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Application of the Lyapunov Exponent Based on Current Vibration Control Parameter (CVC) in Control of an Industrial Robot

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Controlling dynamics of industrial robots is one of the most important and complicated tasks in robotics. In some works [3, 7], there are algorithms of the manipulators steering with flexible joints or arms. However, introducing them to calculation of trajectory results in complicated equations and a longer time of counting. On the other hand, works [4,5,6]show that improvement of the tool path is possible thanks to the previousidentification of the robot errors and their compensation. This text covers application of Largest Lyapunov Exponent (LLE) as a criterion for control performance assessment (CPA) in a real control system. The main task is to find a simple and effective method to search for the best configuration of a controller in a control system. In this context, CPA criterion based on calculation of LLE by means of a new method [9–11] is presented in the article.

Keywords: Largest Lyapunov exponent, robot control, stability, nonlinear dynamics.

1. Introduction

Controlling system with use of the Largest Lyapunov Exponent (LLE) is employed in many different areas of the scientific research [9–25]. Typical criteria of control performance assessment (CPA) used in control engineering are widely known and described in each automatics and dynamic system systems control basic guide [8]. The main scope of this article is to investigate application of the new method presented in [9–11] as the method that can be implemented to allow the control of the industrial robot dynamics.

At present, industrial robots are used mainlyto handle the material, to weld, to cut or to paint. In these applications not accuracy, but repeatability of the robot which is higher by one order of magnitude, is most important. Fig. 1 presents possible combinations of accuracy and repeatability, typical for industrial robots. Most robotsare programmed by PTP method on–line. They are taught the specific points,which are later repeated for many times when there is a given configuration of the robot.

This method is used in typical tasks. However, robots are starting to work in other industry areas, like milling or drilling with the use of specific software like Robotmaster or PowerMill Robot. In these tasks a total accuracy of the robot is important because accuracy is demanded in a drilling or milling process. What is more, the tool paths are generated off-line. Therefore, it is crucial to identify sources of the errors because they have an impact on the real trajectory of the tool[1, 2]. The accuracy and repeatability of the industrial robots are described by ISO 9283 standard. In some works[3, 7], there are algorithms of the manipulators steering with flexible joints or arms. However, introducing them to calculation of trajectory results in complicated equations and a longer time of counting. On the other hand, works [4, 5, 6]show that improvement of the tool path is possible thanks to the previousidentification of the robot errors and their compensation. The main scope of this article is to investigate application of LLE as CPA criterion using simple method for LLE calculation[9].

2. Current vibration control parameter (CVC) – background of the metod

The basic task for any control system is to minimize error of regulation. Error of regulation is a function of time equal to the difference between value of reference signal $\mathbf{y}(t)$ and output signal $\mathbf{x}(t)$ of the system:

$$\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{y}(t) \qquad \mathbf{e},\, \mathbf{x},\, \mathbf{y} \in \mathbf{R}^m \tag{1}$$

where:

$\mathbf{e}(t)$ – error of regulation,
$\mathbf{y}(t)$ – reference signal,
$\mathbf{x}(t)$ – output signal.

In the presented control method, robot and its control system are treated together as one dynamicalsystem. Generally presented control method bases on the analysis of the robot trajectory disturbance changes $d\mathbf{e}(t)$ in direction of the disturbance vector $\mathbf{e}(t)$ (Fig. 1). Vector $\mathbf{e}(t)$ determines difference between the desirable reference robot trajectory $\mathbf{y}(t)$ and the real measured robot trajectory $\mathbf{x}(t)$. Trajectories $\mathbf{y}(t)$ and $\mathbf{x}(t)$ are analyzed in the six dimensional phase space. That dimensions are constituted by displacements and velocities of the effector in the workspace of the robot. It is expected that regulation error attains small values and tends to zero as quick as possible. We propose CVC measure to determine the regulation system behaviour qualitatively and quantitatively. We consider Largest Lyapunov Exponent (LLE) stability measureas the parameter showing quality of the regulation. LLE shows the average changes of perturbation $\mathbf{e}(t)$. The faster $\mathbf{e}(t)$ is decreasing to zero the better is the reaction of the control system on to the disturbance appearance. As the reference system can be treated as desired behavior of the analyzed system, one can introduce (CVC) parameter in the following way (1):

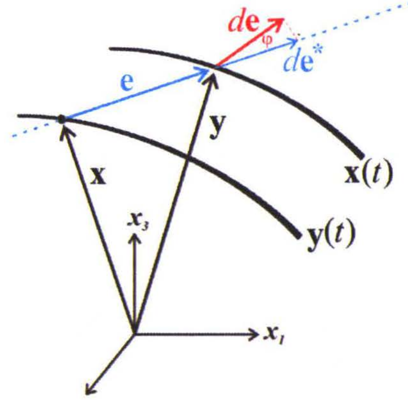

Figure 1 Graphic illustration of the method

While in [38] behavior of e(t) was analyzed with use of the linearized variational equation (2):

$$\frac{d\mathbf{e}}{dt} = \mathbf{U}(\mathbf{y}(t))\mathbf{e} \tag{2}$$

where: $\mathbf{U}(\mathbf{y}(t))$ is the Jacobi matrix in the point $\mathbf{y}(t)$,

in present case one can calculate actual $\mathbf{e}(t)$ and $d\mathbf{e}(t)$ from the behavior of the two real systems $\mathbf{x}(t)$ and $\mathbf{y}(t)$. Thus one can find transformation $\mathbf{U}(\mathbf{y}(t))$ without any knowledge about analysed systems equations.

Analyzing the transformation of the vector ein the direction of the chosen transversal eigenvector $\mathbf{w}^*$ of the $\mathbf{U}$ transformation, it can be written:

$$\frac{d\mathbf{e}^*}{dt} = \lambda^* \mathbf{e}^* \tag{3}$$

After the transformation:

$$\frac{d\mathbf{e}^*}{\mathbf{e}^*} = \lambda^* dt \tag{4}$$

and the integration for $\mathbf{z}^*(0) = \mathbf{z}_0^*$, one obtains:

$$\mathbf{e}^* = \mathbf{e}_0^* \, e^{\lambda^* dt} \tag{5}$$

where:

$\mathbf{z}^*$ – component of the vector $\mathbf{z}$ in the direction of the eigenvector $\mathbf{w}^*$,

λ^* – eigenvalue in the direction of the eigenvector $\mathbf{w}^*$.

Eq. (5) describes a mean transformation of the vector $\mathbf{e}^*$ after time t.

For $t \to \infty$, λ^* is a value of the Lyapunov exponent in the direction of $\mathbf{w}^*$.

In the presented method, Eq. (3) is used for the system stability determination. Using vectors dot product properties (Fig. 1):

$$\mathbf{e} \cdot \frac{d\mathbf{e}}{dt} = |\mathbf{e}||\frac{d\mathbf{e}}{dt}| \cos \varphi \tag{6}$$

After simple transformations:

$$\frac{\frac{d\mathbf{e}^*}{dt}}{\mathbf{e}} = \frac{\mathbf{e} \cdot \frac{d\mathbf{e}}{dt}}{|\mathbf{e}|^2} = \hat{\lambda}^* \tag{7}$$

The average value $\hat{\lambda}^*$ of λ^* is the parameter allowing us to determine stability of the systemalong the direction of the disturbance $\mathbf{e}$. Taking also into account that the direction of the disturbance vector $\mathbf{e}$ is convergent to the eigenvector respective to the largest Lyapunov of the transformation $\mathbf{U}(\mathbf{y}(t))$ in Eq. (2), one can conclude that the obtained value $\hat{\lambda}^*$ is similar to the value of the LLE. That value obtained with use of the procedure given above is the value of the CVC parameter. Introduced CVC parameter allows to control error of regulation $\mathbf{e}(t)$ (1). It is expected that regulation error attains small values and tends to zero quickly. It means at first, that CVC value should be negative. Moreover the lower is the CVC value, the faster $\mathbf{e}(t)$ tends to zero.

3. System overviewMeasuring position

At present, industrial robots are used mainlyto handle the material, to weld, to cut or to paint. In these applications not accuracy, but repeatability of the robot which is higher by one order of magnitude, is most important. Fig. 2 presents possible combinations of accuracy and repeatability, typical for industrial robots.

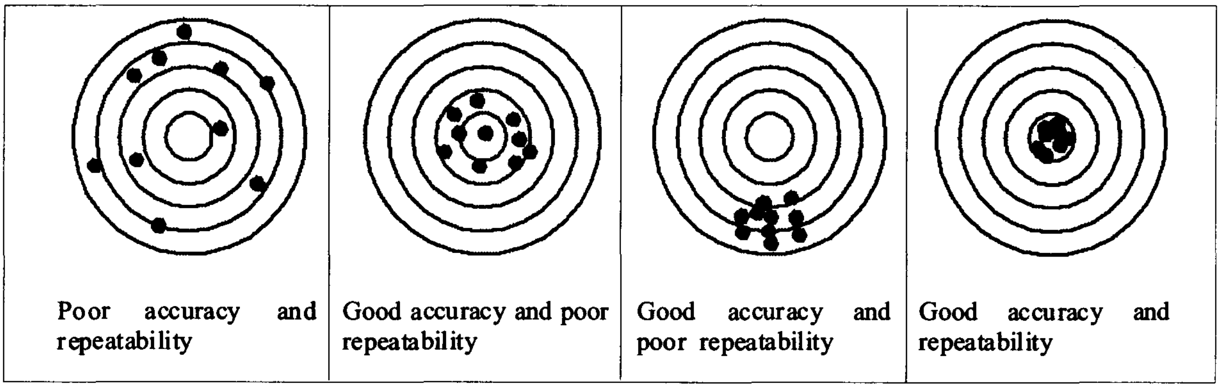

Figure 2 Accuracy versus repeatability [2]

The errors may occur from a number of reasons and their identification enables compensation of the errors in a given area (Tab. 1).

Tab. 2 shows repeatability of the positions and paths of ABB robots. As it is seen, their errors are serious in comparison with typical CNC machines used in industry. It is due to the open kinematic structure of the robot having considerably

Table 1 Causes for accuracy and repeatability

Mechanics	Kinematics	Dynamics
gear errors: backlash, compliance static, dynamic and thermal deformation angle transmission error	link length deviation displacement of axes zero error	dynamic of the real system following error simplified joint controller

Table 2 Position repeatability and Path repeatability for ABB robots

Robot Model	Position repeatability [mm]	Path repeatability [mm]
IRB 1600	0.02 - 0.05	0.06 - 0.19 depending on variant
IRB 2400	0.03	0.11 - 0.15 depending on variant
IRB 6700	0.05 - 0.06	0.06 - 0.14 depending on variant

long arms. Dynamics equations of the robot do not take into account flexibility of joints, etc. The measurements were done on the 6 DOF industrial robot IRB 2400 inInstitute of Machine Tools and Production Engineering of Lodz University of Technology. The robot is not calibrated,equipped with electro-spindlewhich enables mechanical working. A telescopic ball bar QC20°-°W by Renishaw (whose measuring accuracy is 0,5m) with Bluetooth wireless technology was placedin the electro–spindleand used in the tests (Fig. 3). The robot performed a circular interpolation according to the given centre point, taking into account the relevant distance of the radius points which comes from the length of the arm.

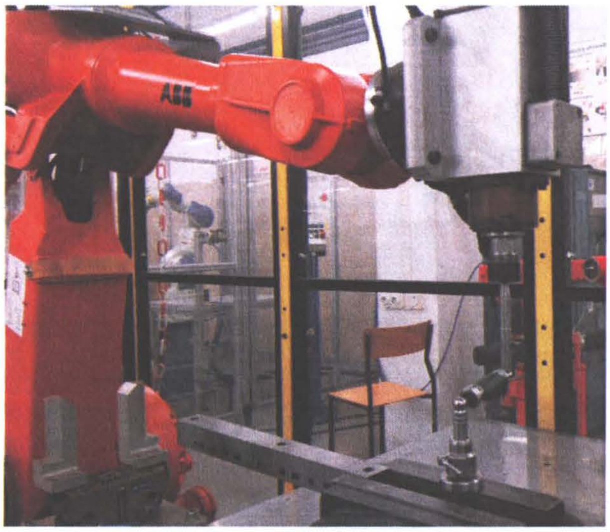

Figure 3 Robot IRB 2400 with telescoping ball bar

4. The results of the research

In the (Fig 4) comparison between the raw measurement data of the error of regulation $\mathbf{e}(t)$ and averaged data in the form of the CVC parameter . While no useful information can be read out from the $\mathbf{e}(t)$ chart, from the CVC chart one can conclude about many aspects of the robot motion. At first one can see that after each perturbation CVC values converge to zero. It means that robot's arm oscillates around the expected position. Moreover, one can see that for each perturbation the way that CVC converges to zero is different. Fthat perturbations were introduced in different robot's positions, or they come from different sources.

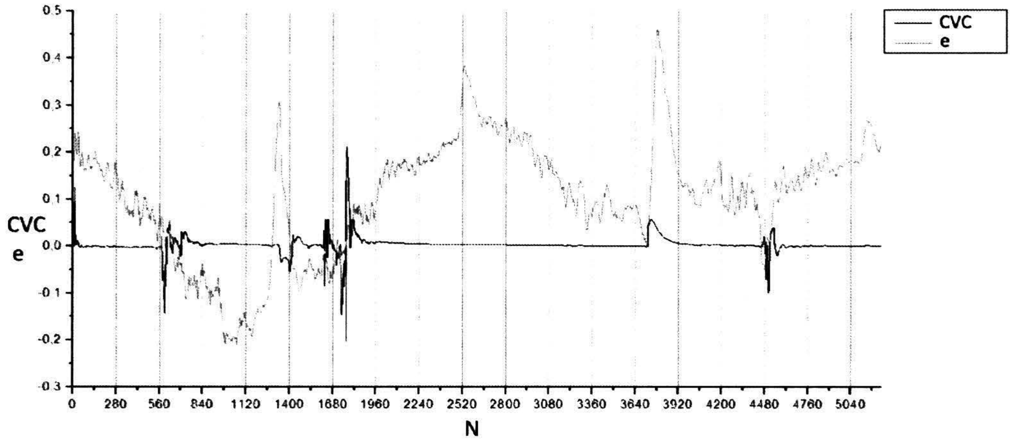

Figure 4 Error of regulation $\mathbf{e}(t)$ and the time – respective CVC parameter.

In the Figs 5 – 14 one can see time series showing the reasons of the disturbances. On each of them one can see the angle positions of chosen links and time-respective CVC values. One can see that perturbations appear in time of the change of direction of some link movement.

Although constructionof the IRB 2400 robot is very complicated, from the presented time series one can conclude that main reasons of disturbances are clearances in the robots joints. Moreover, one can find which joint clearance is responsible for the chosen perturbation and how fast disturbance disappears.

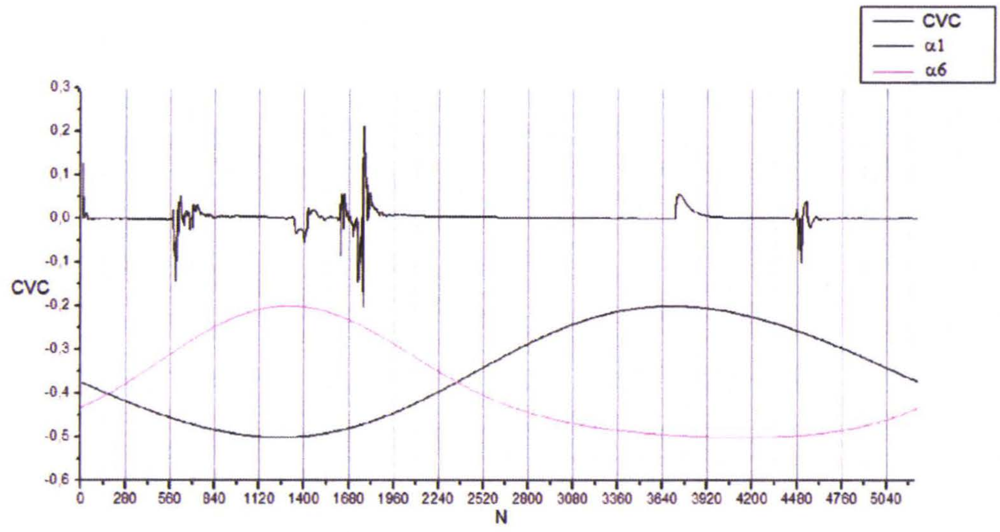

Figure 5 Time series of the links angular displacements and CVC parameter – links 1 and 6

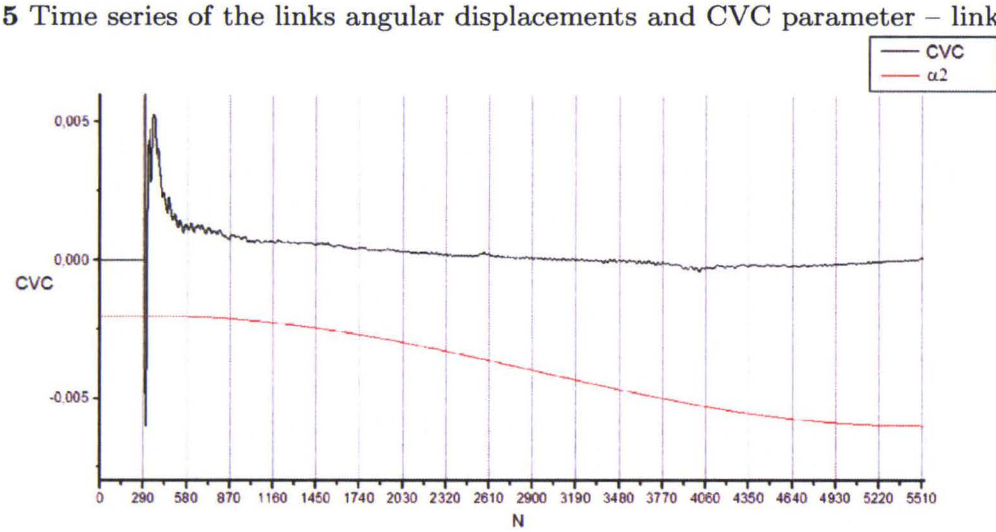

Figure 6 Time series of the links angular displacements and CVC parameter – link 2

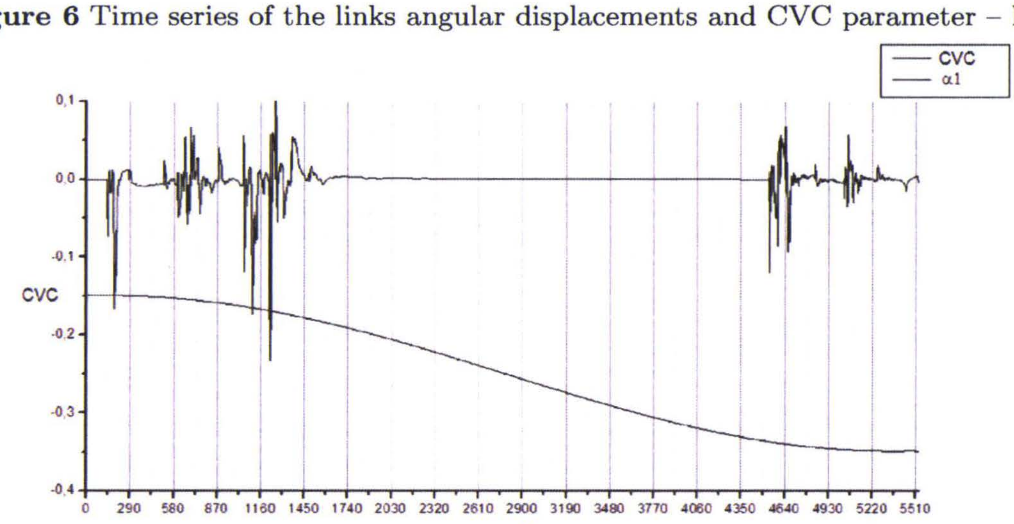

Figure 7 Time series of the links angular displacements and CVC parameter – link 1

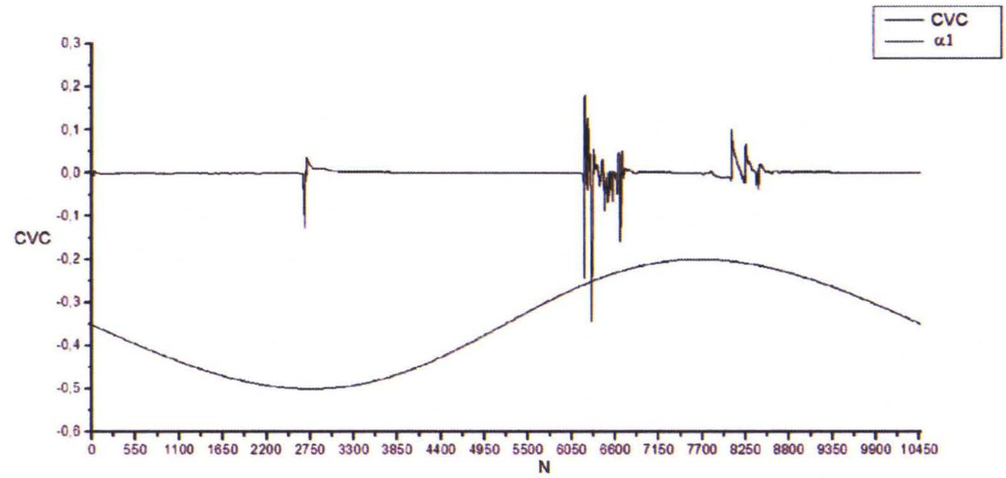

Figure 8 Time series of the links angular displacements and CVC parameter – link 1

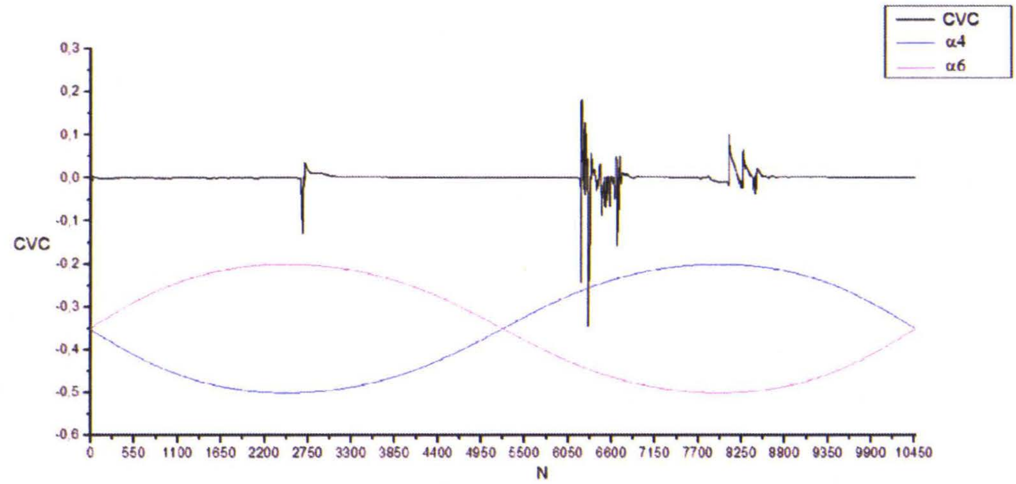

Figure 9 Time series of the links angular displacements and CVC parameter – links 4 and 6

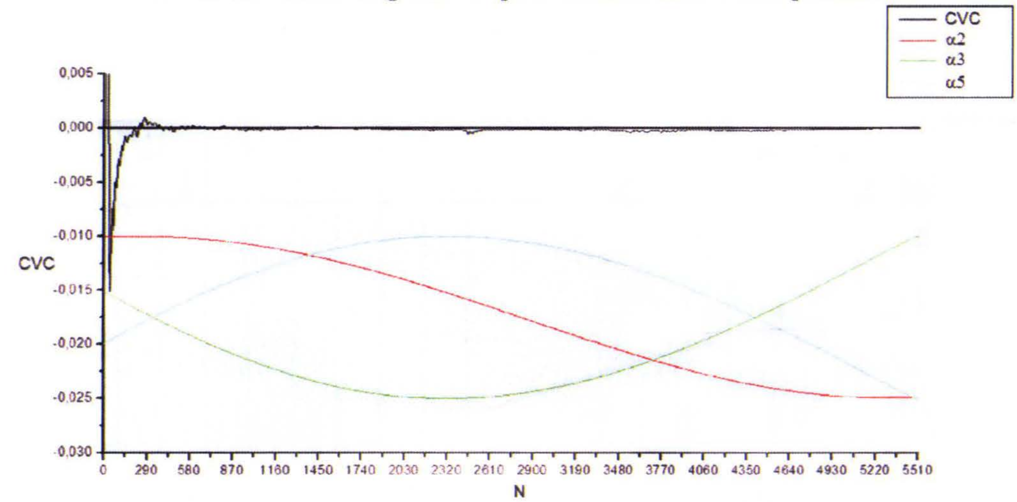

Figure 10 Time series of the links angular displacements and CVC parameter – links 2, 3, 5

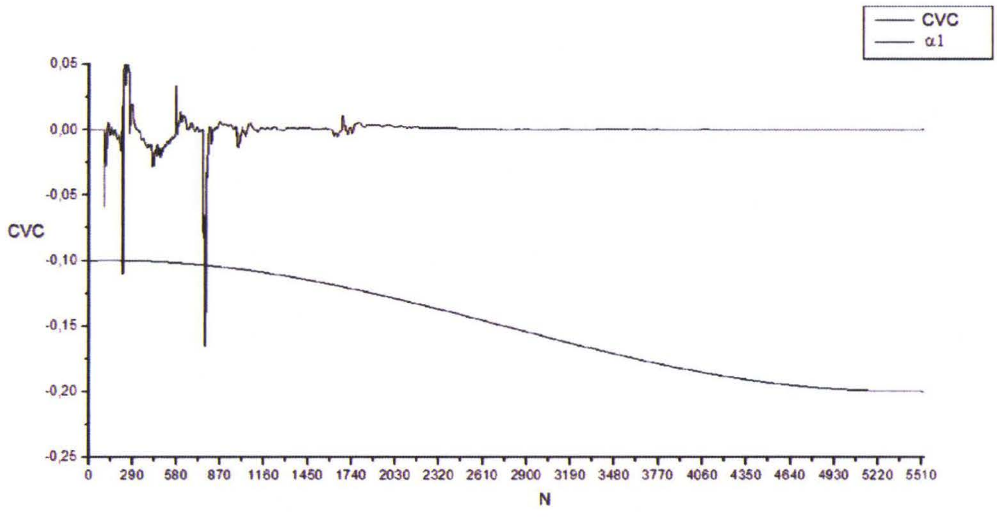

Figure 11 Time series of the links angular displacements and CVC parameter – link 1

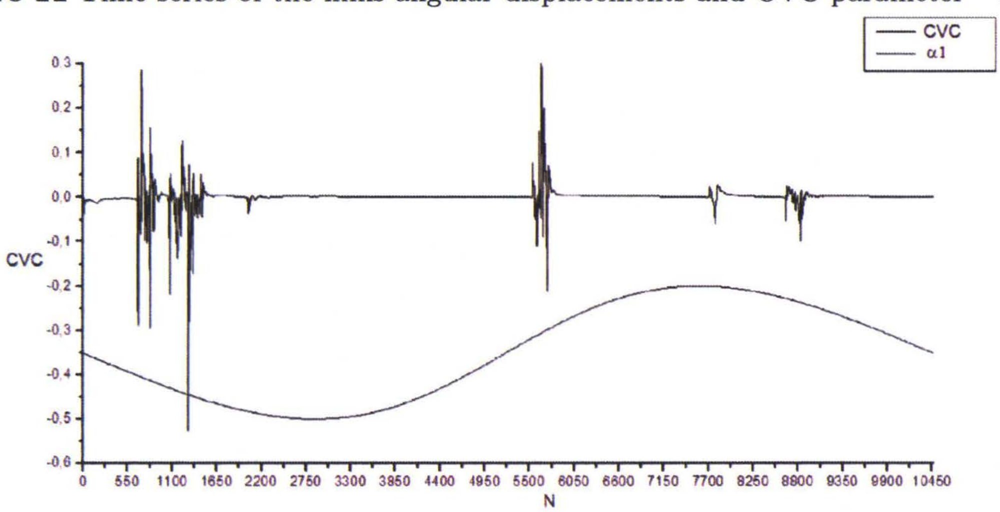

Figure 12 Time series of the links angular displacements and CVC parameter – link 1

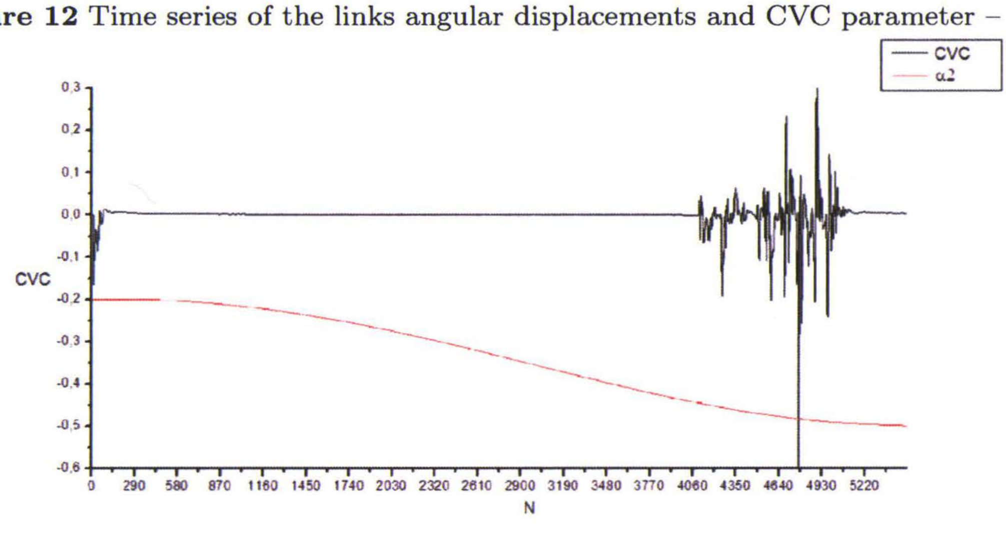

Figure 13 Time series of the links angular displacements and CVC parameter – link 2

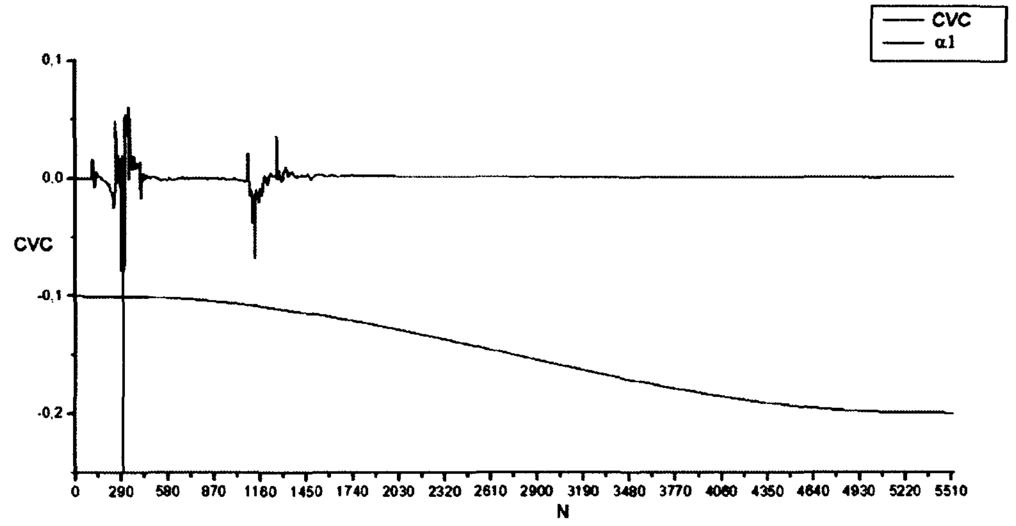

Figure 14 Time series of the links angular displacements and CVC parameter – link 1

5. Conclusions

The main scope of this article was to investigate application of Largest Lyapunov Exponent (LLE) as a criterion for control performance assessment (CPA) in a real control system. It was shown that the method can be implemented to allow the control of the industrial robot dynamics. The next stage of the investigations could be measuring of the CPA convergence to zero value. That double current vibration control parameter (DCVC) could be the quantitative indicator of such dynamical system behavior.

References

[1] **Alban, T. and Janocha, H.**: Dynamic calibration of industrial robots with inertial measurement systems, *Laboratory for Process Automation*, **1999**.

[2] **Shiakolas, P. S., Conrad, K. L. and Yih, T. C.**: On the accuracy, repeatability, and degree of influence of kinematics parameters for industrial robots, *International Journal of Modelling and Simulation*, V. 202, No. 3, **2002**.

[3] **Tchoń,Mazur, K. A., Dulba, I., Hossa, R. and Muszyski, R.**: Manipulatory i roboty mobilne. Modele, planowanie ruchu, sterowanie, *Akademicka Oficyna Wydawnicza PLJ*, Warsaw, **2000**.

[4] **Abele, E., Bauer, J., Pischan, M., v. Stryk, O., Friedmann, M. and Hemker, T.**: Prediction of the tool displacement for robot milling applications using coupled models of an industrial robot and removal simulation, *Proceedings of the CIRP 2nd International Conference Process Machine Interactions*, Vancouver, Canada, **2010**.

[5] **Klimchik, A., Wu, Y., Caro, S., Furet, B. and Pashkevich, A.**: Accuracy Improvement of Robot-Based Milling Using an Enhanced Manipulator Model, *Advances on Theory and Practice of Robots and Manipulators*, 73–81, **2014**.

[6] **Slamani, M., Nubiola, A. and Bonev, I.**: Assessment of the positioning performance of an industrial robot, *Industrial Robot: An International Journal* , 39(1), 57–68, **2012**.

[7] **Moberg, S.**: Modeling and Control of Flexible Manipulators, Linkping, **2010**.

[8] **Dębowski, A.**: Automatyka podstawy teorii, *Industrial Robot: An International Journal* , 39(1), 57–68, **2012**.

[9] **Dąbrowski, A.**: Estimation of the largest Lyapunov exponent from the perturbation vector and its derivative dot product, *Nonlinear Dynamics*, 67, 283–291, **2012**.

[10] **Balcerzak, M., Dąbrowski, A., Kapitaniak, T. and Jach, A.**: Optimization of the Control System Parameters with Use of the New Simple Method of the Largest Lyapunov Exponent Estimation, *Mechanics and Mechanical Engineering*, 17(4), 325–339, **2013**.

[11] **Dąbrowski, A.**: Estimation of the largest Lyapunov exponent from the perturbation vector and its derivative dot product, *Nonlinear Dynamics*, NODY-D-13-01203R3, **2014**.

[12] **Eckmann, J. P., Kamphorst, S.O., Ruelle, D. and Ciliberto, S.**: Lyapunov-Exponents from a time series, *Phys. Rev. Lett.*, 34(9), 4971–9, **1986**.

[13] **Rosenstein, M. T., Collins, J. J. and De Luca, C. J.**: A practical method for calculating largest Lyapunov exponents from small data sets, *Physica D*, 65(1,2), 117–34, **1993**.

[14] **Parlitz, U.**: Identification of true and spurious Lyapunov exponents from time series, *Int. J. Bifurcat. Chaos* , 2(1), 155–65, **1992**.

[15] **Young, L.**: Entropy, Lyapunov exponents, and Hausdorff dimension in differentiable dynamical systems, *IEEE Trans. Circuits Syst.*, 30, 599–607, **1983**.

[16] **Kantz, H.**: A robust method to estimate the maximal Lyapunov exponent of a time series, *Phys. Lett. A*, 185, 77–87, **1994**.

[17] **Kim, B. J. and Choe, G. H.**: High precision numerical estimation of the largest Lyapunov exponent, *Commun Nonlinear Sci.Numer.Simulat.*, 15, 1378–1384, **2010**.

[18] **Stefański, A.**: Estimation of the largest Lyapunov exponent in systems with impacts, *Chaos Solitons Fractals*, 11(15), 2443–2451, **2000**.

[19] **Stefański, A. and Kapitaniak, T.**: Estimation of the dominant Lyapunov exponent of non-smooth systems on the basis of maps synchronization, *Chaos Solitons Fractals* , 15, 233–244, **2003**.

[20] **Stefański, A.,Dąbrowski, A. and Kapitaniak, T.**: Evaluation of the largest Lyapunov exponent in dynamical systems with time delay, *Chaos, Solitons and Fractals*, 23, 1651–1659, **2005**.

[21] **Stefański, A.**: Lyapunov exponents of the systems with noise and fluctuating parameters, *Journal of Theoretical and Applied Mechanics*, 46(3), 665–678, **2008**.

[22] **Yilmaz, D. and Gulerb, N. F.**: Analysis of the Doppler signals using largest Lyapunovexponentand correlation dimension in healthy and stenosed internal carotid artery patients, *Digital Signal Processing*, 20, 401–409, **2010**.

[23] **Gharavia, R. and Anantharamb, V.**: An upper bound for the largest Lyapunov exponent of a Markovian product of nonnegative matrices, *Theoretical Computer Science*, 332, 543–557, **2005**.

[24] **Ronga, H. W., Mengb, G., Wanga, X. D., Xuc, W. and Fangc, T.**: Largest Lyapunov exponent for second-order linear systems under combined harmonic and random parametric excitations, *Journal of Sound and Vibration*, 283, 1250–1256, **2005**.

[25] **Vallejos, R. O. and Anteneodo, C.**: Largest Lyapunov exponent of long-range XY systems, *Physica A*, 340, 178–186, **2004**.