INTERACTIVE BUCKLING OF FGM COLUMNS UNDER COMPRESSION

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The present paper deals with interactive buckling of thin-walled columns, which are made of functionally graded materials (FGMs). FGMs are inhomogeneous composites made up of two constituents: metallic and ceramic phases. The content of the volume of these two phases changes gradually along the thickness of the structures. Al-TiC metal-ceramic material is applied and these FG structures are subjected to axial compression. The structures are simply supported at both ends. The classical laminate plate theory (CLPT) is employed to obtain the governing equations of the thin-walled FG plate equilibrium. A plate model of the FG column has been adopted in the study to describe global buckling which leads to lowering the theoretical value of load carrying capacity. In order to obtain the differential equilibrium equations of individual plates the Hamilton's Principle for the asymptotic analytical-numerical method was applied and the nonlinear theory of composite plates has been modified in such a way that it additionally accounts for the full Green's strain tensor for thin plates and the second Piola-Kirchhoff's stress tensor in Lagrange's description. The study is based on the numerical method of the transition matrix using Godunov's orthogonalization. The solution method assumed in this study allows for interaction analysis of all buckling modes. In the presented considerations, thermal effects are neglected. The most important advantage of this method is that it enables one to describe a complete range of behaviour of thin-walled FG structures under compression. The authors have found no earlier studies on interactive buckling of FGM structures. One can find papers in which the buckling and post-buckling behaviour of thin-walled elements made of FGMs under compression (e.g., plates [1]) is presented. The nonlinear analysis of this type of elements devoted to basic types of loads is covered in [2]. In paper [3], the nonlinear Koiter's theory has been used to explain an effect of the imperfection sign (sense) on the post-buckling equilibrium paths of FG structures. In the case of FG structure, nonzero first-order sectional inner forces that cause an occurrence of nonzero post-buckling coefficients responsible for sensitivity of the system to imperfections appear. It results in the fact that post-buckling equilibrium paths of plate structures made of FGMs are nonsymmetrically stable. This explains the differences in the plate response dependence on the imperfection sign (sense). Using the classical laminate plate theory (CLPT), the stress and moment resultants (**N**, **M**) are defined as:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{1}$$

where: **A**, **B**, **D** - are extensional, coupling and bending stiffness matrices, respectively, for the FG structure. Due to the presence of the nontrivial submatrix **B**, the coupling between extensional and bending deformations exists. The interactive buckling of thin-walled beam-columns with closed and open cross-sections (i.e. trapezoidal, square, top hat and lip channel) are considered (Fig. 1). An interaction of various buckling modes, that is to say, from a two-mode up to four-mode approach has been assumed in the analysis. Attention has been drawn to the effect of the imperfection sign (sense) and the nonsymmetrical stable post-buckling equilibrium path on load carrying capacity. The differences in the behavior of the analyzed columns can be easily explained by different effects of adjacent walls of the column and by the nonsymmetrical stable post-buckling path for FG structures [3-4] on the assumption that the values of imperfection are the same.

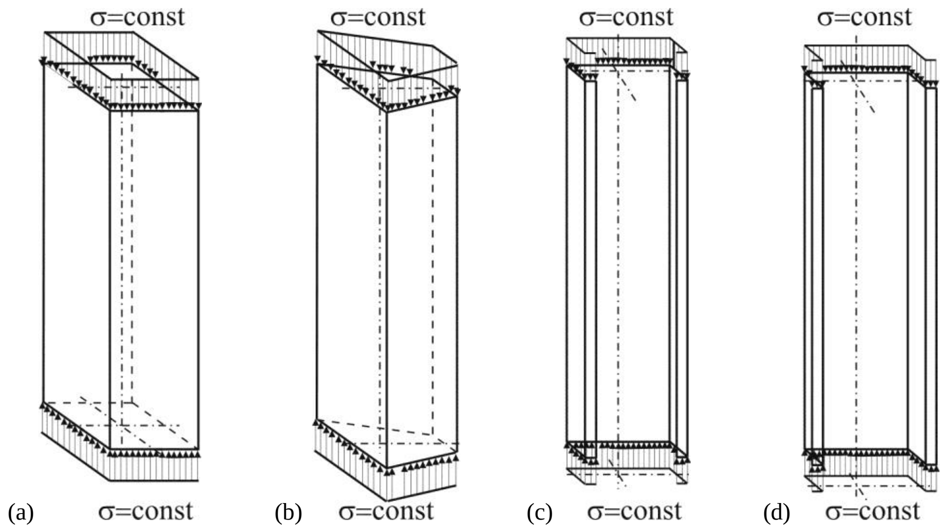

Fig. 1. Thin-walled beam-columns with different cross-sections

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REFERENCES

[1] Jha DK, Tarun Kant, Singh RK, A critical review of recent research on functionally graded plates. Composite Structures 96, 2013, pp. 833–849.

[2] Hui-Shen S. Functionally graded materials - Nonlinear analysis of plates and shells. CRC Press, Taylor & Francis, London, 2009.

[3] Kolakowski Z., Mania R.J., Grudziecki J., Local nonsymmetric post-buckling equilibrium path in thin FGM plate. Eksploatacja i Niezawodnosc/Maintenance and Reliability, 17(1), 2015, pp. 135–142.

[4] Kołakowski Z., Teter A., Static interactive buckling of functionally graded columns with closed cross-sections subjected to axial compression, Composite Structures, 123, 2015, pp. 257–262.