Theoretical analysis of the foil bearings dynamic characteristics
(MAT179-15)

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Abstract: In the theoretical analysis, three systems were identified: a rotor, a gas film and a flexible structure. The mathematical analysis involves formulation of analytical equations for each of these elements and determination of their interactions. It was found that the rotor dynamics was subject to Newton's second law of motion, the gas flow in the bearing could be described by the Reynolds equation, whereas a spring-damper model was selected for the structural analysis. The Reynolds equation is a differential equation the exact solution to which is unknown. The work describes the finite difference method in detail, where the partial derivatives in the Reynolds equation are replaced by a system of algebraic equations. In order to solve the resulting system, the Alternating Direction Implicit method (ADI) was used. Thanks to those calculations, it was possible to determine the bearing dynamic characteristics using both the linear and non-linear method.

1. Introduction

The foil bearing is a modern type of oil–free, aerodynamic bearing used in high speed turbomachines such as high temperature compressors and microturbines. The demand for this type of solution is derived from electricity production for micro combined heat and power (CHP) plants, so–called distributed energy resource (DER), where the power of the installation is usually less than 5 kWe. The small power system can be based on the Organic Rankine Cycle (ORC) with a low boiling working medium. In this solution the basic machine is a small turbine driving an electric generator. In order to reach the nominal power in the reduced size machine the generator needs to rotate at a speed of 10000–100000 rpm. Such high speeds are impossible to reach with the use of conventional rolling–element bearings or common oil–lubricated bearings, since their operation in these conditions may lead to instability of the machine rotating system. Furthermore, in order to maintain the purity of the cycle, we are searching for a hermetic solution, where the machine is equipped with bearings lubricated with the working medium.

The presented here concept of a foil journal bearing is related to these assumptions.

2. General considerations about gas foil bearings

Because of the method of lift generation two types of gas bearings can be distinguished: self–acting, so called aerodynamic bearings and gas powered or aerostatic bearings. The foil bearing is an aerodynamic support, which is made up of compliant surfaces (a bump and a top foil) (see figure 1), [1]. The uniqueness of foil bearing operation results from the fact that the top foil is clenched during the bearing operation on the rotating journal by means of an elastic bump foil. The aerodynamic film of a very low thickness, theoretically close to the cylindrical one, is generated by viscosity effects.

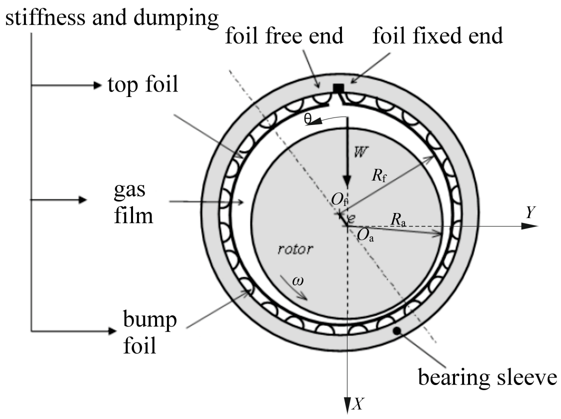

Figure 1. Compliant foil bearing

An important problem of the aerodynamic gas bearings application is related to the start–up and the shut–down in contact with the shaft surface, and thus:

– a high drive moment for the start-up (disputable from the viewpoint of turbine mechanical characteristics) is needed,
– there is a limited number of start-up/shut-down cycles (wear),
– in a case of the heat generation, a cooling system is required.

In general gas bearings have a limited load capacity. Furthermore, the analysis of gas foil bearings is difficult due to interactions between the gas film pressure and the complicated deflection of the top foil and the underlying bump strip support structure.

A great advantage of foil bearings is such that they require no external pressurization system for the working fluid and so the surrounding gas can be used as the lubricant. Other benefits are [2]:

– high rotational speeds,
– little maintenance,
– compliance thanks to the bump foil structure,
– compactness,
– no impact on the machinery environment thanks to the absence of external lubrication system.

3. Theoretical analysis of the gas foil bearing

In order to define a mathematical model for foil bearings, at first, three systems have to be isolated: rotor, gas film and elastic structure. In table 1 physical hypothesis are presented for each system.

Table 1. Hypothesis adopted for foil bearing analysis

System	Hypothesis
Rotor	non-deformable in equilibrium position for given rotational speed
Gas film	thin–film compressible, Newtonian fluid isothermal, continue, laminar flow general curvature of the film is negligible non–slip boundary conditions inertial forces and mass forces are negligible viscosity and density do not vary along film thickness
Elastic structure	elastic deformation quasi-static

The mathematical analysis will therefore apply to each system separately and determine their interactions. This section will discuss analytical equations describing the phenomena occurring in the foil bearing.

3.1. Foil bearing geometry and coordinate systems

Due to the geometry of the bearing it is convenient to define a cylindrical coordinate system $\theta\eta z$ (fig. 2) where the z axis coincides with the Z axis of the bearing global coordinate system. The η axis is the radial coordinate measured from points located on the sleeve surface to shaft surface located points. The angle θ determines the circumferential coordinate. It is assumed that the parallel displacement of the journal axis is characterized by the relative eccentricity of the bearing ϵ, where $\epsilon = e/C$, C being the bearing clearance. In the local coordinate system the angle φ is determined for a given equilibrium position by the centers line, i.e. the axis intersecting points $O_{\mathrm{f}}, O_{\mathrm{a}}$ (the angle between the X and X' axis in. fig. 2).

Then the cylindrical geometry of the gas film in the bearing can be found from equation 1.

$$H = 1 - \varepsilon \cos(\theta - \eta_L) \tag{1}$$

where H is the dimensionless film thickness, ε – the dimensionless eccentricity and η_L – the

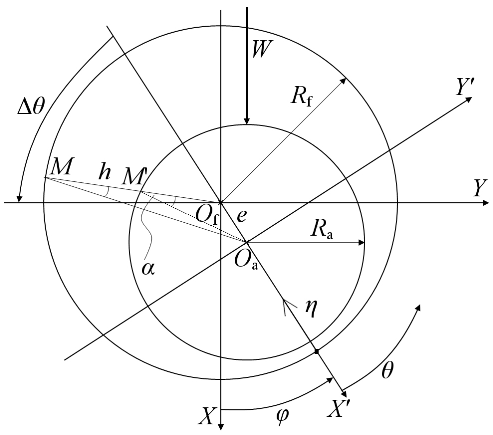

Figure 2. Cylindrical coordinate system for gas film calculation

angle determining the position of the foil locket.

3.2. Hydrodynamic model for gas foil bearings

The theoretical work of Reynolds, [6], is generally considered to be the hydrodynamic lubrication theory basis since it led to the Reynolds Equation. The Reynolds Equation is a partial differential equation derived from the mass and momentum conservation equations (Navier–Stokes equations) governing the pressure distribution of thin viscous fluid films and the lubricant flow. The Reynolds Equation (2) was used to describe the pressure distribution in nearly any type of fluid film bearings under hypothesis cited in table 1.

$$-\frac{\partial}{\partial\theta}\left(\mathbf{PH}^3\frac{\partial\mathbf{P}}{\partial\theta}\right)-\frac{\partial}{\partial\xi}\left(\mathbf{PH}^3\frac{\partial\mathbf{P}}{\partial\xi}\right)+\Lambda\frac{\partial}{\partial\theta}\left(\mathbf{PH}\right)+\frac{\partial}{\partial\tau}\left(\mathbf{PH}\right)=0 \tag{2}$$

where

$\Lambda=\frac{6\mu\omega R^2}{p_\mathrm{a}C^2}$ is the compressibility number,

$\mathbf{P}$ – the dimensionless pressure,

τ – time.

$$\begin{cases} F_x=-\int_{\xi_1}^{\xi_2}\int_0^{2\pi}P\cos\theta\,\mathrm{d}\xi\,\mathrm{d}\theta \\ F_y=-\int_{\xi_1}^{\xi_2}\int_0^{2\pi}P\sin\theta\,\mathrm{d}\xi\,\mathrm{d}\theta \end{cases} \tag{3}$$

When the pressure distribution is known, the aerodynamic lift force can be calculated according to the equation 3.

3.3. Model for the foil structure elastic deformation

The geometry of the flexible structure shown in fig. 3a) is taken into consideration. Its material properties and stresses allow to consider the elastic solid and so the structure can be modelled as series of springs and dampers (see fig. 3b)). Damping properties being originated from friction phenomena.

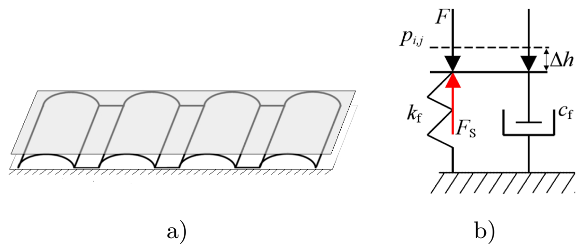

Figure 3. Flexible structure

In order to calculate the deformation of the structure, four hypothesis have been formulated:

H1: The structure deflection depends only on pressure.

H2: The rigidity of the structure is linear and the elastic force is given by the relation $F_{\mathrm{S}} = k_{\mathrm{f}}\Delta h$, where k_{f} is the elasticity coefficient and Δh is a length change of the spring.

H3: There is no axial deformation of the structure.

H4: For given θ and different ξ the deformation has the same value.

Based on these hypothesis the relationship between the change in punctual force, fb_i and foil deflection, h_{ij}, can be expressed by equation 4, [3]

$$fb_i^{t+\mathrm{d}t} - fb_i^t = \frac{h_{ij}^{t+\mathrm{d}t} - h_{ij}^t}{s} + c_{\mathrm{f}}\left(\left.\frac{\mathrm{d}h}{\mathrm{d}t}\right|_{t\to t+\mathrm{d}t} - \left.\frac{\mathrm{d}h}{\mathrm{d}t}\right|_{t-\mathrm{d}t\to t}\right) \tag{4}$$

where $s = 1/k_{\mathrm{f}}$ and c_{f} is a damping coefficient.

3.4. Rotor dynamics

The rotor of mass $2M$ is treated as a rigid structure and it is supported symmetrically in two identical finite lengths bearings (without gyroscopic motion) (see fig. 4). This approach allows to evaluate the influence of the bush behaviour on the rotor trajectory. Forces acting on the rotor result from its own weight, the unbalance, any dynamic (changing in time) loads and the pressure field in the gas film. They depend on the shaft position and the rotational speed.

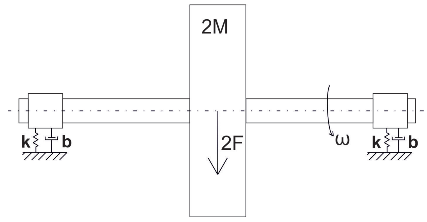

Figure 4. Rotating system model

More specifically, when the journal rotates, each bearing supports, [1]:

– the mass M,

– the static force $W_0 = F$,

– a dynamic force of harmonic type $W_\mathrm{d}(t) = W_\mathrm{d} \sin(\gamma t)$, where γ is a frequency of pulsation,

– a vibrational synchronous excitation due to the unbalance. This excitation is characterised by its eccentricity e_b,

– the aerodynamic forces $F(t)$ calculated from the pressure distribution in the gas film.

This can be expressed by Newton's second law equations:

$$\begin{cases} M\ddot{x} = \sum F_{\mathrm{ext}/x} \\ M\ddot{y} = \sum F_{\mathrm{ext}/y} \end{cases} \tag{5}$$

or with the right–hand side of the equation developed and put in dimensionless form:

$$\begin{cases} \overline{M}\ddot{\overline{x}} = \overline{W}_0 + \overline{W}_{\mathrm{d}/x}(T) + \overline{M} e_\mathrm{b}\omega^2 \cos(T) + \overline{F}_x(T) \\ \overline{M}\ddot{\overline{y}} = \overline{W}_{\mathrm{d}/y}(T) + \overline{M} e_\mathrm{b}\omega^2 \sin(T) + \overline{F}_y(T) \end{cases} \tag{6}$$

where (x, y) are rotor centre coordinates.

According to the linear theory, dynamic properties of aerodynamic gas bearings are usually represented by a set of eight coupled dynamic coefficients (k_{ij}, b_{ij}), linearized around the static equilibrium position of the bearing, [4]. Limited excitation loads allow one to linearize the forces in the bearing film and to determine the equivalent values of coupled bearing stiffness and damping coefficients.

The bearing static equilibrium position determines the initial conditions for an analysis of the bearing dynamics and it is noted by subscript $\cdot_0$. Then, we introduce the position perturbation and the velocity perturbation, whence stiffness and damping coefficients are defined as follows

$$\left.\frac{\partial F_x}{\partial x}\right|_0 = K_{xx},\ \left.\frac{\partial F_x}{\partial y}\right|_0 = K_{xy},\ \left.\frac{\partial F_y}{\partial x}\right|_0 = K_{yx},\ \left.\frac{\partial F_y}{\partial y}\right|_0 = K_{yy} \tag{7}$$

$$\left.\frac{\partial F_x}{\partial \dot{x}}\right|_0 = B_{xx},\ \left.\frac{\partial F_x}{\partial \dot{y}}\right|_0 = B_{xy},\ \left.\frac{\partial F_y}{\partial \dot{x}}\right|_0 = B_{yx},\ \left.\frac{\partial F_y}{\partial \dot{y}}\right|_0 = B_{yy} \tag{8}$$

It should be highlighted that the lack of precise criteria, which allows one to determine the applicability range of the linear method, often leads to serious errors in the dynamic analysis of the rotor–bearings system.

4. Numerical methods

Due to the non–linear nature of the equations of fluid motion, they can be solved in only a few, very specific situations, hence the need for a numerically approximated solution. In order to make possible the approximation process of solving, partial differential equations need to be discretized. As a result of the discretization, differential equations are replaced by an algebraic equations system. There exists different methods of discretization, the most common are: Finite Difference Method (FDM), Finite Element Method (FEM) and Finite Volume Method (FVM). For the project needs the Finite Difference Method was used to solve equations governing the gas film behaviour. All described methods were solved using the structured programming language, Fortran.

By replacing $P^2 = Q$, the Reynolds equation can be rewritten (equation 9). According to the finite difference method, derivatives in partial differential equations can be replaced by central differences, [5]. For example, first derivatives will be approximated according to equations 10 and second derivatives – according to equations 11.

$$\begin{split} 2P\frac{\partial H}{\partial \tau} + \frac{H}{P}\frac{\partial Q}{\partial \tau} + 2\Lambda P\frac{\partial H}{\partial \theta} + \Lambda\frac{H}{P}\frac{\partial Q}{\partial \theta} - H^3\left(\frac{\partial^2 Q}{\partial \xi^2} + \frac{\partial^2 Q}{\partial \theta^2}\right) + \\ -3H^2\left(\frac{\partial H}{\partial \theta}\frac{\partial Q}{\partial \theta} + \frac{\partial H}{\partial \xi}\frac{\partial Q}{\partial \xi}\right) = 0 \end{split} \tag{9}$$

$$\begin{aligned} \frac{\partial H}{\partial \theta} &\approx \frac{H^n_{i,j+1} - H^n_{i,j-1}}{2\Delta\theta} \\ \frac{\partial H}{\partial \xi} &\approx \frac{H^n_{i+1,j} - H^n_{i-1,j}}{2\Delta\xi} \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial^2 Q}{\partial \xi^2} &\approx \frac{Q^n_{i+1,j} - 2Q^n_{ij} + Q^n_{i-1,j}}{(\Delta\xi)^2} \\ \frac{\partial^2 Q}{\partial \theta^2} &\approx \frac{Q^n_{i,j+1} - 2Q^n_{ij} + Q^n_{i,j-1}}{(\Delta\theta)^2} \end{aligned} \tag{11}$$

In order to solve the Reynolds equation, the Alternating Direction Implicit (ADI) method can be used. This method is applied to the finite differences and consists on splitting the calculation into two steps. In the first step derivatives with respect to θ are evaluated while derivatives with respect to ξ are evaluated in the second step. Then, according to the tridiagonal elimination method, the summands are grouped and equations 12 and 14 can be written.

$$A_{ij}^{n} Q_{i,j-1}^{n+1} + B_{ij}^{n} Q_{ij}^{n+1} + C_{ij}^{n} Q_{i,j+1}^{n+1} = f_{ij}^{n} \tag{12}$$

where

$$\begin{aligned} Q_{ij}^{n+1} &= X_{ij} Q_{i,j+1}^{n+1} + Y_{ij} \\ X_{ij} &= \frac{-C_{ij}^{n}}{B_{ij}^{n} + A_{ij}^{n} X_{i,j-1}} \\ Y_{ij} &= \frac{f_{ij}^{n} - A_{ij}^{n} Y_{i,j-1}}{B_{ij}^{n} + A_{ij}^{n} X_{i,j-1}} \\ Q_{i,1}^{n+1} &= Q_{i,N+1}^{n+1} = Q_{i,1}^{n} \end{aligned} \tag{13}$$

$$D_{ij}^{n} Q_{i-1,j}^{n+2} + E_{ij}^{n} Q_{ij}^{n+2} + F_{ij}^{n} Q_{i+1,j}^{n+2} = g_{ij}^{n+1} \tag{14}$$

where

$$\begin{aligned} Q_{ij}^{n+2} &= X_{ij} Q_{i+1,j}^{n+2} + Y_{ij} \\ X_{ij} &= \frac{-F_{ij}^{n}}{E_{ij}^{n} + D_{ij}^{n} X_{i-1,j}} \\ Y_{ij} &= \frac{g_{ij}^{n} - D_{ij}^{n} Y_{i-1,j}}{E_{ij}^{n} + D_{ij}^{n} X_{i-1,j}} \\ Q_{1,j}^{n+2} &= Q_{M,j}^{n+2} = 1 \end{aligned} \tag{15}$$

The calculation of $Q_{1,j}^{n+2}$ ends an iteration cycle. The statements $Q_{ij}^{n} = Q_{ij}^{n+2}$ and $P_{ij}^{n} = \sqrt{Q_{ij}^{n+2}}$ start new iteration process.

5. Rotor trajectory in step function test

First, for the given static force, F_0, the equilibrium position is found. This position is determined by an eccentricity and a centers angle,$(\varepsilon_0, \varphi_0)$ (see fig. 5, green point). Second, for a static force, $F_1 = F_0 + \Delta F$, another equilibrium position is calculated, $(\varepsilon_1, \varphi_1)$. Then, the dynamic calculation has been carried out for the dynamic force, the step function, defined in equation 16.

$$F_d(t) = \begin{cases} F_0 + \Delta F, \ n_t \geq 60 \\ F_0, \ n_t < 60 \end{cases} \tag{16}$$

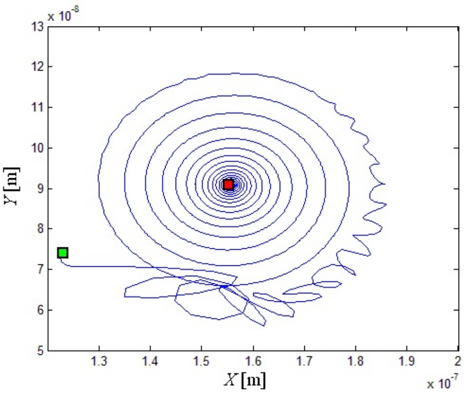

Figure 5. Pressure distribution in the gas film

The rotor trajectory can be observed in fig. 5. The final position, $(\varepsilon_d, \varphi_d)$ (red point) is then compared with $(\varepsilon_1, \varphi_1)$ position. The deviation was less than 1% for φ and about 1.5% for ε.

6. Conclusions

The present paper is a trial to show major equations and methods for the theoretical analysis of a journal foil bearing. The equations were programmed by means of numerical modelization and the model in Fortran have been developed. The ADI numerical method for solving the Reynolds equation has given a good convergence. A choice of the spring–dumper model for the elastic structure deformation is an important simplification, since it requires less computational resources in comparison to finite element method based codes. The rotor trajectory in step function dynamic excitation has been shown. The results pointed to good programme performance. Other numerical tests have revealed a good stability of the foil bearing.

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