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# Geometrically nonlinear vibrations of double-layered nanoplates

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**Abstract:** The geometrically nonlinear vibrations of simply supported double-layered graphene sheet systems are considered in the presented manuscript. The interaction between layers is taken into account due to van der Waals forces. The investigation is based on the nonlocal elasticity theory, Kirchhoff plate theory and von Kármán theory. The governing equations are used in mixed form by introducing the stress Airy function. The analytical presentation of the nonlinear frequency ratio for in-phase vibration and anti-phase vibration modes is presented. It is shown that the nonlocal parameter included in the compatibility equation can significantly change the vibrating characteristics.

**Keywords:** double-layered nanoplate system, the nonlocal elasticity theory, Kármán plate theory, Bubnov-Galerkin method.

## 1. Introduction

The modern industry is developing rapidly in the field of nanotechnology. This fact leads to the need for new studies of the nanostructures (nanobeams, nanoplates, nanoshells) using in the design of sensors, resonators, nanoelectromechanical systems (NEMS), nanooptomechanical structures, energy storage systems, DNA detectors, drug delivery. An important role is played by graphene objects, the study of which is the focus of our work. We employed the nonlocal elasticity theory, which is based on the fact that the stress at a given point is a function of strains at all other points of structure, to study double-layered nanoplate system.

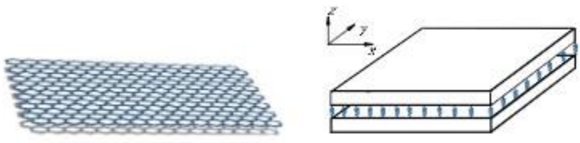

**Fig. 1.** Diagram of double-layered graphene sheet system

According to nonlocal theory the constitutive relation for the nonlocal stress tensor at a point $x$ in an integral form is presented as follows

$$\sigma = \int_V K(|X' - X|, \tau)\sigma'(X')d\,X' \tag{1}$$

where $\sigma, \sigma'$ are nonlocal and local stress tensors, $K(|X' - X|, \tau)$ is the nonlocal modulus, $\tau = e_0 \alpha / l$, $\alpha$ is the internal characteristic length, $e_0$ stands for a constant appreciate to material, whereas $l$ is external characteristics length. Interaction between two layers regards the action of van der Waals force (vdW) . This interaction is often modeled as the Winkler foundation. We assume that both nanoplates have the same mechanical properties.

## 2. Formulation of the problem

The governing equations for each layer are taken as:

$$D\Delta^2 w_1 = (1 - \mu\nabla^2) L(w_1, F_1) - c(w_1 - w_2) - \rho h \frac{\partial^2 w_1}{\partial t^2}), \qquad (2)$$

$$(1 - \mu\nabla^2) \frac{1}{E} \Delta^2 F_1 = -\frac{h}{2} L(w_1, w_1),$$

$$D\Delta^2 w_2 = (1 - \mu\nabla^2) L(w_2, F_2) - c(w_2 - w_1) - \rho h \frac{\partial^2 w_2}{\partial t^2}), \qquad (3)$$

$$(1 - \mu\nabla^2) \frac{1}{E} \Delta^2 F_2 = -\frac{h}{2} L(w_2, w_2),$$

where $\mu$ stands for the nonlocal parameter and $\nabla^2$ is the Laplacian operator, $E$ is Young's modulus, $\nu$ is Poisson's ratio, $D$ is flexural nanoplate rigidity, $\rho$ is density of the plate, $h$ is thickness, whereas $L(w, F), L(w, w)$ are differential operators determined in [1] and $c$ stands for interaction coefficient [2]. Note that we propose using the nonlocal compatibility equation for DLGs, where the nonlocal parameter is introduced based on the nonlocal constitutive relation. System of the governing equations is supplemented with the simply supported boundary conditions.

We present the deflections $w_i\,(x, y, t), i = 1,2$ of each nanoplate as

$$w_i\,(x, y, t) = y_i\,(t) W(x, y), i = 1,2,$$

where $W(x, y)$ is shape function and $y_i\,(t)$ is generalized coordinate for $i$-th layer. Further application of the Bubnov-Galerkin approach gives the system of the coupled second order ordinary differential equations which discussed for in-phase vibration (IPV) mode and anti-phase vibration (APV) mode.

## 3. Concluding Remarks

The nonlinear vibrations of double-layered graphene sheet systems are studied. The governing equations of the problem are based on the nonlocal elasticity theory, Kirchhoff hypothesis, the von Kármán equations and presented in the mixed form with Airy function. Two graphene sheets are bonded by van der Waals force. The investigation of the influence of the nonlocal parameter in the compatibility equation is performed. It is concluded that the nonlocal parameter in the compatibility equation can significantly change the results and should be taken into account in order to achieve reliable results.

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