

## Dynamic Response of Viscoplastic Thin-Walled Griders in Torsion

Leszek CZECHOWSKI  
*Department of Strength of Materials  
Łódź University of Technology  
Stefanowskiego 1/15, 90-924 Łódź, Poland  
leszek.czechowski@p.lodz.pl*

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This work deals with an analysis of isotropic or orthotropic girders subjected to transient dynamic loads. The duration of dynamic loading was assumed to be equal to a period of the natural fundamental flexural vibrations of a structure under analysis. Numerical calculations were performed with the finite element method using ANSYS<sup>®</sup> 11.0 software. The results of computations were presented as maximum angle of the rotation of the girder in a function of the dynamic load factor, DLF (the ratio of pulse loading amplitude to static critical load). In study it has been taken into account apart from the elastic–plastic range of material with isotropic hardening as well as the strain rate effect described by Perzyna model.

*Keywords:* Finite Elements Method, carbon–epoxy laminate, post–critical deformation states, thin-walled beams.

### 1. Introduction

The structural stability problem has been investigated for over a century and is now very extended. An assessment of a behavior of thin-walled structures in the whole range subjected to pulse loading in torsion can bring about many difficulties especially if it uses equations of the classic plate theory. Nowadays, for this purpose the FEM is often and widely applied in the majority of theoretical approach (almost each problem in some approximation can be gained). It allows for an accomplishment of some results in a comparison to the analytical–numerical solution or experimental test. Furthermore, this method of solution is considerably cheaper and more comfortable (in simply way can be conditions of analysis changed). A typical simulation consists of generation of correct numerical model (for example, a way or a quality of division into finite elements, proper boundary conditions, right number of elements or an application of proper loading) corresponding to the structure under investigation. In dynamics, in comparison to static solution, many

additional phenomena need to be considered and investigated. Aspects which are essential for occurring problems can be divided into two groups. The first of all, numerical difficulties including way of method solution, proper damping coefficient of material and for example an obtaining of solution convergence as well as physical interpretation of the structures behaviour.

Proceeding to studied problem and observing particularly literature it can be found many works devoted the dynamic stability of thin-walled structures under pulse loading [1, 3, 4, 5, 9, 10, 11, 12, 14]. During a studying of behaviour of some construction it is usually very desirable to determinate critical dynamic loading (for example related to buckling static loading) for the durations being equal to the period of natural vibrations. Some criteria used by researchers in an assessment of the dynamic stability of thin-walled structures allow for approximate determination of critical dynamic loading. In the literature [1, 3, 4, 5, 9, 10, 12, 14], it is said that dynamic buckling occurs if duration of loading is close to the period of natural frequency of flexural vibrations of the structure.

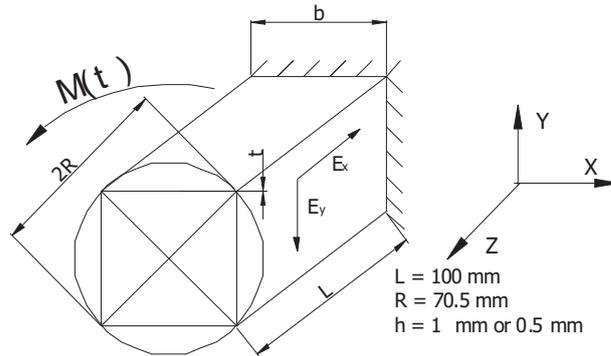
On other hand, looking at the literature, there are a lot of works devoted mainly to cylindrical shells made of different materials, taking into account either an elastic or elastic-plastic range subjected to static or dynamic loading in torsion. However, a few papers describing the plate structure subject to dynamic torsion can be found as well. An analysis of buckling and post-buckling of the cylindrical shell is to found in [18, 19, 20]. Ma et al. [13] dealt with dynamic plastic buckling of circular cylindrical shells under an influence of dynamic torque. They carried out an empirical experiment on the torsional buckling of circular shells and next they discussed the attained results. Cylindrical shells under torsion were also studied in [20]. The authors assumed the Hamiltonian system approach to investigate the propagation of shear wave. They distinguished two types of buckling modes: torsional and helical. Kubiak, Królak and Kołakowski [8] presented the results of analysis of stability and load-carrying capacity of thin-load orthotropic poles subjected to static combined loading. They solved the problem of buckling (stability problem) using the Byskov and Hutchinson asymptotic method within the second order approximation. To verify the computational results, the researchers carried out calculations applying the finite element method within the ANSYS 5.4 code. An analytical-numerical method of transition matrices was also applied in [2], where the authors presented a study of global, local and coupled buckling of structures subjected to different loads. Investigations of dynamic stability of plates and structures built of plates subjected to in-plane pulse loading were conducted in [1, 3, 4, 5, 9, 10, 11, 12, 14] among others. The authors of those papers described the phenomena of dynamic buckling and presented the results of calculations on the basis of the dynamic stability criteria. Mania in his works [11, 12] extended the analysis of dynamic buckling of columns taking into account strain rate effect according to Perzyna model and Stoffel [17] assumed the application of different constitutive laws in the studies of the circular plate high strain rate response. However, in case of torsional loading, only a few publications concern a dynamic loads, even though some papers discuss static loads and analysis of girders.

In this paper, the author studies a behavior of a structure under pulse loading in torsion whose duration equals the fundamental natural vibration period of the given structure. Numerical calculations were conducted to obtain the dynamic response of

such structures under rectangular pulse loading. For computations it was assumed the plate structure with rectangular cross-section considering the materials in elastic or elastic-plastic range and the strain rate effect. The simulation has been carried out in Ansys 11.0 code using finite element method.

## 2. Girders shape and applied material

For purpose of an assessment of the response of the structure it was focused only on the one shape of the column. Namely, the study concerns rectangular girders composed of plates. The diagonal of column and the length of the column amounted 141 [mm] and 100 [mm], respectively. The thickness of wall equaled to 1 [mm] or 0.5 [mm] (Fig. 1). The properties of isotropic and orthotropic materials for elastic range and elastic-plastic range for isotropic material used in the calculations are shown in Tab. 1 and Tab. 2, respectively. For the one-layer composite structure is assumed that the principal directions of orthotropy coincide with longitudinal edges of the column. In this case, material properties of plates are described by:  $E_x$ ,  $E_y$  – Young's moduli along the longitudinal axis of column and in lateral direction, respectively,  $\nu_{21}$ ,  $\nu_{12}$  – Poisson's ratio,  $G_{12}$  – shear modulus in plane of each plate.



**Figure 1** The dimensions of the columns and direction of the moduli

In the further part of work it assumed isotropic material with bilinear characteristics for a different yield stress and for different tangential modulus in elastic-plastic range given in diagrams. For purpose of the simulation with viscoplastic material it was considered the Perzyna model [15, 16] simply described through eq. 1, where  $\dot{\epsilon}^p$  is the strain rate,  $m$  and  $\gamma^*$  mean the constants,  $\sigma_o$  a static yield stress. For the ductile steel, Jones [6] suggests  $m = 0.2$  and  $\gamma^* = 40.4$  and such values have been taken into account in this work.

$$\sigma = \sigma_o \left[ 1 + \left( \frac{\dot{\epsilon}^p}{\gamma^*} \right)^m \right] \quad (1)$$

**Table 1** Material properties for an elastic range

Material property	$E_x$ [GPa]	$E_y$ [GPa]	$\nu_{yx}$ [-]	$G_{12}$ [GPa]	$\rho$ [kg/m <sup>3</sup> ]
mat_1	200	200	0.3	80	7800
mat_2_1	29.523	97.423	0.3	11.818	2000
mat_2_2	97.423	29.523	0.09	11.818	2000

**Table 2** Properties of steel in elastic-plastic range

Young's modulus $E$ [GPa]	E = 200					
Poisson's ratio $\nu$ [-]	0.3					
Density $\rho$ [kg/m <sup>3</sup> ]	7800					
Initial yield stress $s_0$ [MPa]	200	300	400	500	600	800
Tangential modulus $E^t$ [GPa]	20 or 10					

### 3. Numerical simulation

Discrete models of the considered girders are presented in Fig. 2b, which shows the element mesh and the boundary conditions. Along the edges of one end of the girder, zero values of displacement along all directions were adopted (restraint), while at nodes situated at the opposite end of the girder, a load corresponding to the torsional moment was applied, which was implemented by uniform distribution of the forces applied at the nodes along the free edges within a cylindrical coordinates system adopted for these nodes. To ensure that the edges of the girder are straight, a transverse plate was mounted to the model in the plane in which the load was applied, which was several times thicker than the other girder walls. The pulse loading of the torque was rectangular with its duration equaling to the fundamental natural vibration period of the given structure. The calculation was conducted for transient analysis with implicit method of the equations integration.

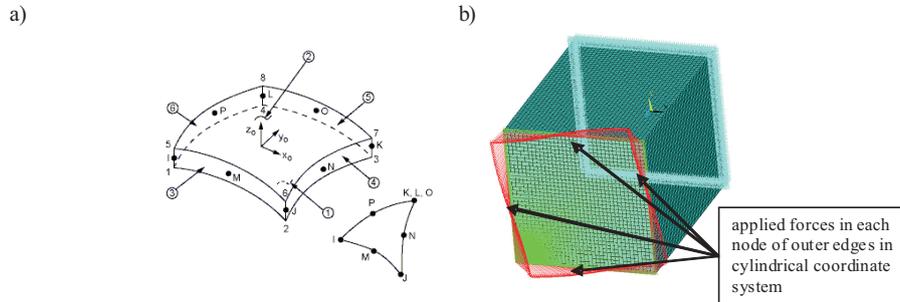
In numerical simulations, it has taken an eight-node element type shell281 with six degrees of freedom at each node (Fig. 2a). This element is well-suited both for linear and particularly for high-rotation or high-strain nonlinear applications.

### 4. Results of numerical simulation

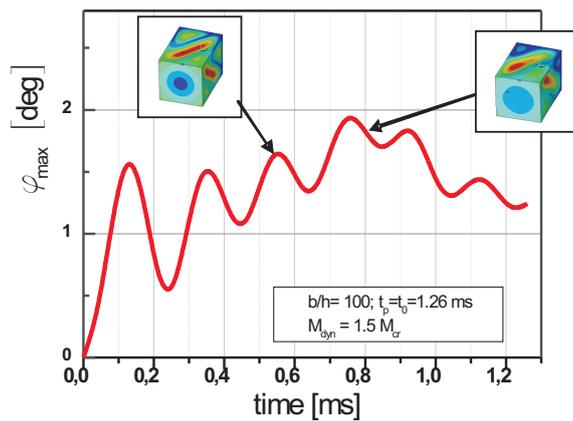
The most of the diagrams presents the relationship between the maximum angles of rotation of the free end of columns obtained in the entire range of the loading as a function of the torque. It seems that this approach of evaluation of dynamic response can be the most reliable. With regard to the lack of proper criterion of dynamic stability of plate structures in torsion, author assumed that column can endure the maximal value until the angle of rotation suddenly increases. On the plots magnitudes mean:

$t_0$  – a period of natural flexural vibrations of given structure,

$t_p$  – pulse duration,



**Figure 2** Drawing of element type shell281 and numerical model with loading and boundary conditions



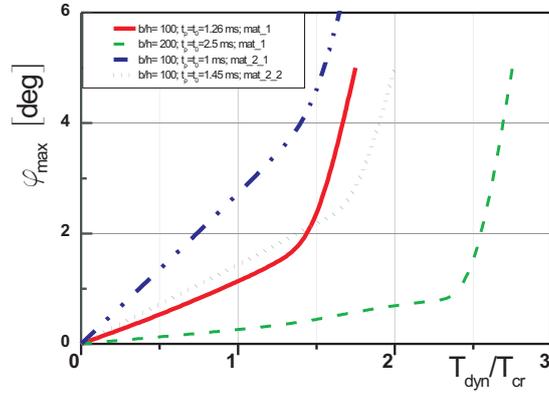
**Figure 3** The course of torsion angle dependent upon the duration of the dynamic torque equaling  $1.5 M_{cr}$

$M_{cr}$  – critical static torque,

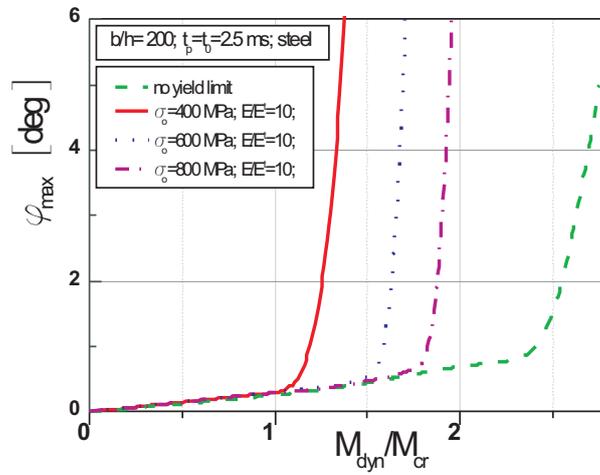
$M_{dyn}$  – dynamic torque or amplitude corresponding to considered structure,

$\varphi_{max}$  – maximal rotation angle under torsional dynamic loading.

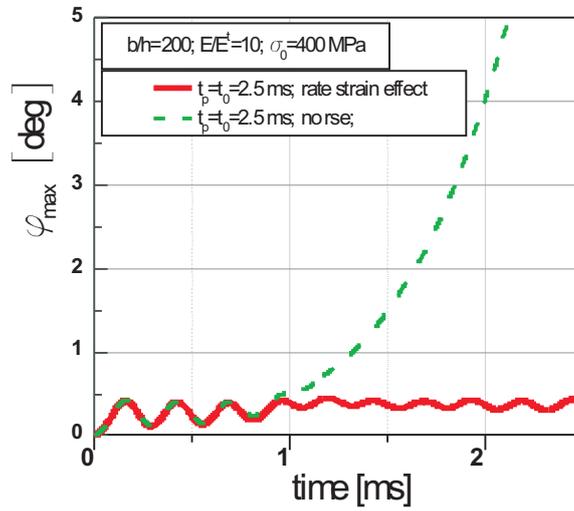
Fig. 3 presents a exemplary course of the angle of rotation of a girder as a function of time of action of the torsional moment at an overload of  $1.5 M_{cr}$ . The curve of the angle changes depending on the dynamic loading, and for low moments (below  $1M_{cr}$ ) the highest angle appeared at  $0.2 \div 0.3 t_p$  of the time of application, while after increasing the amplitude the maximum value was observed at  $0.6 \div 0.8 t_p$ .



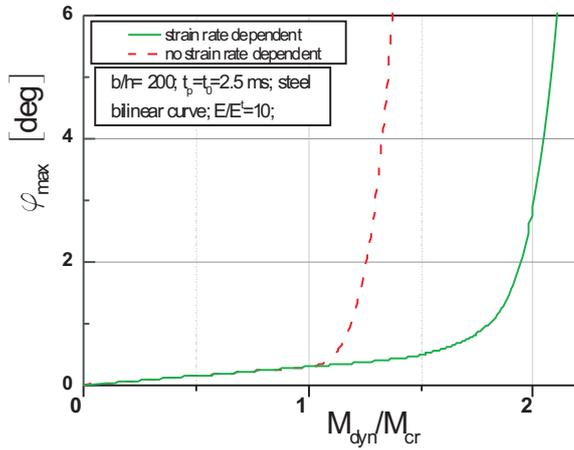
**Figure 4** The courses of maximal torsion angle dependent upon the dynamic torque for isotropic and orthotropic material in elastic range



**Figure 5** Course of maximal torsion angle dependent upon the dynamic torque for isotropic material with bilinear characteristic for different yield stress



**Figure 6** Comparison of curves of maximal torsion angle dependent upon duration of pulse loading with or without consideration of strain rate effect for  $M_{dyn} = 1.5M_{cr}$

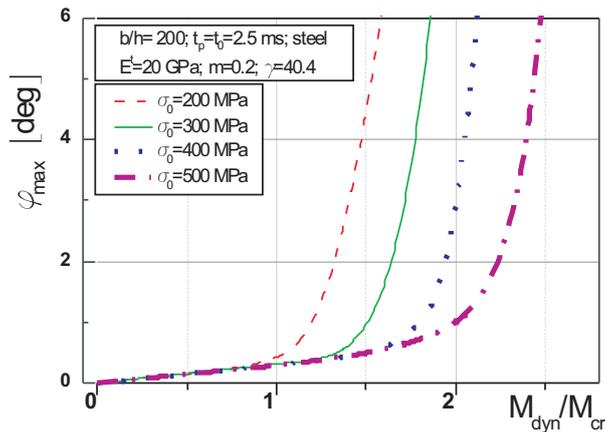


**Figure 7** Comparison of curves of maximal torsion angle with regard to torque with consideration or without consideration of strain rate effect

In Fig. 4, it was observed that the thinner structures carry the greater dynamic torsional loading with respect to its critical loading (even over two times for the thickness  $t = 0.5$  [mm] and about  $1.4 M_{cr}$  for girder with thickness  $t = 1$  [mm]). For moduli along the  $z$ -axis that were higher than those along the transverse direction, the structure may bear the greater dynamic loads (a sharp increase in the angle of the rotation appears at  $1.7 M_{cr}$ ). The next diagram (Fig. 5) shows the influence of the initial yield stress on the maximal angle of the rotation for the column with  $b/h=200$  and  $E^t/E = 10$ , where  $E^t$  means tangential modulus in elastic-plastic range for steel. Obviously, for the greater yield stress of material the structures can endure the greater torque. For steel with  $R_e = 400 MPa$ , the rapid growth of angle rotation in the column appeared at  $1.2 M_{cr}$ .

In Fig. 6 and Fig. 7, diagrams show a response difference of the column with the consideration of the strain rate effect or without. In the first plot it seems that curves are alike till the velocities of strain are small (in the initial stage) but for the further courses diverge (Fig. 6). Taking into account the viscoplastic material constitutive law (Fig. 8) indicates that structures can bear significantly the greater loading in the torsion as well as presented in works [11, 12] under other kinds of loads (about 80 [%]).

Taking a look at the all diagrams it can be observed the similarity between curves for the girders with consideration of SRE at  $R_e = 500 MPa$  (Fig. 8) and curves obtained assuming the only elastic range of material (Fig. 4).



**Figure 8** Curves of maximal torsion angle in the function of torsional pulse loading with consideration of the strain rate effect

## 5. Final remarks and conclusions

The paper presents the results of numerical calculations for girders subjected to dynamic torsion. Analysis was conducted through the transient application of loading to a given structure and the determination of its behavior throughout the pulse duration. In case of torsion of the plate structure, the criteria for determination of critical dynamic loading have not been clearly given in the literature, so here it was assumed the critical dynamic torque is reached when the angle of the rotation grows rapidly.

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