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Nonlinear Dynamics of Forced Oscillator Subjected to a Magnetic Interaction

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Abstract: It is considered the system composed of a cart moving along a linear rolling bearing with harmonic excitation produced by a stepper motor with an unbalanced disk. The magnetic field is generated by a pair of non-point neodymium magnets, one of which is mounted on the cart, whereas another one is fixed on the guide out of the axis of oscillations. The mathematical model for the cart dynamics is derived where the scaled point dipoles approximation of magnetic interaction is used. The numerical and bifurcation analysis of the model presented is carried out and compared to the experimental results.

Keywords: magnetic pendulum, forced oscillations, bifurcations.

1. Introduction

In contrast to the dynamics of linear system, the behaviour of nonlinear systems are much more diverse and complex [1]. It depends on the peculiarities of system's construction, regimes of its operation, and many other reasons. Therefore, the nonlinear systems are the permanent source of new studies and inventions. This work deals with the relatively simple mechanical model, but the incorporation of nonlinear magnetic interaction leads us to the statement of new problems in the field of nonlinear dynamics. Thus, we continue the studies of the system presented in [2] and we develop the physically motivated description for the magnetic interactions produced by non-point magnets.

2. Results and Discussion

The experimental stand (Fig.1a) we treat consists of trolley moving along a linear rolling bearing with periodic forcing realized by the use of rotating unbalanced disk driven by a stepper motor. The stiffness is composed of linear elasticity generated by linear mechanical spring and nonlinear stiffness produced by a pair of repulsive neodymium magnets of axes perpendicular to the direction of system motion. The position of the system is measured by the use of Hall sensors.

The mathematical description of the physical model (Fig. 1b) of the experimental stand (Fig. 1a) reads as follows :

$$m\,\ddot{x} + F_R(\dot{x}, x) + F_S(x) = m_0 e \omega^2 \sin \omega t, \tag{1}$$

where $F_R(\dot{x}, x)$ is a resistance force; $F_S(x)$ is a restoring force. When the magnets are absent, the resistance force incorporates the viscous force $c\dot{x}$ and dry friction T sign $\dot{x}$, whereas $F_S = kx$. Adding the magnets causes the appearance of magnetic repulsive force $\vec{F}_M$ which makes the contribution to F_S and F_R. To estimate $\vec{F}_M$, we adopt the point dipoles approximation. According to this approach, the repulsive magnetic force $\vec{F}_M = -\nabla(\vec{n}\cdot\vec{B})$, where $\vec{B} = \frac{F_0}{n^2|\vec{r}|^3}(3(\vec{n}\cdot\hat{r})\hat{r} - \vec{n})$, the magnetic moment $\vec{n} = (0, n)$, $\vec{r} = (x, z)$, $\hat{r} = \vec{r}/|\vec{r}|$. Thus, we get $\vec{F}_M = (F_M^x, F_M^z) = F_0\left(\frac{3x(4z^2-x^2)}{(x^2+z^2)^{7/2}}, \frac{3z(2z^2-9x^2)}{(x^2+z^2)^{7/2}}\right)$. The components of the vector $\vec{F}_M$ provide the horizontal and orthogonal force projections which are incorporated into the restoring and friction forces: $F_S(x) = kx - F_M^x$ and $F_R = c\dot{x} + T \text{ sign } \dot{x} + \mu \cdot \text{sign}\dot{x} \cdot F_M^z$. To validate the model (1), the results of experiments [2], carried out at fixed $z = 0.01$m and varying frequency ω, are used. It turned out, to describe the cart's dynamic correctly, the expression for the magnetic force should be scaled, i.e. in the equation (1) $\vec{F}_M \to \alpha\vec{F}_M(x/\beta, z)$, where $\alpha = 2.5$, $\beta = 3$, $\mu = 0.001$, $F_0 = 1.4695 \cdot 10^{-8}$ N·m^4, $c = 12.9906$ N·s/m, $k = 929.333$ N/m, $T = 1.2267$ N, $m_0 e = 0.25152$ kg·m, $m = 6.73766$ kg [2]. The corresponding numerical bifurcation diagram for the equation (1) is presented in Fig.1c, which is in very good agreement with the experimental bifurcation diagram (see [2]). From the diagram it follows that equation (1) possesses the coexisting attractors (color points). These attractors undergo only one period doubling bifurcation at $\omega < 12.54$, whereas at $\omega > 18.91$ their bifurcations are described by the period-doubling cascade. Moreover, the jump phenomenon occurs at $\omega \approx 18.91$.

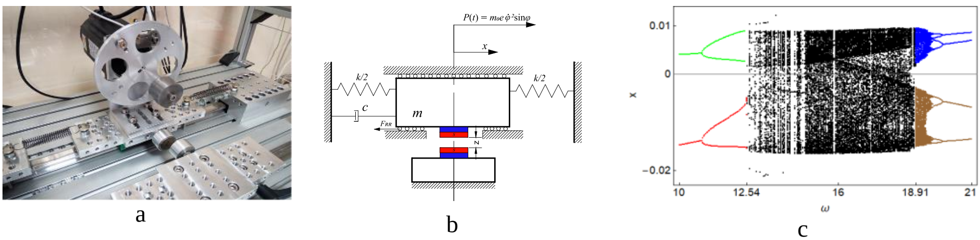

Fig.1. The experimental stand (a) of the oscillator, its physical model (b) and the numerical bifurcation diagram (c) for the equation (1).

3. Concluding Remarks

Thus, the present research deals with the forced oscillator taking into account the influence exerted by field of permanent magnets. To develop the equation of motion for this model, the magnetic interaction is described on the base of the point dipoles approximation. During model validation it is derived the scaled point dipoles approximation describing the experimentally observed regimes in a wide range of frequency interval.

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