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# APPLICATION OF ELECTROOPTIC MODULATORS FOR DIVERGENT LIGHT BEAM 


#### Abstract

Analysis of electrooptic modulators of the light strength working with divergent light beam is performed numerically employing the Jones matrices method. The approach allows to consider small inaccuracies in cutting and alignment of electrooptic crystal and the divergence of entering light beam. The results show that the divergence typical for gas lasers should not affect significantly the work of electrooptic modulators even for large electrooptic crystals, while diode lasers may be used with modulators only in the form of precise modules equipped with colimation optics.


Keywords: electrooptic modulator, divergent light beam, diode lasers, HeNe lasers.

## 1. INTRODUCTION

The electrooptic modulator consisting of the linear polarizer, quarter-wave plate, electrooptic crystal and analyzer is a convenient device to modulate the light strength. Previously, strong influence of small inaccuracies in the crystal cutting and alignment on the modulation index (both at the first and second harmonic of the modulating field) for those configurations where the light propagates in the vicinity of the optic axis has been reported [1,2]. Moreover, significant harmonic distortions in electrooptic modulation due to the small inaccuracies has been numerically predicted [1]. These results were obtained, however, assuming an ideally parallel light beam. According to our best knowledge no estimation of the effect of the light beam divergence on the work of electrooptic modulator has been reported so far. Typically the divergence of the light beam emitted by gas lasers is of the order of magnitude 1 mrad . The divergence of a single mode edge emitting diode laser vary from 20 to 40 degrees in the direction perpendicular to the facet, while the divergence for the
parallel axis is 2.5 to 6 times lower [3,4]. In recent years vertical-cavity surface-emitting lasers (VCSEL) become popular. The light beams emitted by VCSEL lasers has circular cross-section and the divergence may be of the order of several degrees [5], but for commercial lasers it is rather not less than 10 degrees [4].

The aim of this work is to investigate numerically the influence of divergence of the light beam entering electrooptic modulator on its modulation index and harmonic distortions. We allow also for small inaccuracies in the crystal cutting and alignment. In our analysis we focus our attention on configurations with the light propagating in the vicinity of optic axis. According to the results reported previously, the modulators employing such configurations are especially sensitive to any inaccuracies. There are two main mechanisms responsible for the deterioration of the parameters of electrooptic modulator:

1) The phase difference between two refracted waves propagating through the crystal in the same direction is a function of the angle of deviation from the optic axis. Therefore, the working point of the modulator on its phase-amplitude characteristic depends on considered direction of the light. The response of the modulator for the whole divergent light beam can not be represented by a single point on the modulator characteristic, but a continuous distribution must be considered. Additionally, the divergence of the light beam and inaccuracies in the crystal cutting and alignment may move the center of this distribution far away from the optimal linear part of the characteristic. This undesired effects becomes stronger with increasing crystal length.
2) Azimuths of two waves emerging from the crystal depend both on the angle of deviation from the optic axis of the crystal and the plane of the deviation. The importance of this mechanism does not depend on the crystal length.

## 2. METHOD

We consider electrooptic modulator composed of linear polarizer, quarterwave plate, electrooptic crystal, and analyzer. The crystal is assumed to be cut in the form of right parallelepiped. The crystallographic axes are denoted $X, Y, Z$, with the $Z$ axis along the optic axis. We allow for imperfections in the system considered, namely: an imprecision in cutting of the crystal and its misalignment, and a divergence of laser light beam entering the modulator. All of these problems may appear simultaneously. In this work we consider only the case, when the intended direction of the axis of incident light beam is along the $+Z$ axis and is perpendicular to the entrance face of the crystal. In general, any other light directions may be also considered (see, e.g. Ref. [6]).

Inaccuracies in the crystal cutting do not imply any deviation from the normal incidence of light on the entrance face. These inaccuracies may, however, lead to a new axis of incident light beam $+z$ being slightly different from the intended $+Z$. The reference azimuth, which is indicated by the $+X$ axis in the ideal system, now may be not exactly perpendicular to the axis of entering light and must be replaced by a new $+x$ axis. To describe the orientation of the $x y z$ system we use two rotation angles $\beta_{c}$ and $\gamma_{c}$, and the following matrix of transformation from the $X Y Z$ to the $x y z$ coordinates:

$$
\mathbf{b}=\left[\begin{array}{ccc}
\cos \gamma_{c} & 0 & -\sin \gamma_{c}  \tag{1}\\
0 & 1 & 0 \\
\sin \gamma_{c} & 0 & \cos \gamma_{c}
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \beta_{c} & \sin \beta_{c} \\
0 & -\sin \beta_{c} & \cos \beta_{c}
\end{array}\right]
$$

Inaccurate alignment of the crystal affects the angle of incidence. Thus, the axis of the incident light beam $+z^{\prime}$ may differ from the intended main axis $+z$ of the parallelepiped. Misalignment perturbs also the zero azimuth, now indicated by the $+x^{\prime}$ axis. To describe the orientation of these new axes we use two rotation angles $\beta_{\mathrm{a}}$ and $\gamma_{\mathrm{a}}$. Transformation from the $x y z$ to $x^{\prime} y^{\prime} z^{\prime}$ coordinates is given by the matrix:

$$
\mathbf{c}=\left[\begin{array}{ccc}
\cos \gamma_{a} & 0 & -\sin \gamma_{a}  \tag{2}\\
0 & 1 & 0 \\
\sin \gamma_{a} & 0 & \cos \gamma_{a}
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \beta_{a} & \sin \beta_{a} \\
0 & -\sin \beta_{a} & \cos \beta_{a}
\end{array}\right]
$$

In our model we allow for any direction of the low-frequency electric field $\mathbf{E}$ in the crystal. Although the intended field direction is usually given by the components of the $\mathbf{E}$ vector, we also need a matrix description for further derivations. We defined two rotation angles $\beta_{\mathrm{E}}$ and $\gamma_{\mathrm{E}}$ and the matrix d transforming the versor of the $+Z$ axis into the unit vector in the intended direction of $\mathbf{E}$

$$
\mathbf{d}=\left[\begin{array}{ccc}
\cos \gamma_{\mathrm{E}} & 0 & -\sin \gamma_{\mathrm{E}}  \tag{3}\\
0 & 1 & 0 \\
\sin \gamma_{\mathrm{E}} & 0 & \cos \gamma_{\mathrm{E}}
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \beta_{\mathrm{E}} & \sin \beta_{\mathrm{E}} \\
0 & -\sin \beta_{\mathrm{E}} & \cos \beta_{\mathrm{E}}
\end{array}\right] .
$$

Typically electrodes are deposited onto crystal faces and the misalignment may not affect the field components, whereas inaccuracies in the cutting lead to rotation of $\mathbf{E}$ relative to its intended direction. Then the actual components of $\mathbf{E}$ in the $X Y Z$ coordinates are given by

$$
\left[\begin{array}{c}
E_{X}  \tag{4}\\
E_{Y} \\
E_{Z}
\end{array}\right]=\mathbf{b}^{\mathrm{T}} \mathbf{d}^{\mathrm{T}}\left[\begin{array}{c}
0 \\
0 \\
E
\end{array}\right],
$$

where the superscript T denotes matrix transposition.

We assume that the light beam entering the modulator propagate from the point source in any directions with the maximum intensity directed along the $+z^{\prime}$ axis of the modulator. Because the most part of low power laser diodes ( $<200 \mathrm{~mW}$ class) and gas lasers operating with a $\mathrm{TEM}_{00}$ mode have a nearly Gaussian transverse intensity profile [3] we focus here our attention on the Gaussian case

$$
\begin{equation*}
\rho\left(x^{\prime}, y^{\prime}\right)=\exp \left(-\frac{x^{\prime 2}+y^{\prime 2}}{2 r^{2}}\right) \tag{5}
\end{equation*}
$$

however, the model may be also applied for other distributions, including flattened quasi-Gaussian distribution [7]. The symbol $\rho$ applies here to the amplitude of light wave. We define the beam radius $r$, for a certain cross-section, as the distance from the axis at which the light intensity (proportional to $\rho^{2}$ ) falls to $1 / \mathrm{e}$ of the axial value.

The intensity and state of the light passing thought the system of some nondepolarizing plane-parallel plates may be found employing Jones matrix calculus. In its original form the calculus was intended only to the case when the incident light is always normal to the optical elements and no longitudinal components of the electromagnetic field appears inside the plates. Such limitations are too restrictive in the analysis of real electrooptic devices, however, we would not made a significant error in calculations if we allow for a small derogations from the original assumptions [8].

Jones matrix calculus is not applicable directly to an unparallel light beam, but it can be applied to the individual elementary waves propagating in some given directions. Assuming that the energy of that part of emerging light, which are not contained in the aperture of detector or the transmission line is negligible, the effective light strength of emerging beam is given by

$$
\begin{equation*}
\bar{I}=\frac{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho^{2}\left(x^{\prime}, y^{\prime}\right) I\left(x^{\prime}, y^{\prime}\right) \mathrm{d} x^{\prime} \mathrm{d} y^{\prime}}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho^{2}\left(x^{\prime}, y^{\prime}\right) \mathrm{d} x^{\prime} \mathrm{d} y^{\prime}} \tag{6}
\end{equation*}
$$

where $I\left(x^{\prime}, y^{\prime}\right)$ is the light intensity related to the elementary wave propagating through the ( $x^{\prime}, y^{\prime}$ ) point on interference plane.

In order to apply Jones calculus to find $I\left(x^{\prime}, y^{\prime}\right)$ we need first some additional formalism to describe the propagation of waves in some direction selected from the divergent beam. Thus, we introduce another coordinate system $x^{\prime \prime} y^{\prime \prime} z^{\prime \prime}$, where the $+z^{\prime \prime}$ axis indicates the direction of selected incident wave and
$+x^{\prime \prime}$ provides the reference azimuth, which is necessary to characterize the polarization state. The transformation from the coordinates $x^{\prime} y^{\prime} z^{\prime}$ to $x^{\prime \prime} y^{\prime \prime} z^{\prime \prime}$ may be described by the matrix

$$
\mathbf{m}=\left[\begin{array}{ccc}
\cos \gamma_{d} & 0 & -\sin \gamma_{\mathrm{d}}  \tag{7}\\
0 & 1 & 0 \\
\sin \gamma_{\mathrm{d}} & 0 & \cos \gamma_{\mathrm{d}}
\end{array}\right]\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \beta_{\mathrm{d}} & \sin \beta_{\mathrm{d}} \\
0 & -\sin \beta_{\mathrm{d}} & \cos \beta_{\mathrm{d}}
\end{array}\right] .
$$

In the $x y z$ coordinates the unit vector s in the direction of selected incident wave can be found as follows

$$
\left[\begin{array}{l}
s_{x}  \tag{8}\\
s_{y} \\
s_{z}
\end{array}\right]=\mathbf{c}^{\mathrm{T}} \mathbf{m}^{\mathrm{T}}\left[\begin{array}{l}
0 \\
0 \\
1
\end{array}\right] .
$$

Hence, the angle of incidence $\alpha$ on the crystal face can be found from

$$
\begin{equation*}
\sin \alpha=\sqrt{s_{x}^{2}+s_{y}^{2}} . \tag{9}
\end{equation*}
$$

In general double refraction phenomena and the field-induced changes in the directions of refracted waves and rays may give significant contributions to the modulation index of electrooptic system and its harmonic distortions. However, in the case of the light propagating in the vicinity of the optic axis, the directions of the both refracted waves and the both refracted rays differ very slightly from that found employing the Snell law for isotropic medium. Moreover, the field-induced changes in the direction of refraction give negligible effects. Thus, for the light propagating in the vicinity of the optical axis the Snell law of refraction can be applied as a very good approximation [6]. Application of the Snell law allow us to simplify and accelerate our calculational routine. The angle of refraction $\beta$ is

$$
\begin{equation*}
\beta=\arcsin \left(\frac{\sin \alpha}{n_{\mathrm{o}}}\right) . \tag{10}
\end{equation*}
$$

Employing Eq. (10) and the fact, that the plane of refraction contains also the unit vector given by Eq. (8), the following unit vector $\mathbf{p}$ describing the direction of refraction can be found in the $x y z$ coordinates

$$
\left[\begin{array}{l}
p_{x}  \tag{11}\\
p_{y} \\
p_{z}
\end{array}\right]=\left[\begin{array}{c}
\frac{s_{x}}{\sin \alpha} \sin \beta \\
\frac{s_{y}}{\sin \alpha} \sin \beta \\
\cos \beta
\end{array}\right] .
$$

This vector shows the direction in which we have to find the refractive indices $n_{f}$ and $n_{\mathrm{s}}$ of the fast and slow refracted waves, respectively, and their polarization azimuth $\alpha_{f}$ employing optical indicatrix equation. Of course, at this stage of calculations no approximations are made, i.e., the crystal must be regarded as the uniaxial medium for $E=0$ and any perturbations due to the applied field are taken into account.

For the homogeneous, dichroic, linearly birefringent medium, the Jones Matrix has the form $[9,10]$

$$
\mathbf{J}=\left[\begin{array}{cc}
T_{\mathrm{f}} \cos ^{2} \alpha_{\mathrm{f}}+T_{\mathrm{s}} \sin ^{2} \alpha_{\mathrm{f}} \mathrm{e}^{-\mathrm{i} \Gamma} & \sin \alpha_{\mathrm{f}} \cos \alpha_{\mathrm{f}}\left(T_{\mathrm{f}}-T_{\mathrm{s}} \mathrm{e}^{-\mathrm{i} \Gamma}\right)  \tag{12}\\
\sin \alpha_{\mathrm{f}} \cos \alpha_{\mathrm{f}}\left(T_{\mathrm{f}}-T_{\mathrm{s}} \mathrm{e}^{-\mathrm{i} \Gamma}\right) & T_{\mathrm{f}} \sin ^{2} \alpha_{\mathrm{f}}+T_{\mathrm{s}} \cos ^{2} \alpha_{\mathrm{f}} \mathrm{e}^{-\mathrm{i} \Gamma}
\end{array}\right],
$$

where $T_{\mathrm{f}}$ and $T_{\mathrm{s}}$ are the amplitude transmission coefficients for the fast and slow refracted waves, respectively, and $\Gamma$ is the phase difference between the waves. If the Jones vector $\boldsymbol{E}_{0}$ describes the light emerging from a linear polarizer, the light emerging from the modulator is described by the vector $\mathcal{E}$

$$
\begin{equation*}
\mathcal{E}=\mathbf{J}_{\mathrm{a}} \mathbf{J} \mathbf{J}_{4} \mathcal{E}_{0} . \tag{13}
\end{equation*}
$$

Here the Jones matrix $\mathbf{J}$ for the crystal is given by Eq. (12). In this study no absorption is assumed ( $T_{\mathrm{f}}=T_{\mathrm{s}}=1$ ) and

$$
\begin{equation*}
\Gamma=\frac{2 \pi L}{\lambda}\left(n_{\mathrm{s}}-n_{\mathrm{f}}\right) \tag{14}
\end{equation*}
$$

where $L$ is the geometrical path of the light in the crystal, and $\lambda$ is the wavelength. The Jones matrix $\mathbf{J}_{a}$ for the analizer can be found as special case of the matrix in Eq. (12), where $T_{\mathrm{f}}=1, T_{\mathrm{s}}=0$, and $\alpha_{\mathrm{f}}=\pi$ or $\pi / 2$ for the modulator with KDP or $\mathrm{LiNbO}_{3}$ crystal, respectively. The matrix $\mathbf{J}_{4}$ describing the quarter-wave plate can be derived from Eq. (12) with the following substitutions: $T_{\mathrm{f}}=T_{\mathrm{s}}=1, \Gamma=\pi / 2$, and $\alpha_{\mathrm{f}}=\pi / 2$ (for KDP) or 0 (for $\mathrm{LiNbO}_{3}$ ). The vector $\mathcal{E}$ allows us to find the strength $I\left(x^{\prime}, y^{\prime}\right)$ of the emerging light

$$
\begin{equation*}
I\left(x^{\prime}, y^{\prime}\right)=\left|\mathcal{E}_{x}\right|^{2}+\left|\mathcal{E}_{y}\right|^{2} \tag{15}
\end{equation*}
$$

where, to simplify the notation, we omitted the dependency of the $\mathcal{E}$ vector on the $x^{\prime}$ and $y^{\prime}$ coordinates.

Since the integrals in Eq. (6) are on the $x^{\prime}$ and $y^{\prime}$ variables, the rotation angles $\beta_{\mathrm{d}}$ and $\gamma_{\mathrm{d}}$ will not be used directly and we need to write the components of the matrices used in Eq. (7) as an explicit functions of $x^{\prime}$ and $y^{\prime}$

$$
\begin{align*}
& \sin \beta_{\mathrm{d}}=-y^{\prime} / \sqrt{y^{\prime 2}+z^{\prime 2}}  \tag{16}\\
& \cos \beta_{\mathrm{d}}=z^{\prime} / \sqrt{y^{\prime 2}+z^{\prime 2}}  \tag{17}\\
& \sin \gamma_{\mathrm{d}}=x^{\prime} / \sqrt{x^{\prime 2}+y^{\prime 2}+z^{\prime 2}} \tag{18}
\end{align*}
$$

$$
\begin{equation*}
\cos \gamma_{d}=\sqrt{y^{\prime 2}+z^{\prime 2}} / \sqrt{x^{\prime 2}+y^{\prime 2}+z^{\prime 2}} \tag{19}
\end{equation*}
$$

For the Gaussian distribution (5) infinite limits of integration in Eq. (6) may by changed to finite without significant changes in results of calculations. In our calculations we assumed the limits $\pm 3 r$. As the integration in the numerator in Eq. (6) is hard to perform analytically, we made it numerically employing the values of $I\left(x^{\prime}, y^{\prime}\right)$ calculated in the points of $64 \times 64$ square grid

$$
\begin{equation*}
\bar{I}=\frac{\sum_{i=0}^{k-1} \sum_{j=0}^{k-1} \rho^{2}\left(x_{i}^{\prime}, y_{j}^{\prime}\right) I\left(x_{i}^{\prime}, y_{j}^{\prime}\right)}{\sum_{i=0}^{k-1} \sum_{j=0}^{k-1} \rho^{2}\left(x_{i}^{\prime}, y_{j}^{\prime}\right)} \tag{20}
\end{equation*}
$$

where $k=64$ is the number of points along one edge of the grid, and

$$
\begin{align*}
& x_{i}^{\prime}=\left(\frac{2 i}{k-1}-1\right) h_{\max }  \tag{21}\\
& y_{j}^{\prime}=\left(\frac{2 j}{k-1}-1\right) h_{\max } \tag{22}
\end{align*}
$$

The value of the $z^{\prime}$ coordinate represents the distance of the interference plane from the point where the extensions of the rays emerging from the modulator converge. The results of the calculation does not depend on $z^{\prime}$, so one can choose any value, avoiding only the special values such as zero or infinity.

## 3. RESULTS AND DISCUSSION

Our analysis concerns two popular electrooptic crystals, namely KDP and $\mathrm{LiNbO}_{3}$. Calculations for KDP were performed using the values of refractive indices $n_{\mathrm{o}}=1.5075$ and $n_{\mathrm{e}}=1.4670$ for wavelength $\lambda=0.63 \mu \mathrm{~m}$ [11], and the coefficients of the linear electro-optic effect $r_{231}=-8.6 \cdot 10^{-12} \mathrm{mV}^{-1}$ and $r_{123}=$ $-10.5 \cdot 10^{-12} \mathrm{mV}^{-1}$ [12]. Due to the analysis of harmonic distortions performed for KDP crystal, also the coefficients of the quadratic electrooptic effect was taken into account: $g_{1111}=-3.4, g_{1122}=-0.2, g_{3311}=-0.7$ [13], and $g_{1212}=-1.4$ [14], all in units of $10^{-20} \mathrm{~m}^{2} \mathrm{~V}^{-2}$. In the case of $\mathrm{LiNbO}_{3}$ crystal the refractive indices are $n_{o}=2.2885$ and $n_{\mathrm{e}}=2.2014$ for $\lambda=0.63 \mu \mathrm{~m}$, and the coefficients of the linear electro-optic effect $r_{113}=8.6, r_{222}=3.4, r_{333}=30.8$, and $r_{232}=28.0$, all in units of $10^{-12} \mathrm{mV}^{-1}$ [15].

We considered the case of sinusoidal electric field $E(t)=E_{0} \sin (\omega \mathrm{t})$ and performed calculations of the modulation index $A_{\omega}=I_{\omega} / I_{0}$, where $I_{0}$ is the constant component of the averaged light intensity $\bar{I}$ transmitted by the system
and $I_{\omega}$ is the component of the fundamental frequency $\omega$. Then, the relative change $\Delta_{\omega}$ in the modulation index $A_{\omega}$ was calculated

$$
\begin{equation*}
\Delta_{\omega}=\left(A_{\omega}-A_{\omega}^{\mathrm{id}}\right) / A_{\omega}^{\mathrm{id}} \tag{23}
\end{equation*}
$$

Here $A_{\omega}^{\text {id }}$ is the ideal value of the modulation index derived on the assumption of no inaccuracies, no beam divergence, and modulation at the middle of the linear part of the transmission characteristic of the optical system.

Our results for the modulator working with longitudinal modulating field in KDP crystal are shown in Fig. 1. In this system, the modulation index for the ideal case is

$$
\begin{equation*}
A_{\omega}^{\mathrm{id}}=2 \pi n_{\mathrm{o}}^{3} r_{123} U_{0} / \lambda, \tag{24}
\end{equation*}
$$

where $U_{0}$ is the amplitude of the modulating voltage. As an example of the modulator working with transverse field we chose the system with $\mathrm{LiNbO}_{3}$ crystal, where

$$
\begin{equation*}
A_{\omega}^{\mathrm{id}}=2 \pi n_{\mathrm{o}}^{3}\left|r_{112}\right| U_{\mathrm{o}} L /(\lambda T) \tag{25}
\end{equation*}
$$

and $r_{112}=-r_{222}$. The results shown in Fig. 2 were obtained for the crystal in the form of a cube, so geometrical path $L$ of the light is equal to the thickness $T$ of the crystal in the direction of field lines.

The results presented in Figs. 1 and 2 indicate that the divergence $\Psi$ should not have a significant influence on the work of the modulators when a gas laser is applied. The divergence $\Psi$ for such lasers varies from 0.66 to $2.35 \mathrm{mrad}[3,4]$, what corresponds to values $0.038 \div 0.135$ degrees. Thus, we can estimate that the critical size of the crystal, at which the divergence becomes important, is of the order of tens of centimeters, while in typical electrooptic applications the size do not exceed the order of a few centimeters. The results presented correspond to red light $\lambda=630 \mathrm{~nm}$. For visible light and shorter wavelengths, the divergence becomes a little more important, however, this does not change fundamentally our conclusions.

In the case of uncolimated semiconductor lasers the divergence of emitted light beam seems to be too large to use them as a light source for electrooptic modulators without additional collimation optics. A reduction of the crystal size allows one to increase the tolerance of the modulator for divergence of the entering beam. However, to obtain good modulation depth for uncollimated semiconductor lasers, the size of crystal along the light propagation should be many times smaller than in the transverse directions, for which the minimum size is imposed by the diameter of the beam. Thus, additional lenses seems inevitable for lasers applied in electrooptic modulators. The light divergence for
collimated diode laser assemblies does not exceed $0.6 \mathrm{mrad}(\perp) / 1.8 \mathrm{mrad}(\|)$ [3,4]. Unfortunately, the use of lenses is expensive.


Fig. 1. Relative change $\Delta_{\omega}$ in the modulation index caused be the beam divergence $\Psi$ for various lengths $L$ of the KDP crystal in the configuration $\boldsymbol{\sigma}=(0,0,1)$ and $\mathbf{E}=(0,0, E)$. The absence of any inaccuracies in the crystal cutting and alignment was assumed. The amplitude of the field strength depend on the crystal length $E_{0}=U_{0} / L$, where the voltage amplitude on the electrodes is always $U_{0}=10 \mathrm{~V}$


Fig. 2. Relative change $\Delta_{\omega}$ in the modulation index caused be the beam divergence $\Psi$ for various lengths $L$ of the $\mathrm{LiNbO}_{3}$ crystal in the configuration $\boldsymbol{\sigma}=(0,0,1)$ and $\mathbf{E}=(0, E, 0)$. The absence of any inaccuracies in the crystal cutting and alignment was assumed. The amplitude of the field strength depend on the crystal length $E_{0}=U_{0} / L$, where the voltage amplitude on the electrodes is always $U_{0}=10 \mathrm{~V}$

Introduction of an additional beam divergence, far above the level of the natural divergence of typical HeNe lasers, slightly improves the tolerance of electrooptic modulator for inaccuracies in crystal cutting and alignment, but according to our numerical results this effect is so weak (see, Figs. 3 and 4) that it is not worth further interest. This is a negative result, however, with important implication. It allows to indicate the direction for further works related to the improving the accuracy of electro-optic measurements in configurations with the light propagating along the optic axis.

Previously it has been reported in Ref. [1] that small deviations of the light from the optic axis of the crystal may lead to significant harmonic distortions in the modulator reply. As these earlier calculations has been performed only for parallel light beam, we performed harmonic analysis for the case when the beam is simultaneously divergent and its central axis is rotated relative to the optic axis due to small inaccuracies in the crystal cutting and alignment. The results obtained show that in any case divergence of the light does not increase harmonic distortions and may even slightly reduce them. Therefore small inaccuracies in the crystal cutting and alignment appear to be the main source of distortions.


Fig. 3. Relative change $\left|\Delta_{\omega}\right|$ in the modulation index caused by the beam divergence $\Psi$ and (a) inaccuracies in crystal alignment $\beta_{\mathrm{a}}$ and $\gamma_{\mathrm{a}}$ in the absence of inaccuracies in crystal cutting; (b) inaccuracies in crystal cutting $\beta_{c}$ and $\gamma_{c}$ in the absence of inaccuracies in crystal alignment. Conditions $\beta_{\mathrm{a}}=\gamma_{\mathrm{a}}$ and $\beta_{\mathrm{c}}=\gamma_{\mathrm{c}}$ correspond to the direction of the gradient of $\Delta_{\omega}$ in $\left(\beta_{a}, \gamma_{a}\right)$ and ( $\beta_{c}, \gamma_{c}$ ) coordinates, respectively. The intended configuration is: KDP crystal, $\boldsymbol{\sigma}=(0,0,1), \mathbf{E}=(0,0, E), L=1 \mathrm{~cm}$, and $E_{0}=10^{3} \mathrm{~V} / \mathrm{m}$


Fig. 4. Relative change $\left|\Delta_{\omega}\right|$ in the modulation index caused by the beam divergence $\Psi$ and (a) inaccuracies in crystal alignment $\beta_{a}$ for $\gamma_{a}=0$ and the absence of inaccuracies in crystal cutting; (b) inaccuracies in crystal cutting $\beta_{\mathrm{c}}$ for $\gamma_{\mathrm{c}}=0$ and the absence of inaccuracies in crystal alignment. Conditions $\gamma_{\mathrm{a}}=0$ and $\gamma_{\mathrm{c}}=0$ correspond to the direction of the gradient of $\Delta_{\omega}$ in $\left(\beta_{a}, \gamma_{a}\right)$ and $\left(\beta_{c}, \gamma_{c}\right)$ coordinates, respectively. The intended configuration is: $\mathrm{LiNbO}_{3}$ crystal, $\boldsymbol{\sigma}=(0,0,1), \mathbf{E}=(0, E, 0), L=1 \mathrm{~cm}$, and $E_{0}=10^{3} \mathrm{~V} / \mathrm{m}$

## REFERENCES

[1] Izdebski M., Kucharczyk W., ICSSC'2000: Growth, Characterization, and Applications of Single Crystals, Rogalski A., Adamiec K., Madejczyk P. (ed.), SPIE Proc. 4412 (2001) 400.
[2] Izdebski M., Kucharczyk W., Acta Physicae Superficierum 6 (2004) 159.
[3] Catalog of Melles Griot products, Optics, opto-mechanics, lasers, instruments, Melles Griot (1995/96).
[4] Catalog of Thorlabs products, Tools of the trade, volume 19, Thorlabs Inc. (2007).
[5] Osiński M., Nakwaski W., Vertical cavity surface emitting laser devices, Li H.E., Iga K. (ed.), Springer Verlag (2003) 135.
[6] Izdebski M., Kucharczyk W., Raab R.E., J. Opt. Soc. Am. A 21 (2004) 132.
[7] Izdebski M., Kucharczyk W., J. Opt. Soc. Am. A 23 (2006) 1746.
[8] Izdebski M., PhD dissertation, Technical University of Łódź, Institute of Physics (2004).
[9] Ścierski I., Ratajczyk F., Optik 68 (1984) 121.
[10] Ratajczyk F., Optyka ośrodków anizotropowych, PWN, Warszawa (1994); Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław (2000).
[11] Ghosh G.C., Bhar G.C., IEEE J. Quant. Electron. QE 18 (1982) 143.
[12] Hellwege K-H., Hellwege A.M. (ed.), Numerical Data and Functional Relationships in Science and Technology, Landolt-Börnstein, New Series, Group III, vols 11 and 18, Springer (1979 and 1984).
[13] Gunning M.J., Raab R.E., Kucharczyk W., J. Opt. Soc. Am. B 18 (2001) 1092.
[14] Górski P., Mik D., Kucharczyk W., Raab R.E., , Physica B 193 (1994) 17.
[15] Maldonaldo T.A., Gaylord T.K., Appl. Opt. 27 (1988) 5051.

# ZASTOSOWANIE MODULATORÓW ELEKTROOPTYCZNYCH DO ROZBIEŻNYCH WIĄZEK ŚWIATEA 

## Streszczenie

Przeprowadzono numeryczną analizę elektrooptycznych modulatorów natężenia światła pracujących z rozbieżną wiązką światła przy wykorzystaniu rachunku macierzy Jonesa. Zastosowany model dopuszcza małe niedokładności wycięcia i orientacji kryształu elektrooptycznego, które moga być uwzględnione razem z rozbieżnością padającej wiązki światła. Wyniki obliczeń pokazują, że rozbieżność wiązek światła wytwarzanych przez typowe lasery gazowe nie powinna znacząco wpływać na pracę elektrooptycznych modulatorów, nawet wtedy gdy stosowane są duże kryształy o rozmiarach rzędu kilkudziesięciu centymetrów. Diody laserowe nadają się jednak do użycia z modulatorami tylko w postaci precyzyjnych modułów zawierających optykę formującą równoległą wiązkę światła.

