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Higher order asymptotic homogenization for dynamical problems

J. AWREJCEWICZ[1], I.I. ANDRIANOV[2*], A.A. DISKOVSKY[3]

1. Lodz University of Technology, Department of Automation, Biomechanics and Mechatronics, Lodz, Poland [0000-0003-0387-921X]
2. Rhein Energie AG, Cologne, Germany [0000-0001-6419-5423]
3. Dnipropetrovsk State University of Internal Affairs, Department of Economic and Law Security, Dnipro, Ukraine [0000-0003-1673-6437]

* Presenting Author

Abstract: In general, asymptotic homogenization methods are based on the hypothesis of perfect scale separation. In practice, this is not always the case. The problem arises of improving the solution in such a way that it becomes applicable if inhomogeneity parameter is not small. Our study focuses on the higher order asymptotic homogenization for dynamical problems. Systems with continuous and piecewise continuous parameters, discrete systems, and also continuous systems with discrete elements are considered. Both low-frequency and high-frequency vibrations are analyzed. For low-frequency vibrations, several approximations of the asymptotic homogenization method are constructed. The influence of the boundary conditions, the system parameters is investigated

Keywords: Periodically nonhomogeneous structures, dynamics, homogenization, scale separation.

1. Introduction

As noted in [1], the bulk of researches based on the asymptotic homogenization method (AHM) use hypothesis of perfect scale separation. In practice, this hypothesis not always justified. Formally, this means that the first approximation of the AHM does not provide the required accuracy. The problem arises of improving the solution in such a way that it becomes applicable for a not small value of the used inhomogeneity parameter. This conclusion is supported by the results of a number of studies. Xing and Chen [2] analysed the static problems of the periodical composite rod using different order of AHM and FEM. Numerical results show that the second approximation is necessary for accurate analysis of periodical composite structures. Kesavan [3,4] considered the Dirichlet eigenvalue problem for a second-order elliptic operator in the divergence form. Comparison with numerical solutions showed the need to take into account higher approximations. Santosa and Vogelius [5] and Moskow and Vogelius [6] studied the eigenvalue problem associated with the vibration of a composite medium with a periodic microstructure. The investigation was devoted to the first-order correction to the homogenized eigenvalues. It is shown that for the Dirichlet problem, the interaction of the periodic microstructure with the boundary of the medium must be taken into account. The first order Neumann eigenvalue corrections are always zero in one dimension. It vaguely of phenomenon that is reminiscent a frequently occurs in connection with spectral approximation for self adjoint operators: the error in the eigenvalue order is square energy norm error in a corresponding eigenvector [5].

Our study focuses on the higher order asymptotic homogenization for dynamical problems. For systems with continuous and piecewise continuous parameters, discrete systems, and continuous systems with discrete elements, explicit expressions for the second approximations for eigenvalues are constructed.

2. Results and Discussion

Consider for example the eigenvalue Neumann problem

$$\frac{d}{dx}\left[a\left(\frac{x}{\varepsilon}\right)\frac{du}{dx}\right]+pu=0,\quad a\left(\frac{x}{\varepsilon}\right)=a\left(\frac{x}{\varepsilon}+1\right), \tag{1}$$

$$\frac{du}{dx}=0\quad \text{at}\quad x=0, x=1. \tag{2}$$

Going over to fast $\eta=\varepsilon^{-1}x$ and slow x variables we seek solution of Neuman problem (1), (2) as follows

$$u=u_0(x)+\varepsilon u_1(\eta,x)+\varepsilon^2 u_2(\eta,x)+\ldots,\quad p=p_0+\varepsilon p_1+\varepsilon^2 p_2+\ldots \tag{3}$$

Using higher order AHM we obtain

$$p_0=\pi^2 n^2\tilde{a},\quad \tilde{a}=\left[\int_0^1 a^{-1}d\eta\right]^{-1}, \tag{4}$$

$$\begin{gathered} p_1=0,\quad p_2=\pi^2 n^2 p_0\left[\tilde{a}\int_0^1(a^{-1}\int_0^\eta u_{11}(\eta)d\eta)d\eta-\int_0^1(\int_0^\eta u_{11}(\eta)d\eta)d\eta\right], \\ u_{11}=0.5-\eta+\tilde{a}\left[\int_0^\eta a^{-1}d\eta-\int_0^1(\int_0^\eta a^{-1}d\eta)d\eta\right]. \end{gathered} \tag{5}$$

For piecewise-continuous function $a(\eta)=E^{in},\quad 0\le\eta\le c,\quad E^m,\ c\le\eta\le 1$ (4), (5) yield [7]

$$p_0=\pi^2 n^2\frac{E^{in}E^m}{(1-c)E^{in}+cE^m},\quad p_1=0,\quad p_2=-\pi^2 n^2 p_0\frac{c^2(1-c)^2\left(E^{in}-E^m\right)^2}{12\left[(1-c)E^{in}+cE^m\right]^2}.. \tag{6}$$

The area of applicability of the obtained solution are determined by the relation $n<<\varepsilon^{-1}$.

3. Concluding Remarks

Continuous and piecewise continuous parameters, discrete systems, and also continuous systems with discrete elements are considered. Both low-frequency and high-frequency vibrations are analyzed. The use of one- and two-point Padé approximants is proposed to improve the results accuracy.

References

[1] AMEEN MM, PEERLINGS RHJ, GEERS MGD: A quantitative assessment of the scale separation limits of classical and higher-order asymptotic homogenization. *European Journal of Mechanics-A/Solids* 2018, **71**:89–100.

[2] XING YF, CHEN L: Accuracy of multiscale asymptotic expansion method. *Composite Structures* 2014, **112**:38-43.

[3] KESAVAN S: Homogenization of elliptic eigenvalue problems. I. *Applied Mathematics and Optimisation* 1979, **5**(2):153-167.

[4] KESAVAN S: Homogenization of elliptic eigenvalue problems. I. *Applied Mathematics and Optimisation* 1979, **5**(3):197-216.

[5] SANTOSA F, VOGELIUS M: First-order corrections to the homogenized eigenvalues of periodic composite medium. *SIAM Journal on Applied Mathematics* 1993, **53**(6):1636-1668; Erratum: Ibid 1995, **55**(3):684.

[6] MOSKOW SH, VOGELIUS M: First order corrections to the homogenized eigenvalues of a periodic composite medium. The case of Neumann boundary conditions. Preprint, 1996.

[7] ANDRIANOV IV, AWREJCEWICZ J, DANISHEVSKYY VV: *Linear and Nonlinear Waves in Microstructured Solids: Homogenization and Asymptotic Approaches*. CRC Press, Taylor & Francis: Boca Raton, 2021.