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Thermal waves in composite membrane with circular inclusions in hexagonal lattice structures

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Abstract: Non-stationary heat conduction in the fibre composite materials is studied. 2D composite media consists of matrix with circular inclusions in hexagonal lattice structures. The perfect contact between different materials is assumed on the boundary of the fibres. The local temperature field in the unit cell is modeled by the heat equation. Time variable excludes from the original boundary value problem using of Laplace transform. Asymptotic homogenization method allows to reduce original problem for multiply-connected domain to the sequence of boundary value problems in simply-connected domains. Composite material is supposed densely-packed with a high-contrast. In this case, the local boundary value problem includes a small parameter equal to the ratio of the distance between the inclusions to the characteristic cell size. Using of additional small parameter and thin layer asymptotics allows to solve local problem analytically. Closed form expression for effective heat conductivity is obtained.

Keywords: Unsteady heat conduction, fibre composite, effective conductivity, Laplace transform, homogenization approach.

1. Introduction

Problem of non-stationary heat transfer in composite media have attracted the attention of researches due to the wide occurrence of the media in engineering applications. As a rule, we are talking about determining the effective characteristics of the considered inhomogeneous material. For this purpose phenomenological and experimental approaches, mixture theory, self-consistent approximation are used. It is natural to use the asymptotic homogenization method (AHM) to solve the described problem. Allaire and Habibi [1] used AHM to the heat conduction problem in a periodically perforated domain with a nonlinear and nonlocal boundary condition modeling radiative heat transfer in the perforations. In 2D case cell problem is solved numerically. AHM with FEM in many cases allow one to obtain numerically effective characteristics and local fields [2], and construct boundary layer correctors [3]. The analytical solution of the problem on a cell requires the use of additional small parameters. Small parameters typical of low-contrast composites or composites with small volume fraction of inclusin are widely used [4,5]. In our work, an analytical solution for densely-packed high-contrast composite is obtained with a help of thin layer asymptotics [6].

2. Results and Discussion

We consider the problem of nonstationary heat transfer in a composite with periodically distributed inclusions of a circular cross section which are placed in regular hexagonal cells (Fig. 1). We take Newton's law of cooling for matrix and inclusions, $T^{\pm} = T_c^{\pm}(x, y)\exp(-\gamma t)$.

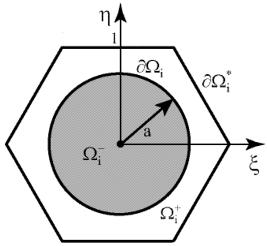

Fig. 1. Structure of the composite cell.

The original eigenvalue problem can be formulated in the following way

$$k_+\left(\frac{\partial^2 T_c^+}{\partial x^2}+\frac{\partial^2 T_c^+}{\partial y^2}\right)=-c_+\rho_+\gamma T_c^+ \quad \text{in} \quad \Omega_i^+, \qquad k_-\left(\frac{\partial^2 T_c^-}{\partial x^2}+\frac{\partial^2 T_c^-}{\partial y^2}\right)=-c_-\rho_-\gamma T_c^- \quad \text{in} \quad \Omega_i^-,$$
$$T_c^+ = T_c^-, \quad k_+\frac{\partial T_c^+}{\partial \mathbf{n}} = k_-\frac{\partial T_c^-}{\partial \mathbf{n}} \quad \text{at} \quad \partial\Omega_i, \qquad T_c^{\pm}=0 \quad \text{at} \quad \partial\Omega . \tag{1}$$

Using the AHM and thin layer asymptotics [6] for the analytical solution of the cell problem, we obtain the homogenized boundary value problem in the following form:

$$q\left(\frac{\partial^2 T_{c0}}{\partial x^2}+\frac{\partial^2 T_{c0}}{\partial y^2}\right)+\tilde{q}\,\gamma_0 T_{c0}=0 \ \text{ in } \ \Omega^*, \qquad T_{c0}=0 \ \text{ at } \ \partial\Omega , \tag{2}$$

where $\frac{c_\pm \rho_\pm}{k_\pm} = \varpi_\pm$, $\tilde{q} = \frac{\varpi_+\left|\Omega_i^+\right| + \varpi_-\left|\Omega_i^-\right|}{\left|\Omega_i^*\right|}$.

3. Concluding Remarks

The obtained solution of a regularly periodic problem has extreme properties with respect to random composites. Thus, it can be used to assess the behavior of real composite materials. Note that the solution at the initial moment of time is rapidly changing not only in spatial variables, but also in time. Description of this state requires additional research.

References

[1] ALLAIRE G, HABIBI Z: Homogenization of a conductive, convective, and radiative heat transfer problem in a heterogeneous domain. *SIAM Journal on Mathematical Analysis* 2013, **45**(3):1136-1178.

[2] WASEEM A, HEUZE T, STAINIER L, GEERS MGD, V. G. KOUZNETSOVA VG: Model reduction in computational homogenization for transient heat conduction. *Computational Mechanics* 2020, **65**:249–266.

[3] LEFIK M, SCHREFLER BA: Modelling of nonstationary heat conduction problems in micro-periodic composites using homogenisation theory with corrective terms. *Archives of Mechanics* 2000, **52**(2):203-223.

[4] MITYUSHEV V, OBNOSOV YU, PESETSKAYA E, ROGOSIN S: Analytical methods for heat conduction in composites. *Mathematical Modelling and Analysis* 2008, **13**(1):67-78.

[5] RYLKO N. Calculation of the effective thermal conductivity of fiber composites in non-stationary case. *Composites* 2005, **5**(4):96-100 (in Polish).

[6] TAYLER AB: *Mathematical Models in Applied Mechanics*. Oxford: Clarendon Press, 2001.