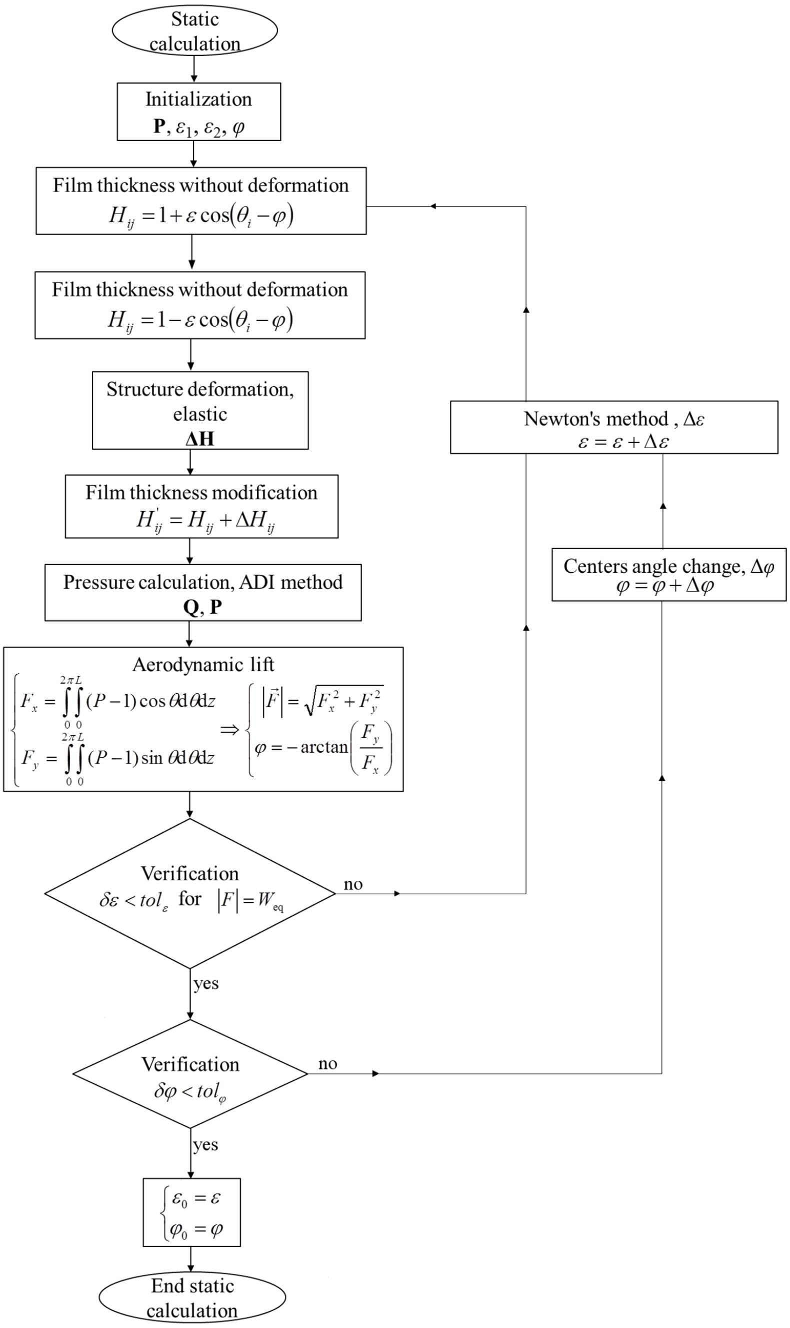


Figure 7. The algorithm for static parameters calculation

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Eliza Tkacz, M.Sc. (Ph.D. student): Lodz University of Technology Institute of Turbomachinery, 90–924 Lodz, 219/223 Wolczanska Street (*eliza.tkacz@p.lodz.pl*). The author gave a presentation of this paper during one of the conference sessions.

Dorota Kozanecka, Professor: Lodz University of Technology Institute of Turbomachinery, 90–924 Lodz, 219/223 Wolczanska Street (*dorota.kozanecka@p.lodz.pl*).

Zbigniew Kozanecki, Professor: Lodz University of Technology Institute of Turbomachinery, 90–924 Lodz, 219/223 Wolczanska Street (*zkozan@p.lodz.pl*).

Jakub Lagodzinski, Ph.D.: Lodz University of Technology Institute of Turbomachinery, 90–924 Lodz, 219/223 Wolczanska Street (*jakub.lagodzinski@p.lodz.pl*).