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MULTIOBJECTIVE OPTIMIZATION OF THE SEMI-OPEN IMPELLER IN A CENTRIFUGAL PUMP BY A MULTILEVEL METHOD

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The complete optimization task for the case of the semi-open impeller with straight blades requires description of its geometry by means of, at least, eighteen design variables. In the case of constant meridional cross-section, required are at least eight design variables. Solution of the task with such a great vector of design variables requires much more time. One of the ways to obtain solutions with great variety of design variables is a comprehensive approach to the task including on the partition into minor subtasks. After decomposing the optimization task, one should choose a procedure for solving it. One of such procedures is parametric optimization, which is a two-stage minimization (maximization) method. This optimization is carried out in two levels. On the lower level, the multi-optimization of the decomposed parts of the tasks, depending on design variables, is being held. The solution of the lower level is used in the upper level (coordinating level) to find optimal coordination variables. It has been shown that the result of multilevel optimization and the whole task optimization is the same in limits of accepted accuracy of calculation of the objective functions. Time of the calculation for the multilevel optimization task is over four times shorter than the time of the undecomposed task.

Key words: centrifugal pump, semi-open impeller, multilevel optimization

1. Introduction

Design methods published in the available literature concern pumps which work in the nominal point. Input data for calculation procedures are nominal point parameters: pump head H_N and capacity Q_N. The point is described by the maximum value of total efficiency $\eta_N = \eta_{max}$. The reference level for calculated efficiency of designed pumps are efficiencies of pumps already existing.

Empirical formulas allow one to determine efficiencies in output data, however they do not answer if could reach higher values. This fact was a direct cause of inserting an algorithm into the design optimization methods describing objective function (efficiency) extremes and being determined on the base of numerical calculus, results of velocity and pressure fields. Oftentimes, an additional demand is the accomplishment of a pump of the minimal available NPSH in the surrounding of its nominal work point. Due to that, the second objective function is minimal of the available NPSH of the pumps first stage.

The methodology of proceeding during the design process of engineering structures is subordinated by practical considerations, whose aim is to achieve some real solutions. Activities tending to receive the optimal solution in the sense of Pareto should contain:

- Normalization of partial objective functions, which represent the essential treatment in the case when partial criteria of evaluation are presented in different units or differ in scale. Normalization is an essential procedure.
- Scalarization of normalized objective functions with use of weighting factors.
- Determination of compromised solutions set.
- Determination of favorite function set.
- Selection from the set of compromised solution, a subset of preferable solutions.
- Coming to a decision in choosing the best solution.

Scalarization of a vector-valued objective function is beneficial treatment leading one to receive practical solutions in optimization tasks. Due to the scalarization, the multi-objective problem is transformed to a single-objective task. It determines task simplification, however it is often used in practice. The scalarization is proceeded on the base of the following formula

$$\overline{F}(x) = \sum_{i=1}^{N} w_i \overline{\overline{F}}_i(x) \tag{1.1}$$

where: w_i is the weighting factor of the i-th partial objective function, N – number of partial objective functions, $\overline{F}_i(x)$ – normalized partial objective functions.

In the engineering practice, there are oftentimes met tasks of bi-optimization (with two factors). The choice of the weighting factor most often results from the primary objective function which appears as the cost of manufacturing and exploitation of the construction. Giving its real value is impossible without information about conditions in which the construction has to work (it is a pump in this very case): what is the working period, which material is used, etc.

The designer often gives a set of solutions for different weighting factors, the so called Pareto optimal set, and the decision is undertaken by decision-maker on the base of information, analysis and design problem formulation by Ostwald (2003).

2. Proceeding in impellers pump design

In the paper, the design method of semi-open impellers is described with special regard to systems with low specific speed ($n_q < 20$)). The main dimensions of the impeller (see Fig. 1) are determined with a method based on the one-dimension flow theory, completed with empirical relationships and dependences, determined by own research.

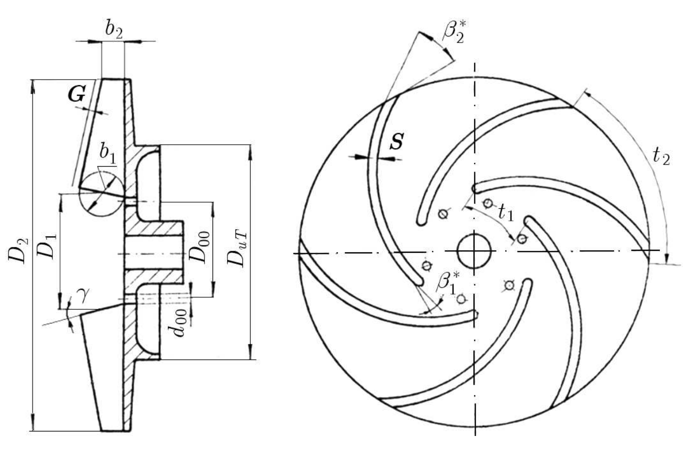

Fig. 1. Geometry of a semi-open impeller – head dimension

In the case of semi-open impellers, the one-dimensional theory is completed with elements describing phenomena responsible for the existence of non-rotating shroud and blade tip (see Fig. 1).

Results of one-dimensional calculus establish a basis for the next design stage based on the three-dimensional flow analysis, during which local fluid parameters (velocity and pressure) are calculated.

Whilst then, the observed disadvantageous hydraulic phenomena, e.g. vortex zone (see Fig. 2), which increment the hydraulic loss, cannot be identified by means of 1D flow method, which establishes a basis for correction shapes in the blade and meridional channel profile. In such a case, calculations are executed once more until satisfactory results are obtained alongside with design assumptions.

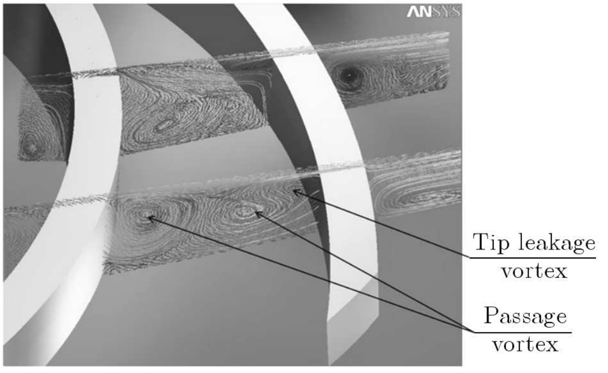

Fig. 2. Secondary flows view received during RANS flow calculus

Such "traditional" use of 3D methods requires capital knowledge and engineer's institution from the designer. This does not guarantee the maximal available efficiency, yet only better than that obtained from calculations based on a 1D model. Formally, the introduction of global optimization methods by definition of an objective function on the base of the results laying out 3D flows using numerical methods for fluid mechanics, allows one to obtain the highest possible efficiency under given limits. The hydraulic loss in channels decides on the pumps efficiency. The reference level for calculated efficiencies of designed pumps are efficiencies of already existing machines (see Fig. 3).

Empirical formulas allow one to determine efficiencies bounded with output data H_N, Q_N for pump design, however they do not answer if could reach higher values. This fact was the direct cause of inserting an algorithm into the design optimization methods describing objective function (efficiency) extremes and being determined on the base of numerical calculus, results of velocity and pressure fields.

Pumps should also be characterized by non-cavitational work in whole range of efficiency change. Impellers are usually designed for nominal parameters of pump operation. It does not guarantee the work of impellers without cavitation in different nominal efficiencies. And that is why the second objective function in this method is the minimal region of the vapour phase in the in-

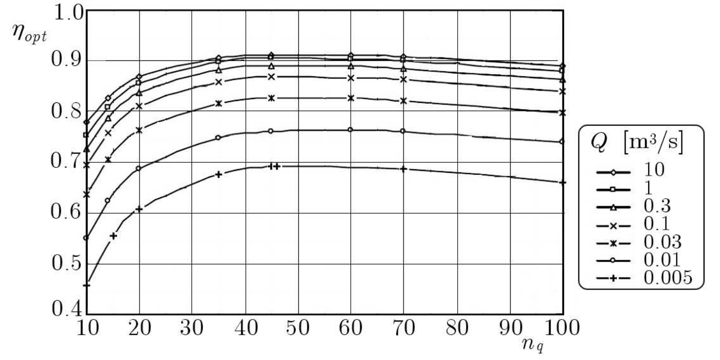

Fig. 3. Obtaining overall efficiencies for single-stage radial pumps depending on the specific speed n_q (Gulich, 2008)

let part of the impeller, which is the standard for NPSH. Both functions are defined on the set of decision parameters, limited with standardized design parameters, such as: thickness of the blade resulting from manufacturing technology and strength of the material and flowability (such as head of pump lift).

Oftentimes, one of the dimensional limits of the impellers development is the pump casing. During modernisation, often the pump casing itself is a certain limit. The casing, for economical reasons, has to stay unchanged. This makes limitations to the dimensions of impellers.

The objective functions in this method are calculated on the base of results obtained from solving the task for double-phase RANS flow, determined with ANSYS-CFX software. Before making up the decision about the method of optimization, some characteristics for given parameters of both objective functions were investigated.

As a result of this analysis, it was found that both of them have a form of non-smooth functions (Papierski, 2008; Thevenin and Janiga, 2008). For the smooth function there was a random noisy function applied. For such an objective function, gradient optimization algorithms are most often used (ex. Broyden-Fletcher-Goldfarb-Shanno algorithm). They could not find the global extreme and stop in one of many local minima.

As the tool for optimization, there were applied algorithms for evaluation of the global extreme of non-smooth functions. Implicit Filtering (Choi *et al.*, 2001; Kelley, 1999) algorithm was implemented, among others, which appeared to be useful for a small number of decision variables. A part from the chosen initial point, it finds the global extreme of the objective function. In the numerical verification procedure, there was used DIRECT algorithm (Kelley, 1999) and Genetic Algorithm (Carroll, 2001), which is more effective and guarantees evaluation of the global extreme, usually for a bigger number of decision parameters than 10 (Harinck *et al.*, 2005; Papierski and Błaszczyk, 2010).

The verification should be understood as a process defining whether the implementation of the model exactly represents conceptual description and its solution (AIAA, 1998). It estimates accurately how the equations of model were exactly solved with computer software and does not define if the model has anything to do with reality.

3. Multilevel optimization algorithms

One of the ways in solving tasks with many decision variables is a system-defined approach to the problem, which consists in partition of it into minor subtasks. In engineering problems, this division is being proceeded in a natural way by explicit differences between separate subsystems. That kind of approach is named the multilevel optimization. The cost of calculations can be considerably reduced if the task is being divided into subtasks which are solved separately. Partial tasks are connected within a certain relation between each other with one parent task. It is used to be named the decomposition and coordination (Findeisen *et al.*, 1980; Ostwald, 2003).

From the theoretical point of view, the decomposition and coordination relies on replacement of the original problem by another problem called the decomposed problem. The specific feature is the conflict between the primary system and subsystems due to the fact that the sum of subsystem solutions is not the solution to the original problem. The coordination depends on the specific relation between these subsystems.

To solve the original problem using the decomposition one should divide the $\boldsymbol{x}$ vector into two components: the vector of coordinating variables $\boldsymbol{y}$ and the vector of decomposed task decision variables $\boldsymbol{z}$. The vector of decomposed decision variables is divided into a set of subsystems $z_1, \dots, zi, \dots, z_N$ with respect to the objective functions set p_i

$$\begin{aligned}\min_{\boldsymbol{x}=(\boldsymbol{y},\boldsymbol{z})\in X} Q(\boldsymbol{x}) &= \min_{\substack{\boldsymbol{y}\in Y, \boldsymbol{z}\in Z\\ (Y\times Z)\in X}} Q(\boldsymbol{y},\boldsymbol{z}) = \\ &= \varPhi\Big[\min_{\substack{\boldsymbol{y}\in Y\\ \boldsymbol{z}^1\in Z^1(\boldsymbol{y})}} p_1(\boldsymbol{y},\boldsymbol{z}^1),\dots,\min_{\substack{\boldsymbol{y}\in Y\\ \boldsymbol{z}^N\in Z^N(\boldsymbol{y})}} p_N(\boldsymbol{y},\boldsymbol{z}^N)\Big]\end{aligned} \tag{3.1}$$

where: $\boldsymbol{x}$ is the vector of decision variables, X – set of acceptable solutions, $\boldsymbol{y}$ – vector of coordination variables, Y – set of acceptable coordination task solutions, $\boldsymbol{z}$ – vector of decomposed decision variables, Z – set of acceptable subsystem solutions, $\boldsymbol{z}^1, \dots, \boldsymbol{z}^i, \dots, \boldsymbol{z}^N$ – separate parts of the vector $\boldsymbol{z}$, $Z_i(y)$ – set of vectors $\boldsymbol{z}^i$ dependent on $\boldsymbol{y}$ $(i = 1, 2, \dots, N)$, Q – objective function of the decomposed task, $\varPhi$ – global objective function of the decomposed task, p_i – objective function of the i-th subsystem $(i = 1, 2, \dots, N)$.

After decomposing the optimization problem one should choose a methodology of task solution. One of them is the parametric optimization method (Findeisen *et al.*, 1980; Ostwald, 2003) which is a two-stage minimization (maximization) method.

This optimization depends on the fact that it is executed in two stages shown in Fig. 4. On the lower level, there is a multiple optimization of decomposed parts of the task carried out, in relation to their decision variables. The result of optimization on the lower stage is used in the upper level (coordinative) used for finding the optimal values of coordinating variables.

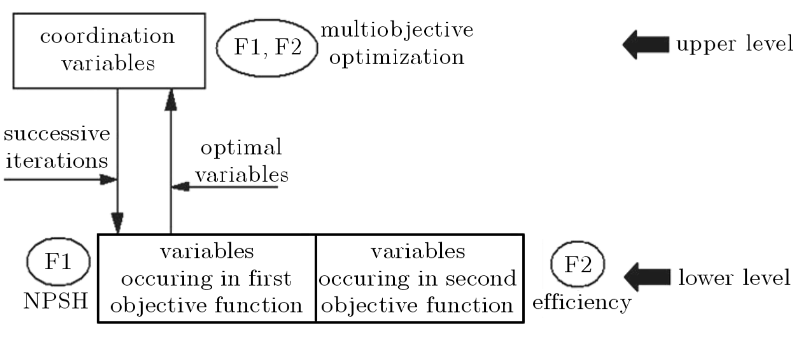

Fig. 4. Scheme of multilevel parametric optimization

3.1. Multilevel optimization in applications for pump impellers

In the case of optimization which concerns the design of pumps impeller, the division of the task into subsystems is easy to be realised. With respect to suction (cavitational) properties of the impeller, the most significant are the following parameters describing the shape of blades (Fig. 1):

β_1^* – inlet blade angle,

$\%M', \beta^*$ – both coordinates of the third-order Bezier control point which describes the blade angle (see Fig. 5), where $\%M'$ is non-dimensional blade length in the meridional section, expressed in percents,

G – shroud gap width,

D_1 – blade leading edge inlet diameter,

γ – lean blades angle in the meridional section,

b_1 – blade leading edge width,

z – number of blades.

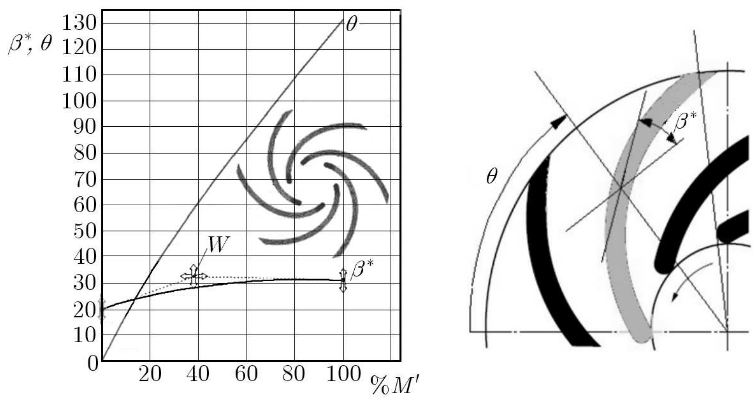

Fig. 5. Shape of blades described with the tracing angle β^*

In terms of the efficiency, the most significant are the following parameters describing the shape of blades:

β_2^* – outlet blade angle,

$\%M', \beta^*$ – both coordinates of the third-order Bezier control point which describes the blade angle tracing,

G – shroud gap width,

D_2 – blade trailing edge diameter,

b_1, b_2 – blade leading and trailing edge width,

z – number of blades.

Some of the above-mentioned parameters are important for the efficiency as well as for suction characteristics. These parameters are: G, $\%M'$, β^*, b_1 and z.

These parameters function as the coordinating variables in the method of multilevel optimization.

The number of blades, due to the fact that it can be only an integer and there is a narrow range of possible change of this parameter, was subtracted from the set of decision variables and treated as the primary variable to be found out if the solution for the number of blades $z-1$ and $z+1$ gives a better solution. In the analysed case the value $z=6$ was optimal.

The rest of values are treated as parameters creating the set for the one-dimensional flow method.

Obviously, becouse of the increase of available computational power, there is a possibility to enlarge the vector of decision variables.

Additionally, in a later validation, the width at the inlet and outlet of the blade palisade was subtracted from the set of decision variables. It resulted from the fact that with change in the meridional profile the casing of measuring head changes as well, which significantly raises the costs of research (validation).

This very case suites the shape optimization of the impeller without changing the rest of pump components, especially the pump casing.

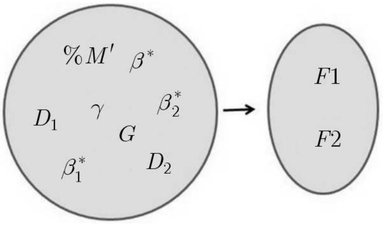

Fig. 6. One-level optimization; (a) space of decision variables, (b) space of objective functions

Within the thus formulated such described optimization task, one have eight decision variables determining the shape of the impellers blade.

Taking into account that each of eight decision variables within the range of acceptable changes (in the unit area) receive 10 values, which is the number of acceptable combinations of different geometry, the so called power of the acceptable set of solutions for one weight value equals 10^8.

The making of the decomposition (partition) of the task into two stages depends on the influence of defined objective functions (Fig. 7):

– Three decision variables β_1^*, D_1, γ having the highest influence on the first objective function. These are the suction criteria;

– Two decision variables β_2^*, D_2 having significant influence on the second objective function (efficiency criteria);

– Coordinating variables G, $\%M'$, β^* having the influence on both objective functions.

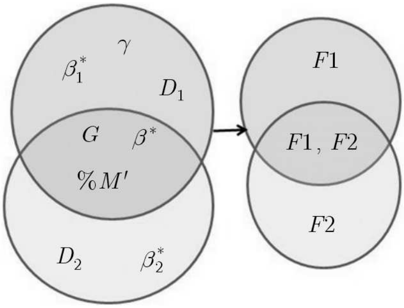

Fig. 7. Multilevel (bilevel) optimizationa); (a) space of decision varaiables, (b) space of objective functions

If just like before, the coordinating variables will receive only 10 values, the amount of the acceptable set of solutions will be $10^3(10^2 + 10^3)$. This number is 91 times lower than that corresponding to the undecomposed set.

There was the following multilevel optimization algorithm processed (Fig. 4):

- The coordinating values G, $\%M'$, β^* were established according to the results obtained from the 1D flow method.
- Optimization of the first objective function, which is the NPSH characteristic, was carried out in relation to the decision variables β_1^*, D_1, γ. Decision variables β_2^*, D_2 were taken from the previous iterative stage (for the first time they were found from the 1D flow method).
- Secondly, an optimization of the second objective function, i.e. the efficiency in relation to the decision variables β_2^*, D_2 was done. Decision variables β_1^*, D_1, γ were left with values obtained in point 2 of this algorithm.
- In the upper coordinative stage, the multi-criteria optimization was established with the weight coefficient equal to 0.5.
- The calculations were repeated from point 2 until the difference of standardized objective function between the last and previous iteration was smaller than the assumed accuracy. The assumed accuracy was 0.01. This value was determined experimentally.

  Higher accuracy caused certain changes resulting from the objective function, which was burdened with random perturbation.

The comparison between the multilevel optimization and original optimization is shown in the Pareto sense in Fig. 8.

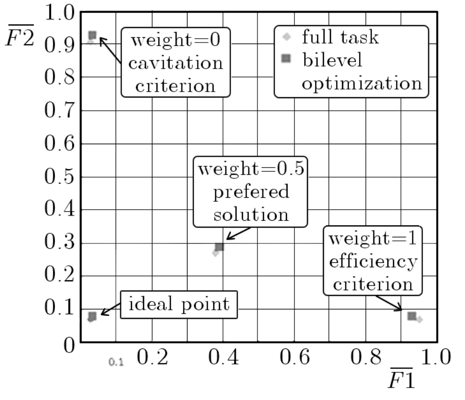

Fig. 8. Value of partial objective function $\overline{F}_1$, $\overline{F}_2$ for full optimization and multilevel optimization. Partial objective functions were normalized with min-max method

4. Validation of design method

The obtained numerical solution is based on the double-phase RANS flow model. It was established on the grid compromising the accuracy and the rate of the calculations. On the research stand (see Fig. 9), some measurements

Fig. 9. General view of test stand for semi-open impeller research at the Institute of Turbomachinery, Technical University of Lodz

aiming at the comparison between optimized parameters of impellers with the real values and at the comparison of parameters found from the traditional one-dimensional method with those obtained by the multilevel optimization method were done.

The result of optimization was the subject for final experimental verification (validation) comparing real parameters of the pump with the impeller designed on the base of one-dimensional method using optimization.

Results of the research are presented in Figs. 10-13. The real efficiency of the pump with optimized impellers is higher by 3.5%, yet the net positive suction head (NPSH3) is lower by about 0.65 m.

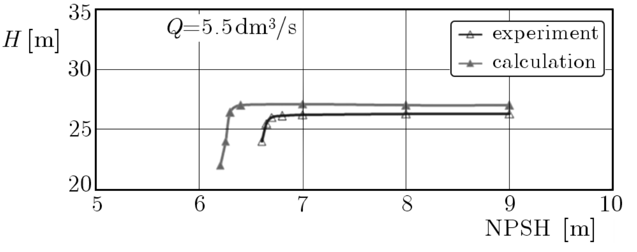

Fig. 10. Comparison of the numerical and measurement results of NPSH characteristics

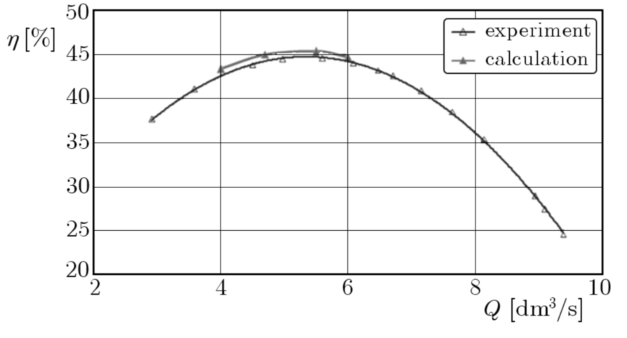

Fig. 11. Comparison of the numerical and measurement results of the efficiency

5. Conclusions

It has been shown (Fig. 8) that the result of multilevel optimization and the full optimization task is the same as far as the accepted accuracy of the objective functions is concerned. The calculation time for the multilevel optimization task is over four times shorter than the time of the undecomposed task.

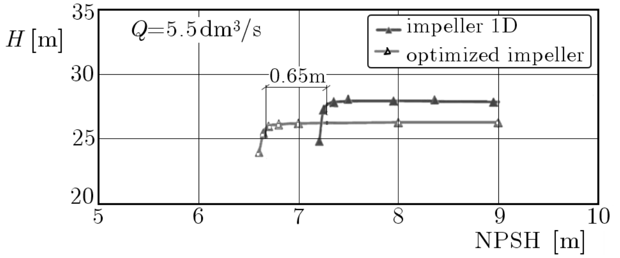

Fig. 12. Comparison of NPSH properness of the impellers designed by the 1D method and with the optimization tool

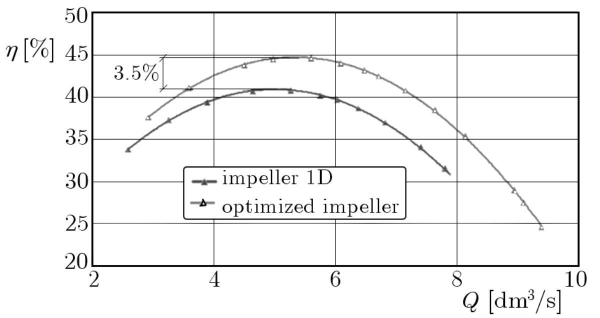

Fig. 13. Comparison of the impeller efficiency designed using the 1D method and the optimization tools

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Wielokryterialna optymalizacja półotwartego wirnika odśrodkowej pompy metodą wielopoziomową

Streszczenie

Pełne zadanie optymalizacyjne dla przypadku półotwartego wirnika wymaga z łopatkami o pojedynczej krzywiźnie wymaga opisania jego geometrii za pomocą co najmniej 18 zmiennymi decyzyjnymi, a w przypadku niezmiennego przekroju merydionalnego potrzeba co najmniej 8 zmiennych decyzyjnych. Czas rozwiązania zadania o tak wielkim wymiarze wektora zmiennych decyzyjnych jest bardzo duży. Jednym ze sposobów rozwiązania zadań z dużą ilością zmiennych decyzyjnych jest systemowe podejście do zagadnienia polegające na podziale problemu na mniejsze części.

Po zdekomponowaniu problemu optymalizacyjnego należy wybrać metodę rozwiązania zadania. Jedną z takich metod jest metoda optymalizacji parametrycznej, która jest dwuetapową metodą minimalizacji (maksymalizacji). Optymalizacja ta polega na tym, że dokonujemy jej na dwóch poziomach. Na poziomie dolnym przeprowadza się wielokrotną optymalizację zdekomponowanych części problemu względem ich zmiennych decyzyjnych. Wynik optymalizacji na poziomie dolnym jest wykorzystywany na poziomie górnym, zwanym koordynacyjnym, do znajdowania optymalnych wartości zmiennych koordynacyjnych. Wykazano, że wynik optymalizacji wielopoziomowej i pełnego zadania optymalizacji jest taki sam w granicach przyjętej dokładności obliczeń funkcji celu. Czas obliczeń zadania optymalizacji wielopoziomowej jest ponad czterokrotnie mniejszy niż zadania niezdekompowanego.

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