

## Magnetic oscillator under excitation with controlled initial phase

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**Abstract:** The work contains the results of numerical simulations of the dynamics of a magnetic pendulum system subjected to excitation with a varying initial phase. A magnetic pendulum consists of a magnet fixed to the end of its arm and an electric coil below it. The initial phase of the excitation is presented as a linear function of the dynamic variable of the system, which is the angular position of the pendulum. The obtained basins of attraction indicate the occurrence of multi-periodic solutions in the system depending on the changes in the parameters of this system. The periodicities of the observed solutions are quantified mostly by odd numbers.

**Keywords:** pendulum, magnetic field, basins of attraction, controlled excitation, nonlinear dynamics

### 1. Introduction

Magnetic oscillators and their dynamics have been a frequently undertaken research topic in recent years, which is confirmed by the following works [1–3]. The dynamics of such systems is highly nonlinear, which generates surprising behaviour of the system. Following the example of work [4], we decided to check how the dynamics of the magnetic pendulum is influenced by the dependence of the exciting torque on the dynamic variable – angular position. The general equation of motion of the considered system is presented in the following dimensionless form

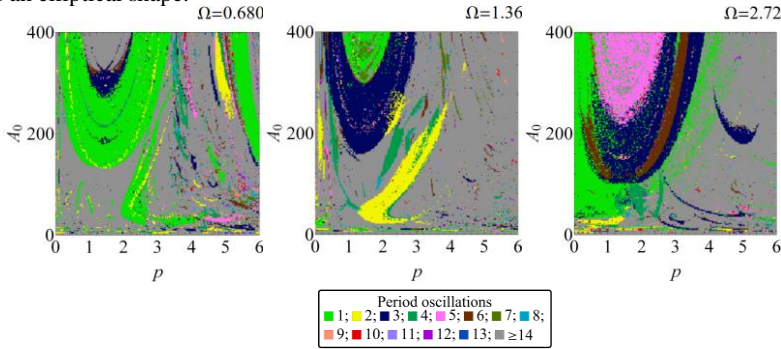
$$\theta'' + \beta\theta' + \alpha\theta + \gamma \sin\left(\frac{1}{\gamma}\theta\right) + \left[\delta + \zeta \exp\left(\nu(\theta')^2\right)\right] \tanh(\sigma\theta') = A_0 \exp(-\theta^2) \sin(\Omega\tau + \phi_0), \quad (1)$$

where:  $\theta$  is the dimensionless angular position of the pendulum,  $\phi_0$  represents an initial phase, and rest of the coefficient, i.e.,  $\beta$ ,  $\alpha$ ,  $\gamma$ ,  $\delta$ ,  $\sigma$ ,  $A_0$  are constant values. The values of these parameters were determined on the constructed experimental setup and are as follows:  $\alpha = 0.323$ ,  $\beta = 0.036$ ,  $\gamma = 7.468$ ,  $\delta = 0.068$ ,  $\zeta = 0.135$ ,  $\nu = -1.029$ ,  $\sigma = 5.549$ ,  $A_0 = 402.569$ . We studied the case where the initial phase was a linear function  $\phi_0(\theta) = p\theta$ , where  $p$  is a slope coefficient. Figure 1 shows the numerically obtained basins of attraction calculated for the studied system for a different value of  $A_0$  and  $p$  parameters, and constant values of  $\Omega$ . The initial conditions used in the calculation of the periodicity of the solution were the same for all parameter configurations and were  $\theta = 0.001$ ,  $\theta' = 0$ .

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## 2. Results and Discussion

The basins of attractions indicate the existence of a range of different multi-periodic solutions. The overwhelming majority of solutions are period-14 or chaotic. Solutions with smaller periodicity create areas of an elliptical shape.



**Fig. 1.** Basins of attraction computed for system (1) in terms of different values of  $A_0$  and  $p$  parameters for three cases of  $\Omega$ .

It is worth noting that most of these solutions are odd periodicity, i.e., period-1, period-3, period-5. It is also important that for higher values of the  $p$  parameter, obtaining a solution with a periodicity of less than 14 significantly decreases.

## 3. Concluding Remarks

The presented magnetic pendulum is an example of a highly nonlinear mechatronic system whose dynamics are rich with regard to both periodic and chaotic solutions. The linear dependence of the initial phase of the excitation on the dynamic variable (angular position) yields significant changes in the solutions.

Analysis of the basins of attraction shows that for certain ranges of parameter  $p$  (slope of the linear function describing initial phase) the variety of solutions increases, however exceeding a certain threshold value ( $p \approx 4$ ) causes the diversity of periodicity to disappear.

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