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Mathematical Modelling of an extended Swinging Atwood Machine

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Abstract: An extended model for a variable-length pendulum's mechanical application is being derived from the Swinging Atwood Machine (SAM). An electrical component consisting of an electromagnet and armature coil is attached on the link connected to the counterweight mass on the left-hand side of the modified SAM to provide an excitation force for the system when an electric current is induced. The extended SAM presents a novel SAM concept being derived from a variable-length double pendulum with a suspension between the two pendulums. The equations of motion are simulated to see the trajectory of the two pendulums. The results of original numerical simulations show some compact regions of attraction at some regimes. Therefore, the extended SAM's nonlinear dynamics presented in the current work can be thoroughly studied, and more modifications can be achieved. The new technique can reduce residual vibrations through damping when the desired level of the crane is reached. It can also be used in simple mechatronic and robotic systems.

Keywords: variable-length pendulum, swinging Atwood machine, suspension, vibration, damping

1. Introduction

Mathematical concepts gives the ability to transfer knowledge from one setting to another, which will significantly be enhanced and easy modifications and application for the real-time implementation of engineering projects.

The variable-length pendulum is a physical concept associated with parametric oscillations governed by certain forms of differential equations and functional principles [1]. A parametric oscillator can be treated as a harmonic oscillator whose physical features change over time [2]. The presented mathematical model is derived from the SAM illustrated in [2].

2. System description

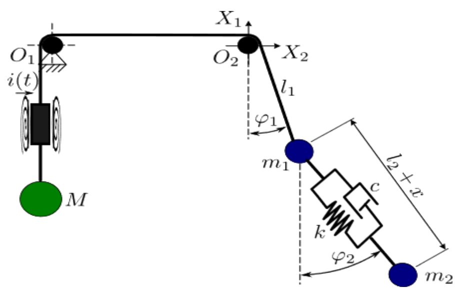

Fig. 1. Schematic diagram of the proposed modification of the SAM model

Figure 1 shows the modified SAM. A suspension system with a stiffness k and a damper c placed between the two pendulums with masses m_1 and m_2. Point 0_1 is fixed, while 0_2 is movable and can oscillate in the plane (X_1,X_2), which allow the variation of the length l_1 and the double pendulum couplings. An electrical component consisting of an electromagnet and armature coil is attached on the link connected to the counterweight mass on the left-hand side of the modified SAM to provide an excitation force for the system when an electric current is induced.

3. Results and Discussion

Kinetic Energy:

$$T = \frac{1}{2}M(\dot{l}_1^2 + \dot{x}^2) + \frac{1}{2}m_1(\dot{l}_1^2 + l_1^2\dot{\varphi}_1^2) + \frac{1}{2}m_2(\dot{x}^2 + (l_2 + x)^2\dot{\varphi}_2^2) \quad (1)$$

Potential Energy:

$$U = \frac{1}{2}kx^2 + Mg(l_1 + (l_2 + x)) - m_1 g l_1 \cos\varphi_1 - m_2 g(l_1 \cos\varphi_1 + (l_2 + x)\cos\varphi_2) \quad (2)$$

With the Lagrange equation, $L = T - U$, we find four degrees of freedom (l_1, x, φ_1, and φ_2). The following equations are obtained:

$$(M + m_1)\ddot{l}_1 + \ gM - v_0\cos(\omega t) - g(m_1 + m_2)\cos\varphi(t) - m_1 l_1 \dot{\varphi}_1^2 \quad (3)$$

where, v_0 – voltage in the armature winding, ω – excitation frequency.

$$(M + m_2)\ddot{x} + m_2(l_2 + x)\dot{\varphi}_2^2 + c_1\dot{x} + kx - g(M - m_2\cos\varphi_2) = 0 \quad (4)$$

$$m_1 l_1^2\ddot{\varphi}_1 + 2m_1\dot{l}_1\dot{\varphi}_1 + l_1(g(m_1 + m_2)\sin\varphi_1) = 0 \quad (5)$$

$$m_2(l_2 + x)^2\ddot{\varphi}_2 + 2m_2(l_2 + x)\dot{x}\dot{\varphi}_2 + c_2\dot{\varphi}_2 + gm_2\sin\varphi_2\,(l_2 + x) = 0 \quad (6)$$

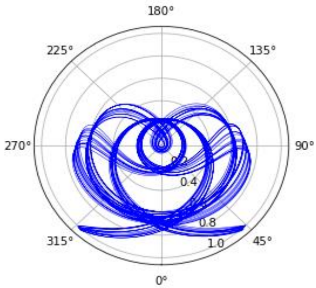

Fig. 2. An orbit of the Modified SAM (φ_1, l_1)

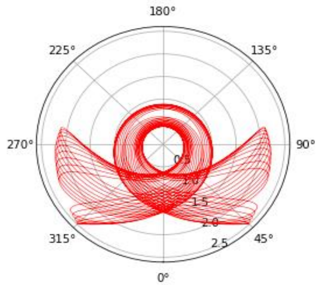

Fig. 3. An orbit of the Modified SAM (φ_2, $(l_2 + x)$)

4. Concluding Remarks

The presented results show the nonsingular orbit under swinging, with no physical contact between the swinging assemble and the fixed points. Interestingly, in some regimes, compact regions of attraction as can be seen in Figure 2 and 3 appear in the system. Therefore, the nonlinear dynamics of the presented modified SAM can be thoroughly studied, and more modification can be achieved.

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