Dynamics of articulated vehicles by means of multibody methods (MAT143-15)

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Abstract: The paper presents modelling of an articulated vehicle by means of joint coordinates, which enable us to describe the motion of the system with a minimal number of generalised coordinates. We consider a model of a semi-trailer formulated using joint coordinates and homogenous transformations. Such an approach enables us to treat the vehicle as a kinematic chain consisting of three single units with an even number of wheels. This means that a single unit vehicle has a structure of an open kinematic chain in a tree form. The contact of wheels with the road is modelled by the Dugoff-Uffelman model. The model is validated by comparing the simulation results with those obtained from experimental measurements. Friction in the fifth wheel is one of the important parameters influencing the motion of a tractor with a semi-trailer. The model presented enables us to analyse different models of friction in the fifth wheel. The influence of those friction models on the results is presented and discussed.

1. Introduction

When articulated vehicles are modelled, they are usually treated as a system of single vehicle units connected in various ways [1-3]. The division of a vehicle into several elements (rigid and/or flexible) is a natural way of dealing with such a complex multibody system. The motion of rigid elements is described by six generalized coordinates ; these elements are then connected by means of either constraint equations or spring-damping elements. When the equations of motion are integrated, not only the generalized coordinates and velocities but also reactions and moments of reactions in joints are calculated. The mass matrix of the system is in a block-diagonal form, which shortens numerical calculation time. However, the number of generalized coordinates is large, which slightly reduces the advantages of the method when constraint equations need to be stabilized. Such an approach is used in commercial software packages for simulation of vehicle dynamics (DADS, DYTRAN). On the other hand there are methods using joint coordinates in which the dynamics of a system is described by a minimal number of generalized coordinates. The disadvantage of such an approach is a full mass matrix and the lack of direct calculations of reactions in joints.

Friction occurring in couplings of any multibody system influences its motion. There is also a large number of papers in which different models of friction are discussed; however, the examples are usually limited to mechanisms with one or very few degrees of freedom [4,5]. Dynamic models of articulated vehicles, even when some simplifications are assumed, usually have several numbers of degrees of freedom. Thus consideration of friction is especially difficult. Friction is usually taken into account in models of contact of the tires with the road [6,7]. Friction in couplings of units is considered less often [8].

This paper is concerned with phenomena accompanying friction in rotary couplings in articulated vehicles such as semi-trailers. Knowledge of the influence of friction in the couplings on the motion of the vehicles is important when the stability of the vehicles is examined. Simplified models are formulated using the joint coordinates for simulation of motion of vehicles. In order to derive the equations of motion, homogenous transformations are used, while the Newton-Euler algorithm for an inverse dynamic problem is applied to calculate the reaction at couplings. This enable us to analyze the influence of different models of friction in the coupling on the motion of the vehicle. The models elaborated have been validated by comparing the simulation results with those obtained from experimental measurements. The results of numerical simulations for different friction models are also presented and the conclusions about the influence of friction on the motion are formulated.

2. Model of articulated vehicles

The model of any articulated vehicle can be formulated using homogenous transformations and joint coordinates by combining models of single units. Dynamic analysis requires a physical model which takes into consideration components of a vehicle such as a vehicle body, suspensions, wheels and a steering system. For the purpose of this paper a model of a semi-trailer is composed of models of three single units. A simplified model of a single unit consists of an uneven number of rigid bodies, one of which represents a vehicle body. Motion of the rigid body (single unit) is be described by one to six degrees of freedom describing the motion of the unit with respect to the preceding unit. Other elements represent wheels, while the flexibility of suspensions is reduced to the contact point between the tire and the road. The motion of a vehicle is performed on a planar, horizontal and undeformable road surface. The steering is reflected by additional functions describing the change in time of the steering angle of the wheels.

The equations of motion are derived using the Lagrange equations. All external forces and moments acting on the vehicle unit are included by means of the generalized forces. The following forces are taken into account: air resistance, drive, braking and aligning torques as well as forces describing the contact between the wheel and the road surface. The latter are derived using the Dugoff-Uffelmann model.

The model of a tractor with semi-trailer consists of three units which are treated as rigid bodies (Fig.1). The motion of each unit is described using its own generalised coordinates and the generalised coordinates of the preceding links in the kinematic chain.

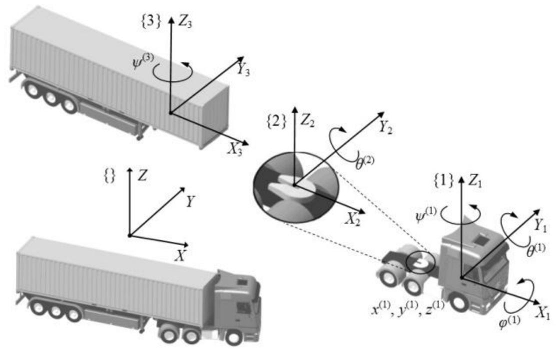


Figure 1. Tractor with semi-trailer as a system of three units

The generalised coordinates describing the motion of each link are the components of the following vectors:

1) tractor

$$\mathbf{q}^{(1)} = \tilde{\mathbf{q}}^{(1)} = \begin{bmatrix} x^{(1)} & y^{(1)} & z^{(1)} & \psi^{(1)} & \theta^{(1)} & \varphi^{(1)} & \varphi_1^{(1)} & \varphi_2^{(1)} & \varphi_3^{(1)} & \varphi_4^{(1)} \end{bmatrix}^T \tag{1.1}$$

2) fifth wheel

$$\mathbf{q}^{(2)} = \begin{bmatrix} \mathbf{q}^{(1)T} & \tilde{\mathbf{q}}^{(2)T} \end{bmatrix}^T = \begin{bmatrix} \mathbf{q}^{(1)T} & \tilde{\mathbf{q}}^{(2)T} \end{bmatrix}^T, \tilde{\mathbf{q}}^{(2)} = \begin{bmatrix} \theta^{(2)} \end{bmatrix} \tag{1.2}$$

3) semi-trailer

$$\mathbf{q}^{(3)} = \begin{bmatrix} \mathbf{q}^{(2)T} & \tilde{\mathbf{q}}^{(3)T} \end{bmatrix}^T, \tilde{\mathbf{q}}^{(3)} = \begin{bmatrix} \psi^{(3)} & \varphi_1^{(3)} & \varphi_2^{(3)} & \varphi_3^{(3)} & \varphi_4^{(3)} & \varphi_5^{(3)} & \varphi_6^{(3)} \end{bmatrix}^T \tag{1.3}$$

The equations of motion of the whole vehicle can be written in the compact form:

$$\mathbf{A}\ddot{\mathbf{q}} = \mathbf{f} \tag{2}$$

The number of equations of motion is

$$n = \tilde{n}^{(1)} + \tilde{n}^{(2)} + \tilde{n}^{(3)} + n_{w1} + n_{w2} + n_{w3} = 6+1+1+4+0+6=18$$

Integration of equations of motion (2) requires initial conditions to be determined, so an appropriate quasi–static problem has to be solved, which can be done in the way presented in [9].

2.1. Coulomb model of friction in the coupling

Modelling of articulated vehicles becomes more complex when friction in connections is taken into consideration. The model of the coupling between units m and m-1 is assumed as in Fig.2.

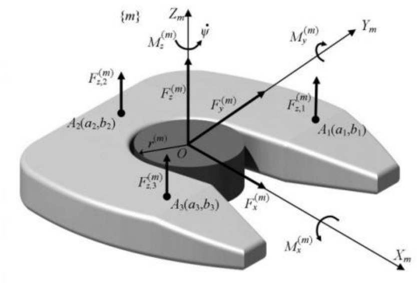


Figure 2. Connection of link m-1 and m with dry friction

Let $\mathbf{F}^{(m)} = \left[F_x^{(m)}, \quad F_y^{(m)}, \quad F_z^{(m)}, \quad 0\right]^T$ and $\mathbf{M}^{(m)} = \left[M_x^{(m)}, \quad M_y^{(m)}, \quad M_z^{(m)}, \quad 0\right]^T$ be respectively the force and the moment about point O with which the link m-1 acts on link m. Following the convention used in robotics, it is assumed that force $\mathbf{F}^{(m)}$ and moment $\mathbf{M}^{(m)}$ are defined with respect to system $\{m\}$ and are balanced by inertial forces and moments of body m and external forces and moments acting on this body.

Let us assume that force $\mathbf{F}^{(m)}$ and components $M_x^{(m)}$ and $M_y^{(m)}$ are known and moment $M_z^{(m)} = M_F$ is the moment of kinetic friction in the connection $(\dot{\psi}^{(m)} \neq 0)$. Then an additional moment:

$$M_{z,k}^{(m)} = -\mathrm{sgn}\dot{\psi}^{(m)} M_F \tag{3}$$

will act on link m. It is obvious that the value of moment M_F depends on $F_x^{(m)}, F_y^{(m)}, F_z^{(m)}$ as well as on $M_x^{(m)}$ and $M_y^{(m)}$. The friction moment on force $\mathbf{F}^{(m)}$ and components $M_x^{(m)}$ and $M_y^{(m)}$ of moment $\mathbf{M}^{(m)}$ can be calculated according to the following formula:

$$\begin{aligned} M_F &= \mu \cdot r^{(m)} \sqrt{(F_x^{(m)})^2 + (F_y^{(m)})^2} \\ &+ \mu \left|F_{z,1}^{(m)}\right| \sqrt{(a_1)^2 + (b_1)^2} + \mu \left|F_{z,2}^{(m)}\right| \sqrt{(a_2)^2 + (b_2)^2} + \mu \left|F_{z,3}^{(m)}\right| \sqrt{(a_3)^2 + (b_3)^2} \end{aligned} \tag{4}$$

where $F_{z,1}^{(m)}, F_{z,2}^{(m)}, F_{z,3}^{(m)}$ are calculated as in [9], a_i, b_i are coordinates of points A_i in local coordinate system $\{m\}$. If in this formula μ is replaced by μ_k (coefficient of kinetic friction), the moment of friction in the case of kinetic friction can be calculated from (3) after substituting (4). When $\mu = \mu_s$

(coefficient of static friction), formula (4) gives the maximal value of the friction moment which can be transferred by the connection.

In order to use formula (4), the phase of friction in the connection and values of forces $F_x^{(m)}, F_y^{(m)}, F_z^{(m)}$ and moments $M_x^{(m)}, M_y^{(m)}$ have to be determined. This problem is also addressed in other research concerned with dry friction, for example [10].

Let us introduce the following index defining the phase of friction in the connection:

$$i_m = \begin{cases} 0 & \text{if} \quad \dot{\psi}^{(m)} = 0 \ - \text{static friction} \\ 1 & \text{if} \quad \dot{\psi}^{(m)} \neq 0 \ - \text{kinetic friction} \end{cases} \tag{5}$$

During motion, index i_m changes in the following way:

<u>If $i_m = 0$:</u> $M_z^{(m)}$ is an additional unknown and the additional constraint equation is formulated:

$$\dot{\psi}^{(m)} = 0 \tag{6.1}$$

Transition into the kinetic friction phase ($i_m = 1$) takes place when the following relation is fulfilled:

$$\left| M_z^{(m)} \right| \geq M_F \tag{6.2}$$

where M_F is defined in (4) assuming $\mu = \mu_s$.

<u>If $i_m = 1$:</u> The moment of friction can be calculated according to formulae (3) and (4). The change in the state of motion in the connection is expected when velocity $\dot{\psi}_m$ changes its sign. Moment $M_{z,k}^{(m)}$ has to be taken into account in the equations of motion of the vehicle. For the semi-trailer the relative motion of unit *m* with respect to unit *m*-1 is defined by angle $\psi^{(3)}$ (*m*=3). The above model of friction is called a Coulomb model with stiction and it is used for comparative analysis as the reference ('C').

In robotics calculation of force $\mathbf{F}^{(m)}$ and moment $\mathbf{M}^{(m)}$ in the coupling is carried out by means of the Newton-Euler procedure for solving inverse dynamics [11]. However, it is impossible to use this approach directly in the case of articulated vehicles because of the specific tree structure of articulated vehicles and the way of assigning the coordinate systems. Detailed description of this algorithm is presented in [9].

3. Other models of friction

There are many different models of friction used in simulations of dynamics of multibody systems. Since friction is a complex phenomenon, the analysis of its influence on the motion of systems is very often limited to simple mechanisms with only a few degrees of freedom. Articulated vehicles are examples of multibody systems with several degrees of freedom and therefore consideration of friction is a difficult task. The most popular model of friction assumes two phases of relative motion in the coupling, so called stick-slip motion, and it is described in section 2.1 as a Coulomb model with stiction. Due to the discontinuity of the friction force at zero relative velocity, this model causes computational problems, so that some researchers consider continuous functions in the vicinity of zero. For the comparative analysis we consider some other models of kinetic friction described below.

'P' – model with continuous function of the third order:

It is assumed that the moment of friction is calculated as follows:

$$M_{z,k}^{(m)} = \begin{cases} \operatorname{sgn}\dot{\psi}^{(m)} M_F & \text{if } \left|\dot{\psi}^{(m)}\right| > \Delta\dot{\psi}^{(m)} \\ \operatorname{sgn}\dot{\psi}^{(m)} M_F \ p_3\left(M_F, \dot{\psi}^{(m)}\right) & \text{if } \left|\dot{\psi}^{(m)}\right| \le \Delta\dot{\psi}^{(m)} \end{cases} \tag{7.1}$$

where $p_3\left(M_F, \dot{\psi}^{(m)}\right)$ is a polynomial of the third degree calculated according to the formula:

$$p_3 = \operatorname{sgn}\dot{\psi}^{(m)} \left(\left(\frac{\dot{\psi}^{(m)}}{\Delta\dot{\psi}^{(m)}} \right)^3 - 3\left(\frac{\dot{\psi}^{(m)}}{\Delta\dot{\psi}^{(m)}} \right)^2 + 3\left| \frac{\dot{\psi}^{(m)}}{\Delta\dot{\psi}^{(m)}} \right| \right) M_F \tag{7.2}$$

'A' – Awrejcewicz model [12]:

The moment of kinetic friction $M_{z,k}^{(m)}$ can be calculated according to the following formula:

$$M_{z,k}^{(m)} = \begin{cases} \operatorname{sgn}\dot{\psi}^{(m)} M_F & : \left|\dot{\psi}^{(m)}\right| > \Delta\dot{\psi}^{(m)} \\ \operatorname{sgn}M_z^{(m)} M_F & : \left|\dot{\psi}^{(m)}\right| \le \Delta\dot{\psi}^{(m)} \wedge \left|M_z^{(m)}\right| > M_F \wedge \operatorname{sgn}\left(\dot{\psi}^{(m)} M_z^{(m)}\right) \ge 0 \\ (2A-1)\operatorname{sgn}\dot{\psi}^{(m)} M_F & : \left|\dot{\psi}^{(m)}\right| \le \Delta\dot{\psi}^{(m)} \wedge \left|M_z^{(m)}\right| > M_F \wedge \operatorname{sgn}\left(\dot{\psi}^{(m)} M_z^{(m)}\right) < 0 \\ A(-M_z^{(m)} + \operatorname{sgn}\dot{\psi}^{(m)} M_F) + M_z^{(m)} & : \left|\dot{\psi}^{(m)}\right| \le \Delta\dot{\psi}^{(m)} \wedge \left|M_z^{(m)}\right| \le M_F \end{cases} \tag{8}$$

where $A = \left(\frac{\dot{\psi}^{(m)}}{\Delta\dot{\psi}^{(m)}} \right)^2 \left(3 - 2\left| \frac{\dot{\psi}^{(m)}}{\Delta\dot{\psi}^{(m)}} \right| \right)$, $M_z^{(m)}$ is the moment calculated from the Newton-Euler procedure.

'T' – Threlfall model [13]:

$$M_{z,k}^{(m)} = \begin{cases} \operatorname{sgn}\dot{\psi}^{(m)} M_F & \text{if } \left|\dot{\psi}^{(m)}\right| > \Delta\dot{\psi}^{(m)} \\ \operatorname{sgn}\dot{\psi}^{(m)} M_F \left(1 - e^{\frac{3\left|\dot{\psi}^{(m)}\right|}{\Delta\dot{\psi}^{(m)}}} \right) & \text{if } \left|\dot{\psi}^{(m)}\right| \le \Delta\dot{\psi}^{(m)} \end{cases} \tag{9}$$

All the above models depend on arbitrary parameter $\Delta\dot{\psi}^{(m)}$ which is a velocity tolerance.

4. Numerical simulations

The model of a semi-trailer presented in section 2 was validated by comparing the computer simulation results with the experimental measurements carried out in [9].

In order to examine the influence of the assumed friction model and friction coefficients on the dynamics of the semitrailer, several numerical experiments were carried out, assuming an idealized steering angle of front wheels of the tractor in such a way that the mass center of the tractor moves 8.5m in a lateral direction when there is no friction in the coupling analyzed (Fig.3).

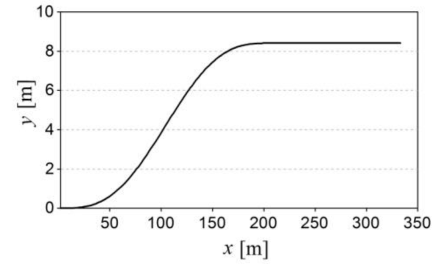


Figure 3. Trajectory of the mass center of the tractor without friction in the coupling

Having performed some numerical simulations, it is assumed that the velocity tolerance is $\Delta\dot{\psi}^{(m)} = 0.001$. The comparison of the trajectory of the tractor calculated for different models with different values of friction coefficient is presented in Fig.4.

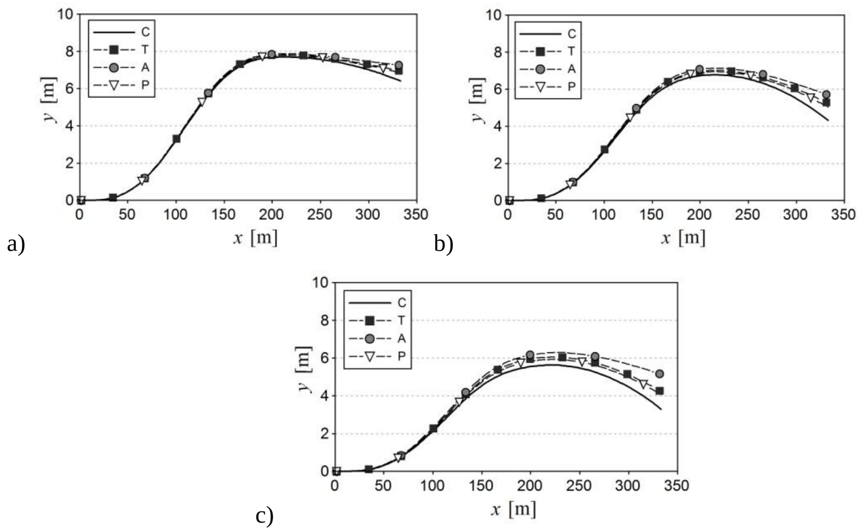


Figure 4. Trajectory of the tractor for different values of the friction coefficient
a) $\mu = 0.1$ b) $\mu = 0.2$ c) $\mu = 0.3$

The results show that the choice of the friction model influences the resulting trajectory of the tractor. The larger the friction coefficient, the larger is the difference in the trajectory. The moment of friction calculated using the Newton-Euler algorithm for different friction coefficients is presented in Fig.5.

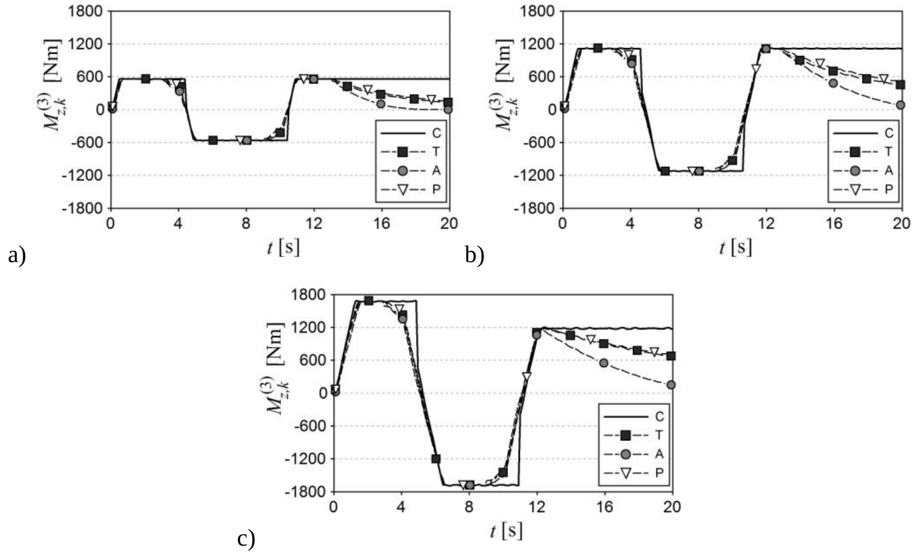


Figure 5. Friction torque in the fifth wheel a) $\mu = 0.1$, b) $\mu = 0.2$, c) $\mu = 0.3$

The larger the friction coefficient, the smaller are the differences between model T and P. The friction force depends on the friction coefficient, but each model depends on it in a different way.

Since friction is considered in the coupling, the next figures present both the relative rotation and the relative angular velocity in the fifth wheel for different friction models and friction coefficients.

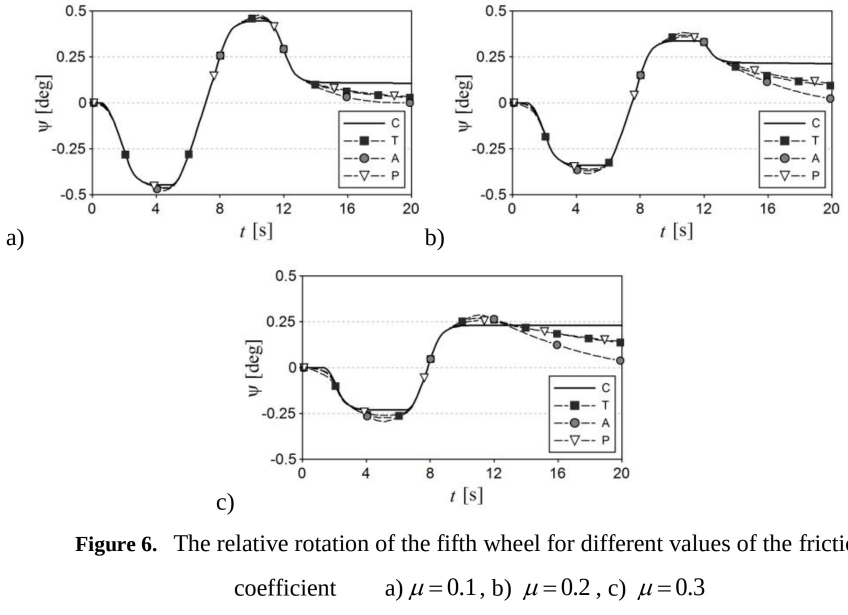
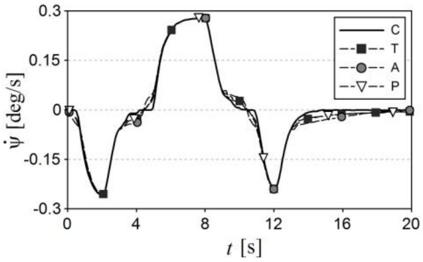
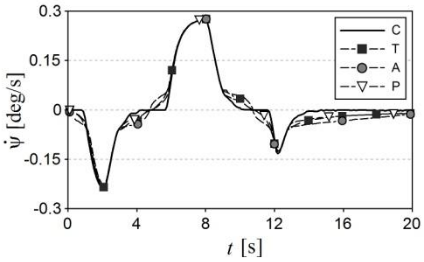


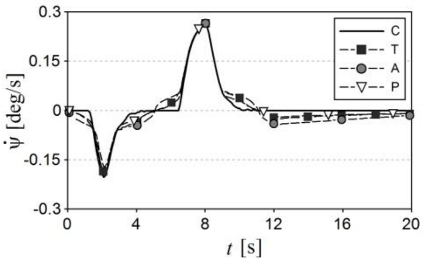
Figure 6. The relative rotation of the fifth wheel for different values of the friction coefficient a) $\mu = 0.1$, b) $\mu = 0.2$, c) $\mu = 0.3$



a)



b)



c)

Figure 7. The relative velocity in the fifth wheel for different values of the friction coefficient a) $\mu = 0.01$, b) $\mu = 0.02$, c) $\mu = 0.03$

5. Conclusions

The paper presents a validated model of an articulated vehicle formulated using multibody methods, with homogenous transformations and joint coordinates. The analysis presented is concerned with the influence of friction in the coupling between the tractor and the semitrailer. In order to calculate the forces and moments acting in the fifth wheel, the Newton-Euler algorithm for solving an inverse dynamic problem is used. The Coulomb model of friction with two phases of relative motion is described and used as a background for comparative analysis of different models of friction in which discontinuity at zero velocity is replaced by continuous functions. It is shown that the larger the coefficient of friction, the larger is the influence of the choice of the friction model on the results of simulations of motion. Owing to the development of active safety systems in vehicles, the mathematical model of a vehicle implemented in these systems is very important. In articulated lorries the joint systems are essential when vehicle motion in extreme conditions is considered. One of the tasks of active safety systems is to prevent these vehicles knifing during some maneuvers, for example during braking. The analysis of the accepted friction model in the fifth wheel carried out in this article points to its influence on the trajectory and relative rotation of the tractor and trailer during motion. These differences are particularly noticeable in friction torque values and relative rotation angles during the last phase of the maneuver. Our analysis reveals the necessity of experimental research to make the proper choice of the friction model in the fifth wheel.

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