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**PAWEŁ ADAMCZYK**

**Technical University of Łódź**
**Institute of Automatic Control**

# OPTIMAL SLIDING MODE CONTROL OF AN ELECTROHYDRAULIC SERVOMECHANISM

Reviewer: **Prof. dr hab. Zbigniew Nowacki**

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*In this paper sliding mode control with acceleration constraint is proposed to achieve robustness and optimal performance of electrohydraulic servomechanism. The main feature of the proposed control is application of a time-varying switching plane. Initially, the plane passes through the system representative point in the error state space and then it moves with a constant velocity to the origin of the space. Having reached the origin the plane stops moving and remains fixed. The plane parameters (determining angles of inclination and the velocity of its motion) are selected to ensure the minimum integral absolute error without violating acceleration constraints. The simulation results show, that the proposed control ensures accuracy, and robust performance.*

## 1. INTRODUCTION

In recent years much of the research in the area of control systems theory focused on the design of a discontinuous feedback which switches the structure of the system according to the evolution of its state vector. This technique, usually called sliding mode control, provides an effective and robust means of controlling nonlinear plants [7, 8, 12, 19, 22]. The main advantage of this technique is that once the system state reaches a sliding surface, the system dynamics remain insensitive to a class of parameter variations and disturbances.

However, robust tracking is assured only after the system state hits the sliding surface, i.e. the robustness is not guaranteed during the reaching phase. In order to overcome these problems the idea of the time-varying switching lines applied for the sliding mode control of the second order systems was introduced in [1, 2, 5, 10, 11, 14, 17, 18, 20]. The sliding mode control of the third order, nonlinear and time-varying system was considered in [3, 4, 5, 6, 15, 16]. On the other hand, sliding mode controllers for electrohydraulic servomechanisms were considered in [9, 13].

In this paper the sliding mode control with time-varying switching plane for the same plant is proposed. The switching plane has constant angles of inclination and originally passes through the system representative point in the error state space. Initially the plane moves with a constant velocity to the origin of the space. Once the plane reaches the origin, it stops moving and remains fixed (time-invariant). The switching plane parameters are selected to ensure the minimization of the integral absolute error in the controlled system subject to the acceleration constraint. The proposed approach ensures favourable dynamic properties of the proposed control system and – more importantly – its robustness with respect to the external load torque from the very beginning of the control process.

## 2. SYTEM DESCRIPTION

The plant considered in this paper is an electrohydraulic position servo system. It usually consists of two components: hydraulic actuator and servo valve. The actuator may be linear or rotary. Linear actuator is the cylinder with moving piston, which slides inside the cylinder and divides the actuator into two chambers. Piston movement is caused by the pressure difference between the two chambers. Typical applications of linear actuators include hydraulic presses and injection moulding machines. However, since rotary actuators are probably more popular than the linear ones, in this paper an electrohydraulic position servo system with rotary actuator is considered. The rotary actuators can be classified as gear, piston, vane or deri sine type. Further in this paper gear and semi rotary piston actuators will be presented and gear motor will be considered in detail.

In gear motors, hydraulic fluid follows from inlet to outlet around the outside of the gears in the space between the teeth. One of the two gears is coupled to the output shaft. This is schematically shown in Fig. 1.

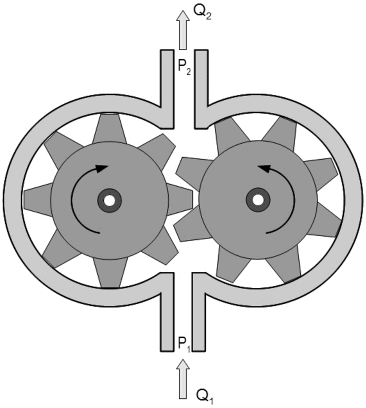


Fig. 1. Schematic diagram of gear hydraulic motor

Semi rotary piston actuator produces very large torque at low speed. The fluid enters into the chamber between fixed and moving vanes. Moving vane is coupled with the output shaft. This type of actuators can have one or more sets of vanes. In Fig. 2 actuators with two sets (a) and one set of vanes (b) are shown. They provide approximately $180^0$ and less than $300^0$ of rotation range respectively.

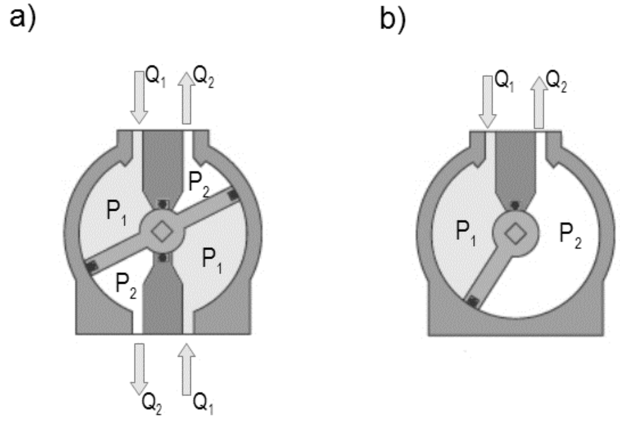


Fig. 2. Schematic diagram of semi rotary piston actuators

Equations, describing the considered hydraulic motor can be expressed as follows. The motor chamber equation can be derived using flow continuity property

$$Q_l = D_m \cdot \dot{\theta}_m + C_{tm} \cdot P_l + \left(V_t / 4 \cdot \beta_e\right) \cdot \dot{P}_l \tag{1}$$

Furthermore, the torque balance equation

$$P_l \cdot D_m = J_t \cdot \ddot{\theta}_m + B_m \cdot \dot{\theta}_m + G \cdot \theta_m + T_l \tag{2}$$

where:

$D_m$ – volumetric displacement of motor,
$C_{tm}$ – total leakage coefficient,
$\beta_e$ – effective bulk modulus,
$V_t$ – total compressed volume
$B_m$ – viscous damping coefficient
$P_l = P_1 - P_2$ – load pressure,
$Q_l$ – load flow,
$\theta_m$ – angular position of motor shaft,
$T_l$ – load torque,
$J_t$ – total motor and load inertia,
$G$ – torsional spring gradient.

Another significant component of the considered system is the electrohydraulic servo valve. Its role is to control pressurized fluid flow in order to move the actuator towards the desired position. The input of the servo valve is an electric current, supplied by the amplifier. The current powers an electromagnetic torque motor which in turn moves the spool valve. The spool valve allows fluid to pass from supply via actuator to the return with a controlled flow rate. Shifting the spool direction enables the fluid flow direction change. Since the relation between spool valve displacement and the control voltage is approximately linear the control voltage is usually taken as the input signal. The electrohydraulic servo valve is schematically presented in Fig. 3. Symbols in this figure correspond to the symbols used earlier in this chapter in Figs. 1 and 2.

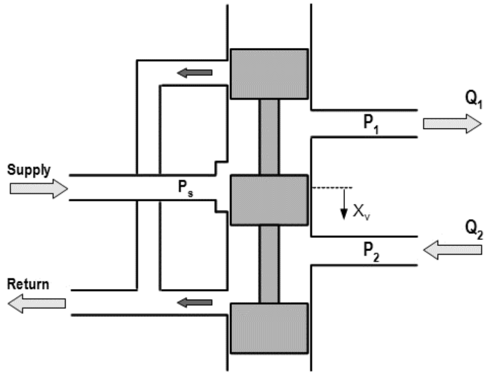


Fig. 3. Schematic representation of electrohydraulic servo valve

The following equations describe electrohydraulic servo valve. The relation between spool displacement and the load flow rate may be represented as

$$Q_l = K_q \cdot X_v - K_c \cdot P_l \tag{3}$$

The valve flow gain, depends on operating point

$$K_q = C_d \cdot W \cdot \sqrt{\left[P_s - \operatorname{sgn}(X_v) \cdot P_l\right]/\rho} \tag{4}$$

where:

$K_c$ – flow – pressure coefficient,
$K_q$ – servovalve gain,
$W$ – area gradient,
$X_v$ – valve displacement,
$P_s$ – supply pressure,
$\rho$ – fluid mass density,
$C_d$ – discharge coefficient.

Combining equations 1-4, describing both actuator and servo valve dynamics, we get the following state equations of the system

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= x_3(t) \\
\dot{x}_3(t) &= -a_1 \cdot x_1(t) - a_2 \cdot x_2(t) - a_3 \cdot x_3(t) + b(\boldsymbol{x},t) \cdot u(t) - d(t)
\end{aligned} \tag{5}$$

where:

$$\begin{aligned}
\boldsymbol{x}(t) &= \left[x_1(t) \quad x_2(t) \quad x_3(t)\right]^T = \left[\theta_m(t) \quad \dot{\theta}_m(t) \quad \ddot{\theta}_m(t)\right]^T \\
a_1 &= (4 \cdot \beta_e/V_t) \cdot (K_{ce}/J_t) \cdot G \\
a_2 &= G / J_t + (4 \cdot \beta_e / V_t) \cdot (D^2{}_m / J_t) + (4 \cdot \beta_e / V_t) \cdot (K_{ce} / J_t) \cdot B_m \\
a_3 &= B_m/J_t + (4 \cdot \beta_e/V_t) \cdot K_{ce} \\
b(\boldsymbol{x},t) &= (4 \cdot \beta_e/V_t) \cdot (D_m/J_t) \cdot K_q \cdot K_v \\
d(t) &= (4 \cdot \beta_e/V_t) \cdot (K_{ce}/J_t) \cdot T_l + (1/J_t) \cdot \dot{T}_l
\end{aligned} \tag{6}$$

In these equations $K_{ce} = K_c + C_{tm}$ is the total flow–pressure coefficient and $K_v$ represents the servovalve amplifier gain. From the above equations it can be seen, that the considered system is nonlinear and subject to the external disturbance so it requires a robust and possibly nonlinear control scheme. In this paper the sliding mode control scheme with time varying switching plane is proposed.

## 3. CONTROL SYSTEM

The trajectory tracking error is defined by the following vector $\boldsymbol{e}(t) = [e_1(t)\ \ e_2(t)\ \ e_3(t)]^T = \boldsymbol{x}(t) - \boldsymbol{x}_d(t)$. Hence, we have $e_1(t) = x_1(t) - x_{1d}(t)$, $e_2(t) = x_2(t) - x_{2d}(t)$, $e_3(t) = x_3(t) - x_{3d}(t)$. In this paper it is assumed that at the initial time $t = t_0$, the tracking error and the error derivatives $e_1(t_0) = e_0$, $e_2(t_0) = 0$, $e_3(t_0) = 0$. Let us consider a time varying switching plane with the constant angle of inclination. Originally the plane moves uniformly (i.e. with a constant velocity) in the state space and then it stops at the time instant $t_f$. Consequently, for any $t \le t_f$ the switching plane is described by the following equation

$$s(\boldsymbol{e},\ t) = 0, \text{ where } s(\boldsymbol{e},t) = e_3(t) + c_2 \cdot e_2(t) + c_1 \cdot e_1(t) + A + B \cdot t \tag{8}$$

where $c_1$, $c_2$, $A$ and $B$ are some constants. The selection of these constants will be considered further in this section. Since the plane stops at the time $t_f$, for any $t \ge t_f$

$$s(\boldsymbol{e},\ t) = 0, \text{ where } s(\boldsymbol{e},t) = e_3(t) + c_2 \cdot e_2(t) + c_1 \cdot e_1(t) \tag{9}$$

First, the constants $c_1$, $c_2$, $A$ and $B$ should be chosen in such a way that the representative point of the system at the initial time $t = t_0$ belongs to the switching plane. For that purpose, the following condition must be satisfied

$$s(\boldsymbol{e}(t_0), t_0)) = e_3(t_0) + c_2 \cdot e_2(t_0) + c_1 \cdot e_1(t_0) + A + B \cdot t_0 \tag{10}$$

Notice that the input signal

$$u = \frac{-f(\boldsymbol{x},t) - c_2 \cdot e_3(t) - c_1 \cdot e_2(t) + \dot{x}_{3d}(t) - B - \gamma \cdot \operatorname{sgn}[s(\boldsymbol{e},t)]}{b(\boldsymbol{x},t)} \tag{11}$$

where $\gamma = \eta + \mu$ and $\eta$ is a strictly positive constant, ensures the stability of the sliding motion on the switching plane (8). We demand tracking error to be an aperiodic function of time, i.e. we wish to avoid undesirable overshoots and oscillations. Furthermore, since we require the system to exhibit good dynamic properties, our objective is to minimize the control quality criterion

$$J = \int_{t_0}^{\infty} \left| e_1(t) \right| dt \tag{12}$$

subject to acceleration constraint.

According to [4] the optimal switching plane parameters subject to acceleration constraint are given as

$$B = \left[ 2(a_{\max})^{3/2} \cdot \operatorname{sgn}(e_0) \right] / \sqrt{|e_0|} \tag{13}$$

$$c_1 = \left( 4 \cdot B / e_0 \right)^{2/3} \tag{14}$$

$$A = -c_1 \cdot e_0 \tag{15}$$

$$c_2 = 2 \cdot \sqrt{c_1} \tag{16}$$

$$t_f = \left( e_0 \cdot c_1 \right) / B \tag{17}$$

where $a_{max}$ is the maximum admissible acceleration value.

# 4. SIMULATION RESULTS

We simulated dynamic behaviour of the electrohydraulic servomechanism, controlled by the presented algorithm. Parameters of the system are given in the following table [13].

| PARAMETER | VALUE | UNIT |
|---|---|---|
| $P_S$ | 13.79 | MPa |
| $K_q$ | $6.45 \cdot \sqrt{\left[ P_s - \operatorname{sgn}(X_v) \cdot P_l \right] / \rho} \cdot 10^{-6}$ | $m^2 s^{-1}$ |
| $\beta_e$ | 344.74 | MPa |
| $V_t$ | 1.64 | $m^3$ |
| $K_{ce}$ | $1.13 \cdot 10^{-6}$ | $m^3 \cdot s \cdot Pa$ |
| $D_m$ | $8.19 \cdot 10^{-6}$ | $m^3 \cdot rad^{-1}$ |
| $J_t$ | $5.65 \cdot 10^{-2}$ | $kg \cdot m^2$ |
| $B_m$ | 8.47 | $N \cdot m \cdot s$ |
| $K_v$ | 0.51 | $m \cdot V^{-1}$ |
| G | $1.13 \cdot 10^{-3}$ | $N \cdot m \cdot rad^{-1}$ |

The maximum admissible actuator acceleration value $a_{max} = 10$ rad/s$^2$. Initial actuator position $x_1(t_0) = 10$ rad, and the main goal to achieve is to set the final position to zero. It makes tracking error $e_1$ equal to $x_1$. From equations (13) – (17) we get the following switching plane parameters: $c_1 \approx 10.87$ 1/s$^2$, $c_2 \approx 6.594$ 1/s, $B \approx 89.63$ rad/s$^3$, $A \approx -108.7$ rad/s$^2$ and the stop time $t_f \approx 1.213$s. The load torque is the sine like function with amplitude 2.825 N·m. According to this load we choose the coefficient $\gamma = 10^4$ rad/s$^3$. In order to avoid the chattering, we replace sign function in equation (11) with saturation function with the threshold value equal to 0.2 rad/s$^2$. Simulation results are shown in figures 4 – 6.

Figure 4 shows actuator position and velocity. Figure 5 presents its acceleration and figure 6 demonstrates the control voltage. It can be seen from these figures that the acceleration constraint $a_{max} = 10$rad/s$^2$ is satisfied and the actuator position converges to zero without oscillations. The system is also insensitive with respect to external disturbance (load torque) from the very beginning of the control process.

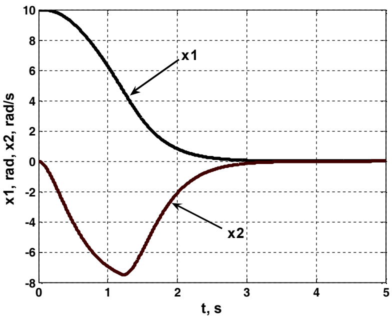


Fig. 4. Actuator position ($x_1$) and velocity ($x_2$)

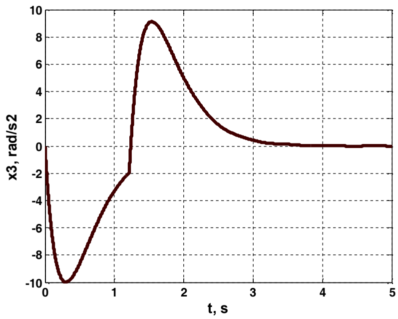


Fig. 5. Actuator acceleration

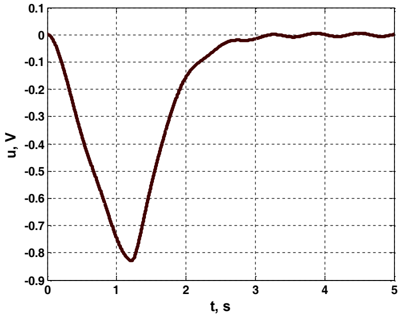


Fig. 6. Control voltage

# 5. CONCLUSIONS

In this paper sliding mode control with time varying switching plane for electrohydraulic servomechanism has been considered. The switching plane parameters are selected to ensure the minimum integral absolute error without violating acceleration constraint. It has been demonstrated that applying time varying switching plane makes the system insensitive to external disturbance from the very beginning of the control action. Furthermore, the proposed control scheme assures good system dynamics and its non-oscillatory response.

# 6. ACKNOWLEDGEMENT

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# OPTYMALNE ŚLIZGOWE STEROWANIE ELEKTROHYDRAULICZNYM SERWOMECHANIZMEM POZYCYJNYM

## Streszczenie

W artykule przedstawiono ślizgowe sterowanie elektrohydraulicznym serwomechanizmem pozycyjnym, który został zamodelowany jako nieliniowy i niestacjonarny obiekt dynamiczny trzeciego rzędu. Zaproponowane sterowanie wykorzystuje ruchomą płaszczyznę przełączeń o stałym nachyleniu. Płaszczyzna przemieszcza się ruchem jednostajnym (ze stałą prędkością) ze swojego pierwotnego położenia określonego przez wartość uchybu regulacji w chwili rozpoczęcia procesu regulacji do początku układu współrzędnych w przestrzeni stanu. Parametry płaszczyzny zostały wyznaczone tak, aby zapewnić minimalizację całki modułu uchybu regulacji, przy uwzględnieniu ograniczenia przyśpieszenia elementu wykonawczego. Badania symulacyjne wykazały, że zastosowane sterowanie zapewnia pożądaną dokładność oraz niewrażliwość na zewnętrzny moment oporowy od początku trwania procesu regulacji.

Paweł Adamczyk
Politechnika Łódzka, Instytut Automatyki