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Selected Problems of Determining Critical Loads in Sructures with Stable Post–Critical Behaviour

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This paper presents selected cases of inapplicability of theory based methods of determining critical loads in thin – walled, composite tubes. 8^{th} layered composite tubes with square cross-section were being subjected to static compression and in order to register experimental data two measuring equipment were employed: strain-gauges and Digital Image Correlation system ARAMIS ®. When measurement data were collected five different theory based methods were applied in order to determine critical loads. Cases where it was impossible to apply certain methods or some doubts about correctness of the results occurred were presented and analyzed. Moreover in cases where it was possible, the theory was equivalently transformed, in such a way to fit experimental data and calculate the critical loads.

Keywords: buckling experiments, critical load determinations, stability investigations, strain–gauges, Digital Image Correlation.

1. Introduction

Problems of structural stability are commonly known since more than two ages. The first scientist who described problem of buckling was Swiss mathematician Leonard Euler [12] in XVIIIth century. However this research field had to wait almost two centuries for expansion of analytical, numerical and experimental methods. Investigations of buckling phenomena, especially experimental, started to be very important with the growth of industry where the need for new designs and materials is very visible. This problem refers to many branches of mechanics such as automotive, aircraft or medical industries [4], [21], [22], [25]. Design in these fields requires to produce very light and high strength structures in order to provide appropriate maintenance. Metals usually provide high strength but are very heavy and cannot be used in form of solid blocks. The response to this requirements is to design thin – walled structures [19], [27], using alternative materials such as composites [13] or combining both [17]. The main advantage of applying thin – walled structures in the different types of designs is reduced weight of the whole system. However designing in such a way must be done very carefully, especially in terms of the structural analysis. In case of compression or bending the phenomenon of buckling can be very dangerous. This forces engineers to conduct appropriate calculations using numerical methods [18] or Finite Element Method [11]. The second method of decreasing the weight of the structures is applying innovative materials such as composites. The main advantage of using this types of materials is huge adjustability of material properties only by means of proper layer arrangements [7]. The disadvantages are very high costs of manufacturing processes which will provide high quality and predictability of future material properties [3], [5]. Designing thin – walled composites combines all advantages and disadvantages of composites and constructions with thin walls what provides big amount of possibilities for potential, future applications. High adjustability of pre- and post-critical stiffness causes that composites can be used as carrying elements (high stiffness) or energy absorbers (low stiffness) [7], [15], [29]. Thin – walled composites are also commonly used as stiffeners in different aerospace structures – especially Fibered Metal Laminates (FMLs) [2], [15] are commonly used for stiffening wings or body in aeroplanes. But before the elements will be mounted in the place of proper usage they have to be tested by means of verification of the appropriate numerical models with series of proper experiments [8], [11]. Experimental buckling analysis, especially for the structures with stable post-critical states, it is always challenging task [9],[10]. This comes from the fact that there is no standard algorithm for determining critical loads but there exist certain amount of theory–based methods [24],[26], which may give different results or in some cases of experimental data sets are not applicable. This paper presents how to deal with some cases of inapplicability of theory based methods for determining critical loads for stable post–critical states. The literature survey indicates some papers dealing with problems with experimental data such as Paszkiewicz & Kubiak [23] for compressed and bent C–shaped composite profiles but certain problems are still not emphasized which authors had to deal with [8].

2. Experimental investigations and measuring equipment

Experimental tests have been performed to determine buckling load and postbuckling behavior of thin–walled composite tubes with square cross – section. The exact numerical and experimental results were presented in the paper of the authors [8]. However this paper is focused on some problems with buckling load determinations and it has been decided to explain how the experimental test were performed. The tubes were produced from eight layers of pre–impregnated tape consisting of E–type unidirectional oriented glass fibers and thermosetting epoxy resin (SE70 Gurit) and subjected to autoclaving process. The length of the column was equal to 250 mm. The dimensions of the cross–section (width $\times$ height $\times$ thickness of the wall) were: $82 \times 82 \times 2$ mm (see Fig. 1 a) and b)).

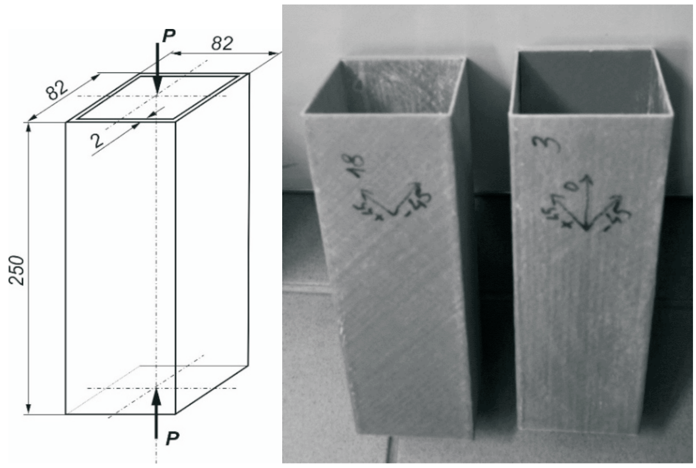

Figure 1 a) nominal dimensions of the tube b) photo of real tubes

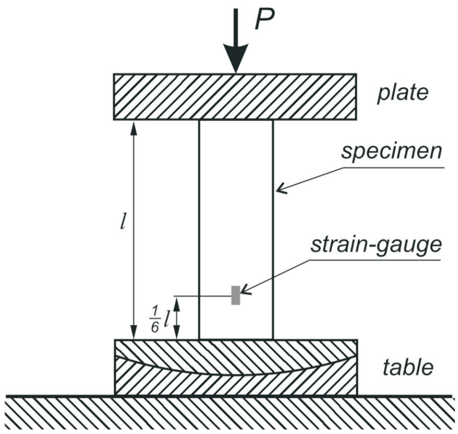

Figure 2 Scheme of applied load and placement of strain – gauges on the tubes

Tubes were subjected to static compression using universal test stand machine INSTRON after Zwick Roel modernization. In order to determine the critical loads and equilibrium paths two measuring systems were employed. The first measuring equipment were strain–gauges glued back to back on one of the walls of the tube (see Fig. 2) at the level of 1/6 of length of the tube – the place was chosen on the basis of the numerical calculations where maximal amplitude of the deflection should appear. The second measuring equipment used for investigations was Digital Image Corellation System ARAMIS ® produced by GOM company. Two neighbouring walls of the specimens were painted and during measurements the photos with frequency of 1 Hz were captured by two cameras what afterwards, thanks to stereographic transformations [14], [20], enabled to create the map of deflections and create the equilibrium path plot for any point of the mapped area.

The tubes were placed in the test machine between two parallel plates – the lower plate was on the spherical bearing in order to provide uniform load distribution on the edges. The traverse beam of the machine was moving with constant velocity of 2 mm/min what was treated as quasi-static compression. In total 24 specimens were investigated – 4 specimens in 6 layer groups [8].

3. Theory based methods of determining buckling load for stable post–buckling states

The structure of thin – walled composite tube with square cross – section had always stable post – equilibrium behavior so in order to calculate the critical loads the criteria for this type of behavior have to be applied. Buckling loads in case of these investigations were determined using five methods which operating principles will be briefly reminded in the subchapters. For the data coming from strain–gauges glued back to back it is assumed that difference in strains is proportional to the deflection of the wall.

3.1. *Inflection point method (IPM)*

The inflection point method had been firstly proposed by Coan [6] and bases on the fact that structure with not excessive geometric imperfections (in comparison with thickness of the wall) should be similar to this as in the Fig. 3. The inflection point corresponds to the critical load.

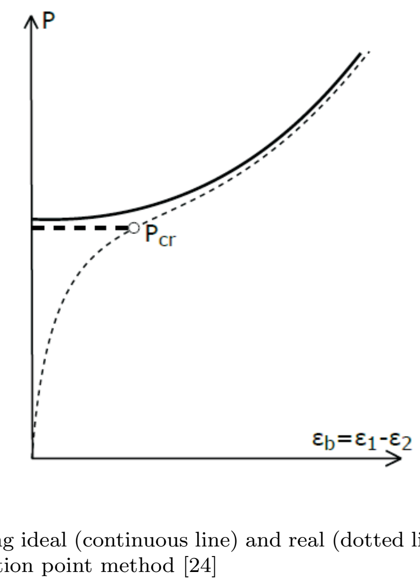

Figure 3 Plot representing ideal (continuous line) and real (dotted line) behavior of the structure and applicability of inflection point method [24]

3.2. *P–w or Koiter's method*

According to the theory of plates and shells [27], [28] and energy approach, for the compressed plate, force is a quadratic function of deflection of the wall hence post–buckling state can be approximated by second order polynomial. For the ideal structure, without initial imperfections, the minimum of the parabola should be placed on the load–axis (b – coefficient equal to 0) and correspond to the buckling load. For non – ideal structure the minimum of the parabola should be taken as the critical load (see Fig. 4).

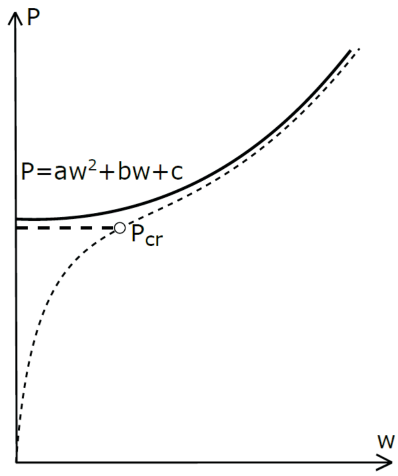

Figure 4 Determining critical load using Koiter's method [24] – the behavior of ideal structure is represented by continuous line, the real by dotted one

3.3. *P–w^2 method*

As it was mentioned in the previous chapter the post-buckling state can be approximated by second order polynomial on the load – deflection diagram. This means that if deflection will be transformed into quantity of squared deflection post – buckling state should become the straight line (see Fig. 5). The intersection of the linear approximation of post – buckling range with load axis is treated as the critical load.

3.4. *Averaged strain method (AVS)*

Averaged strain method can be only applied to the strain – gauges measuring data. The operating principle in this method is to present averaged strain curve on the load strain – diagram and therefore to divide it into the pre- and post – buckling state, linearly approximate it and find intersection of these two lines (see Fig. 6). The intersection of this two lines corresponds to the critical load.

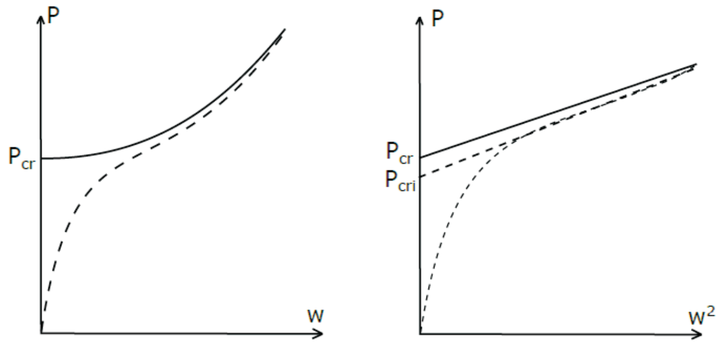

Figure 5 a) load - deflection diagram b) load - squared deflection diagram for ideal (continuous lines) and real (dotted lines) structure and applicability of $P - w^2$ method [24]

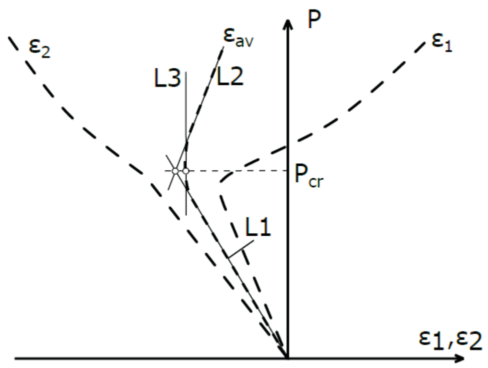

Figure 6 Strains (ε_1 and ε_2), averaged strain (ε_{av}) curves and methodology of finding critical load using averaged strain method (lines L1 and L2) and vertical tangent method (line L3) [24]

3.5. *Vertical tangent method (VTM)*

Vertical tangent method, similarly to the averaged strain method, can be only applied to the strain – gauges measurement data. The averaged strain curve is plotted on the load – strain diagram. Therefore the vertical tangent, if exists, is searched (see Fig. 6).

4. Robustness and selected limitations of applicability

In this chapter selected cases of inapplicability of methods presented in chapter 3 will be presented. Experimentally obtained equilibrium paths, where it was impossible to determine critical load using these methods directly or it was impossible are presented. In the first case – direct inapplicability of methods, the suggestions how to equivalently transform the theory in order to fit the experimental data are presented.

4.1. *Inflection point method*

4.1.1. *Order of polynomial*

The author experience in this field had shown, that order of polynomial used to approximate the region where inflection point should be found can strongly affect the determined critical load. Although the elastic energy of the plate can be expressed by fourth order polynomial equation with deflection as variable, what implies that force can be expressed as third order equation with respect to the deflection, approximating of the experimental curve can be done using higher order polynomials. Using higher order polynomials is explainable due to the fact that the researcher want to have the best mathematical description of the curve fitting to all experimental data what explains correctness of this approach. According to the theory, the lowest order of polynomial, at the same time allowing to find inflection point, is equal three. At the same time, this region can be approximated by higher order polynomials – in this case authors were considering orders of polynomials from three to six (see Fig. 7).

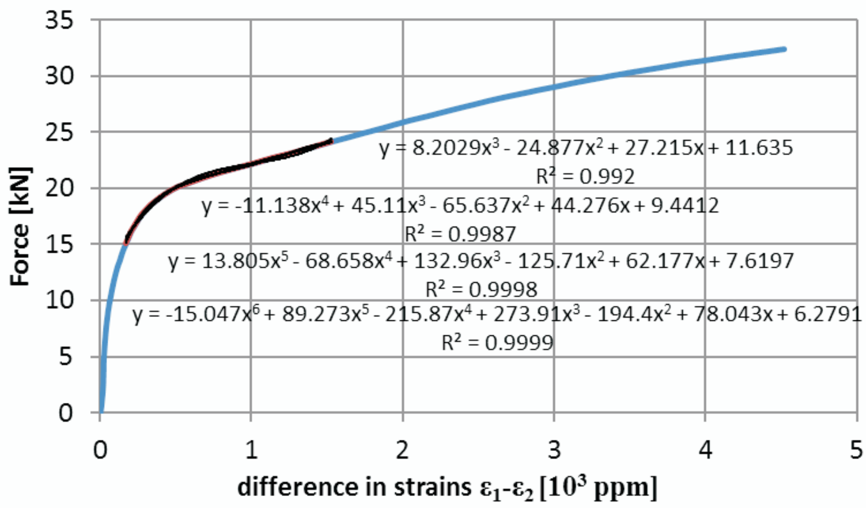

Figure 7 Determining critical load from strain–gauge data using inflection point method by approximating fragment of the curve using different order polynomials

For all cases the value of determination coefficient R^2 was higher than 0.99 what, according to the theory, indicates very good fitting of polynomial approximation to the experimental data. It can be also noticed that for higher order polynomial approximations the value of determination coefficient grows.

Third order polynomial:

$$y = 8.2029x^3 - 24.877x^2 + 27.215x + 11.635 \tag{1}$$

gives one inflection point:

$$(x; y) = (1.010; 22.194) \tag{2}$$

Fourth order polynomial:

$$y = -11.13x^4 + 45.11x^3 - 65.63x^2 + 44.27x + 9.441 \tag{3}$$

has two inflection points:

$$\begin{aligned} (x_1; y_1) &= (0.803; 21.403) \\ (x_2; y_2) &= (1.222; 23.036) \end{aligned} \tag{4}$$

Fifth order polynomial:

$$y = 13.8x^5 - 68.65x^4 + 132.9x^3 - 125.7x^2 + 62.17x + 7.619 \tag{5}$$

has three inflection points:

$$\begin{aligned} (x_1; y_1) &= (0.762; 21.207) \\ (x_2; y_2) &= (0.912; 21.794) \\ (x_3; y_3) &= (1.311; 23.187) \end{aligned} \tag{6}$$

Sixth order polynomial:

$$y = -15.04x^6 + 89.27x^5 - 215.8x^4 + 273.9x^3 - 194.4x^2 + 78.04x + 6.279 \tag{7}$$

has two inflection points (but it could have up to four):

$$\begin{aligned} (x_1; y_1) &= (1.044; 22.418) \\ (x_2; y_2) &= (1.429; 24.093) \end{aligned} \tag{8}$$

The author decided to consider always the minimal value of inflection point (being inside the investigated interval) coming from different polynomial approximation and present this values in the Tab. 1.

Table 1 a) nominal dimensions of the tube b) photo of real tubes

Order of polynomial	III	IV	V	VI
Min. result [kN]	22.194	21.403	21.207	22.418

Although the value of determination coefficient R^2 is in all cases very close to 1 for all orders of polynomials (higher than 0.9926) the minimal inflection points are different - for the final value of critical load, the lowest value is taken (5^{th} order of polynomial). Moreover, mean, standard deviation and absolute difference between highest and lowest results are equal, respectively:

$$\begin{aligned} &\bar{x} = 21.806 \\ &\sigma = 0.590 \\ &|x_{\max} - x_{\min}| = 1.211 \end{aligned} \tag{9}$$

The ratio of absolute difference and mean is equal to 5.55% what indicates that using different order polynomials does not have significant influence on the results.

4.1.2. *Inflection point does not exist*

The authors experience had shown that inflection point is very robust method of finding the critical load. But, in some cases, it does not work. An example of such a situation, which author had met was lack of existence of inflection point (see Fig. 8). In such a form the inflection point methodology cannot be directly applied.

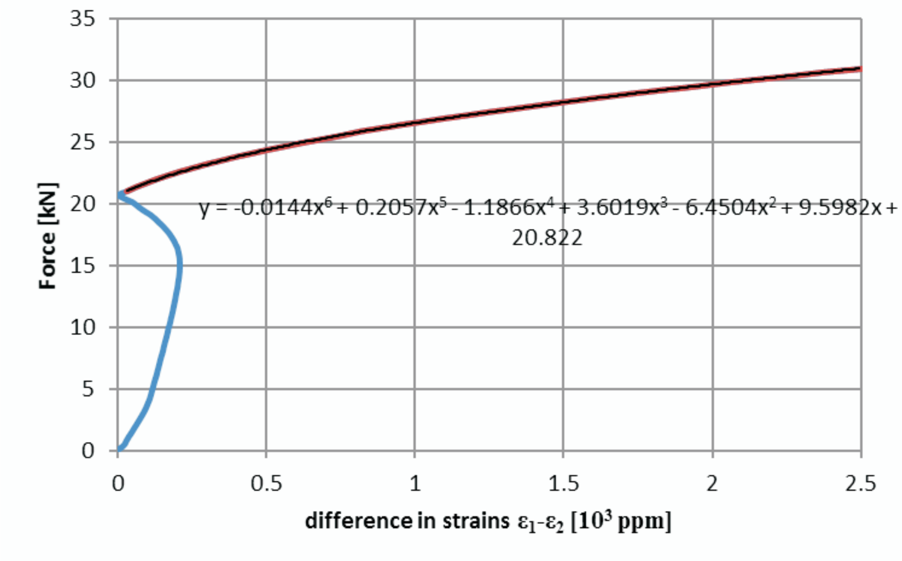

Figure 8 Data series where inflection point method is not applicable

The origin of such a behavior as presented in the Fig. 8 can come from the changing buckling mode during the experiment. For example, initial buckling mode was corresponding to appearance of two halfwaves but under increasing load the jump into three halfwaves occurred. This transition from one buckling mode to another can be treated as a loss of stability. The suggestion of the authors is to approximate the post-buckling state by best fitting to the curve polynomial and find the intersection with the load axis. This point can be treated as the critical load (20.822 kN in the Fig. 8).

4.2. *P–w or Koiter's method*

4.2.1. *Too high flatness of the parabola*

One of the problems connected with applying Koiter's method can be concave parabola with too high flatness which results in the placement of the parabola minimum in other quarter of the coordinate system then equilibrium path is placed (see example in the Fig. 9).

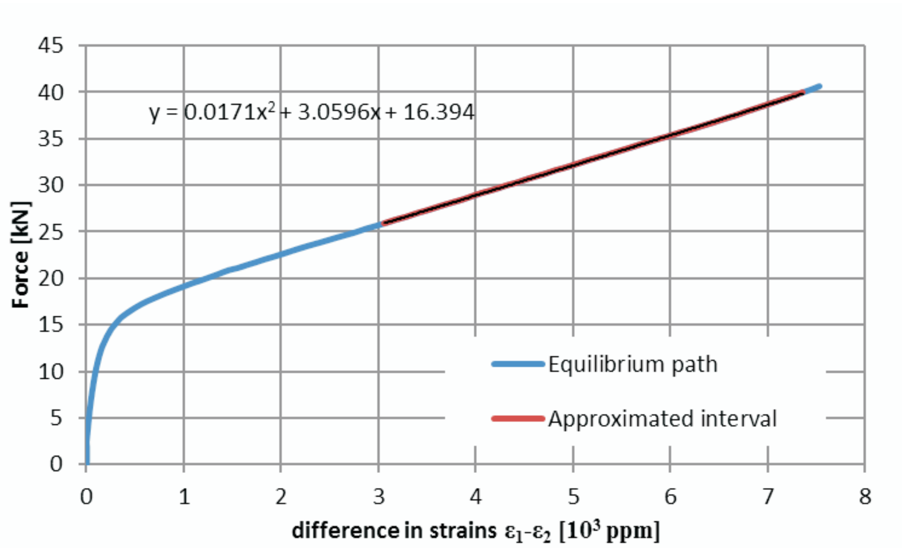

Figure 9 Parabola with very high flatness resulting in the placement of minimum in the third quarter of coordinate system

The polynomial:

$$y = 0.0171x^2 + 3.0596x + 16.394 \tag{10}$$

has got minimum at:

$$(x; y) = (-89.462;\ -120.465) \tag{11}$$

The pair of numbers being a minimum of the parabola does not make physical sense. The point defined by these coordinates is placed in the third quarter of the coordinate system while equilibrium path is in the first one. Hence the value of -120.465 kN cannot be treated as critical load. The suggestion of the authors is to consider as critical load the value of the function at $x = 0$. Therefore searched value of the critical load is equal to c – coefficient of the polynomial. The correctness of this approach can be explained by the fact that for ideal plate structure minimum is placed on the load axis (b – coefficient of the polynomial equal to zero). For any other parabola having the minimum located elsewhere than first quarter it must be noticed that point of intersection with load axis is the lower bound for the point laying on the curve in the first quarter, where critical load should be placed. For presented case the critical load is equal to 16.394 kN.

4.2.2. *Convexity of parabola*

Author's experience in dealing with the experimental data and using Koiter's method had shown that the main problem which occurs is convexity of parabola in post-buckling state. In such a case (see an example presented in the Fig. 10) it is impossible to apply this method. The reason of inapplicability is lack of physical sense of such a behavior. Convex parabola suggests unstable postbuckling behavior while in case of plate structures stable should be considered.

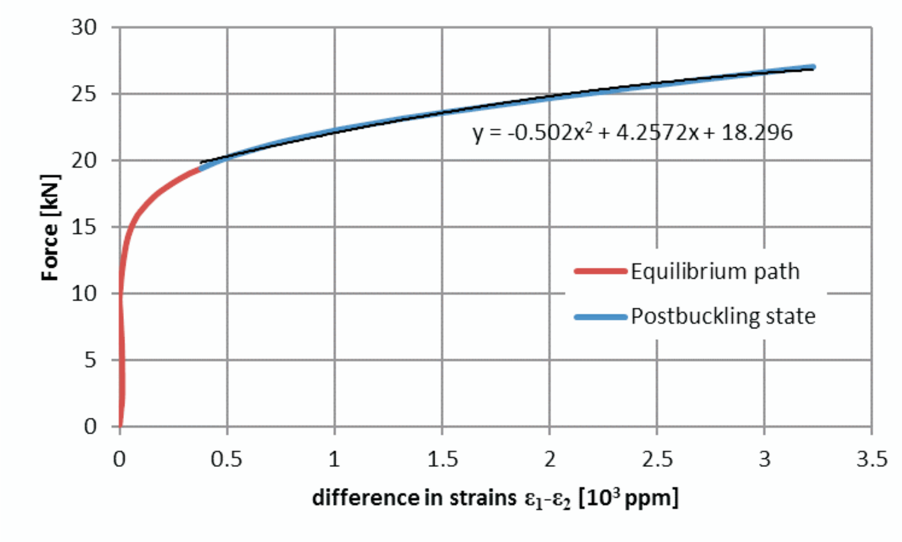

Figure 10 Example of inapplicability of Koiter's method – convex parabola

4.3. *P–w^2 method*

4.3.1. *Different ranges of linear approximation in post-buckling state*

$P - w^2$ method turned out to be very robust method and worked in all cases which author had to deal with. However, the method is very researcher sensitive and result may be affected by the researcher. The problem comes from the fact that in the assumptions of the method it is not clearly said which part of post–buckling state should be approximated and taken under inspection. Such a case is presented in the Fig. 11 a) and b).

According to the first approximation the critical load is equal to 17.399 kN, while according to the second one 14.741 kN. This proves high impact of the human to the result. However, it is worth to discuss, what was an origin of such a behavior. In first analyzed case presented in the Fig. 11 a) under inspection was taken whole post–buckling state, which is a big averaging of whole interval. In the second case, the beginning of the post–buckling state was taken and, indeed lower critical force was obtained. The correctness of choosing second result for further considerations can be explained by means of taking the lower value as a lower bound for critical force.

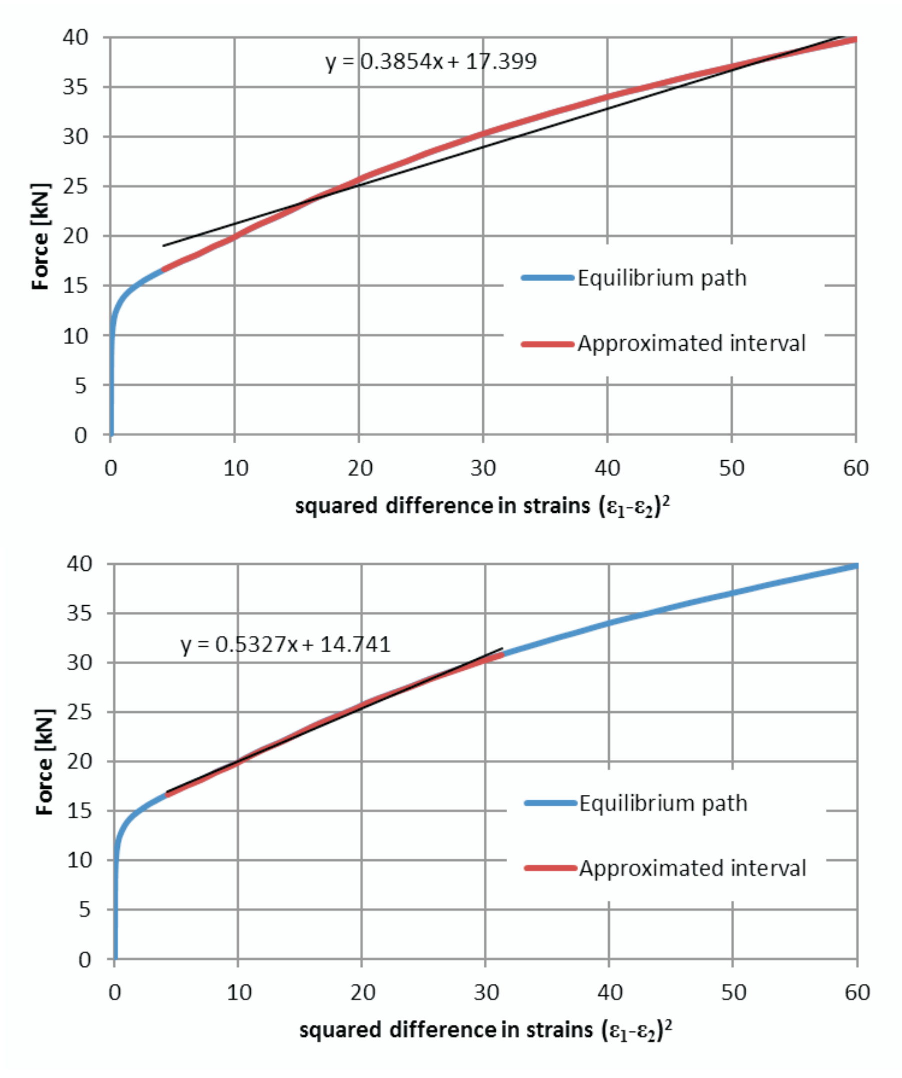

Figure 11 a) and b) Plots representing the same equilibrium path on the load - squared difference in strains diagram with different linear approximations of post–buckling ranges

4.4. *Averaged strain method*

4.4.1. *Impossibility of dividing averaged strain curve into pre– and post–buckling state*

Averaged strain method bases on dividing averaged strain curve into pre– and post–buckling state, making linear approximation of this regions and finding the intersection of this lines. However, sometimes it is impossible to do so as e.g. in the case presented in the Fig. 12. The origins of such a problems may be very different – the investigations could be interrupted in order to avoid destruction of the specimens resulting in post–critical interval being too short to approximate or either strain – gauges could be glued in a place where very high deflections appeared and strain – gauges stopped to register data.

When applying averaged strain method it is always good to plot data series from two strain – gauges. This creates new opportunities. If strain – gauges are glued at the same point of the wall but back to back to each other it means, that during buckling one strain gauge will be under tension and the second one – under compression. Such a situation takes indeed place in the Fig. 12. Up to certain moment both strain gauges (SG-1 and SG-2) are being under compression but when buckling appears SG-1 is under tension and SG-2 under higher compression. This fact is enough to propose the equivalent solution of finding the critical load using averaged strain method. Instead of analyzing one average strain – curve two strain curves can be analyzed separately and afterwards mean value can be calculated. The methodology is presented in the Fig. 13.

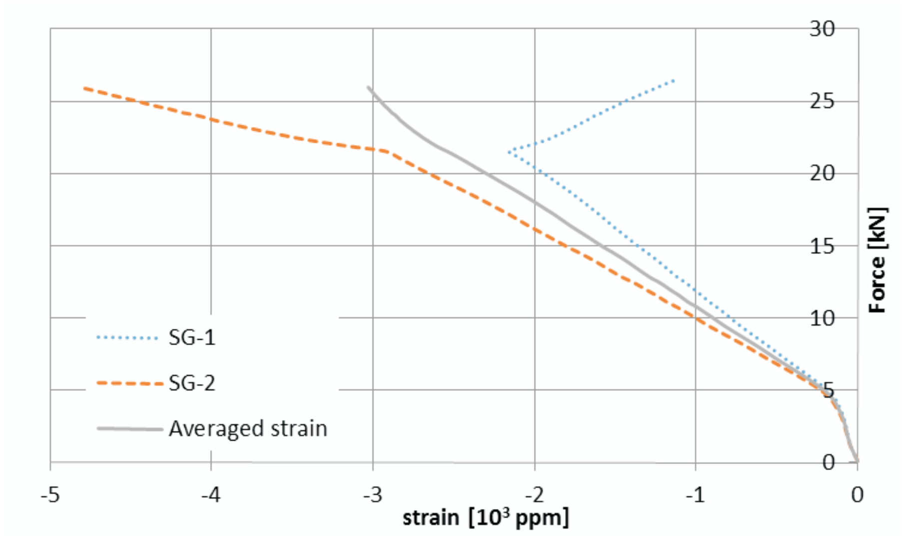

Figure 12 Example of situation when averaged strain curve cannot be divided into pre- and post–buckling state

First strain curve can be divided into pre– and post–buckling state. The equations of linear approximation of these two intervals are:

$$\begin{cases} y = 5.1189x + 32.293 \\ y = -8.567x + 3.3414 \end{cases} \tag{12}$$

and the solution of this set of equations:

$$(x_1; y_1) = (2.115;\ 21.464) \tag{13}$$

The second strain curve gives following set of equations:

$$\begin{cases} y = -2.4706 + 13.932 \\ y = -6.1153 + 3.8471 \end{cases} \tag{14}$$

and solutions are:

$$(x_1; y_1) = (2.767;\ 20.768) \tag{15}$$

Therefore, an average value can be calculated:

$$y_{av} = F_{cr} = \frac{y_1 + y_2}{2} = \frac{21.464 + 20.768}{2} = 21.116 \tag{16}$$

Hence, this value can be treated as a critical load.

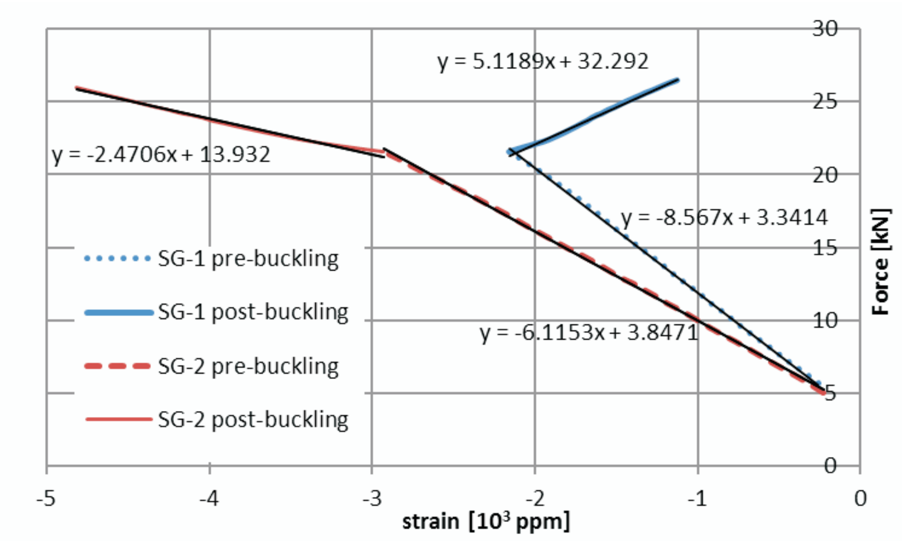

Figure 13 Determining critical load from two strain curves instead of one averaged strain curve

4.5. *Vertical tangent method*

4.5.1. *Vertical tangent line does not exist*

Due to many similarities in two methods – averaged strain and vertical tangent one they can be compared to each other. Under inspection the same case as in the Fig. 12 will be taken. From presented chart it is purely visible that averaged strain curve does not have the vertical tangent line and method is not applicable. The only curve for which vertical tangent can be determined is SG-1, but data from one strain–gauge does not provide small uncertainty level.

5. Comparison of exemplary critical loads obtained from measurement data sets

Under inspection will be taken data series coming from the same measurement but two measuring devices. This allows to evaluate the real critical load.

Table 2 Scheme of applied load and placement of strain–gauges on the tubes

Name of the method	AVS	VTM	IPM 3^{rd} order	IPM 6^{th} order	$P-w$	$P-w^2$	Mean	SD
Strain-gauges	14.58	16.69	16.79	15.85	12.42 (11.43)*	15.89	15.27	1.82
DIC system	-	-	18.98	17.27	14.87 (1.97)*	19.77	17.30	2.45
Strain gauges and DIC system							16.04	2.17

*the value in the bracket is calculated from the minimum of parabola laying else where than first quarter of coordinate system

According to the results presented in the Table 2 some conclusions can be drawn. Critical loads obtained from strain–gauges are giving comparable results using different methods. The ratio of standard deviation to the mean is equal to c.a. 12% what can be treated as satisfactory result. For DIC system results the ratio of standard deviation to the mean was equal to c.a. 14% what means higher dispersion of the results coming from different critical load determination methods. For this particular case it can be also noticed that results coming from DIC system are higher than those from strain-gauges, however it is not a rule that DIC gives higher results than strain-gauges. Moreover the absolute difference between means obtained from two measuring devices is equal 2.03 kN what proves good agreement between critical loads determined from two different measuring devices data. For the $P-w$ method it can be noticed that for both – strain–gauges and DIC system data, critical loads determined using this method are the lowest in comparison with other methods. This considerations can be concluded by stating that it is a mistake to rely only on one method.

6. Conclusions

Investigations of nature of buckling phenomena are very important in the industry. Although the research conducted by the authors was on composites, it can be applied to any other materials and had shown that critical loads in stability investigations should be always determined using different methods and, if it is possible, using different measuring equipment. On that basis the researcher can evaluate the interval where searched critical force should be found and the precision of calculations. The authors had an opportunity to work with many experimental data and can share with some observations concerning the methods themselves. The most robust method – applicable in almost all cases turned out to be $P-w^2$ method, however high impact of the human on the final result is observed. Inflection point method provides also very good robustness except for cases where change of the buckling mode can be observed. $P-w$ method is also very precise however in some cases it cannot be directly applied but after small, equivalent modifications in the methodology it becomes robust. This three methods refer to both: strain-gauges and DIC system data. In case of the strictly applied only to strain – gauges methods averaged strain and vertical tangent ones it can be said that averaged strain one can be used almost always – directly or by considering two strain curves and finding the mean value. When vertical tangent line does not exist the method cannot be applied and data about critical load cannot be determined.

Determining critical loads from experimental data is always challenging task and should be done very carefully. The main problem seems to be a fact that universal algorithm for finding critical loads does not exist. It is a researcher task to evaluate experimental data. This includes estimating the applicability of the method, setting proper ranges of intervals and finally verifying the results. In cases where researcher has got doubts, proper literature with different cases and problems should be read and analyzed. After analysis of many cases the researcher can decide which results is robust or not and evaluate the interval where real buckling load is placed.

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