9.

Post-ultimate behaviour of thin-walled cold-formed steel open-section members in pure and eccentric compression

This chapter is a review of research in recent years as a result of collaboration between the Department of Steel Structures and Structural Mechanics of Politehnica University of Timisoara and Department of Strength of Materials of Łódź University of Technology related to a load carrying capacity, post-failure behaviour and plastic mechanisms of failure of thin-walled cold-forms steel (TWCFS) members subjected to compression and/or bending.

9.1. Introduction

The EN 1993-1-1 [9.1] code defines four classes of cross-sections of steel structural elements as shown in Fig. 9.1. The role of cross-section classification is to identify the extent to which the resistance and rotation capacity of cross-sections is limited by its local buckling resistance.

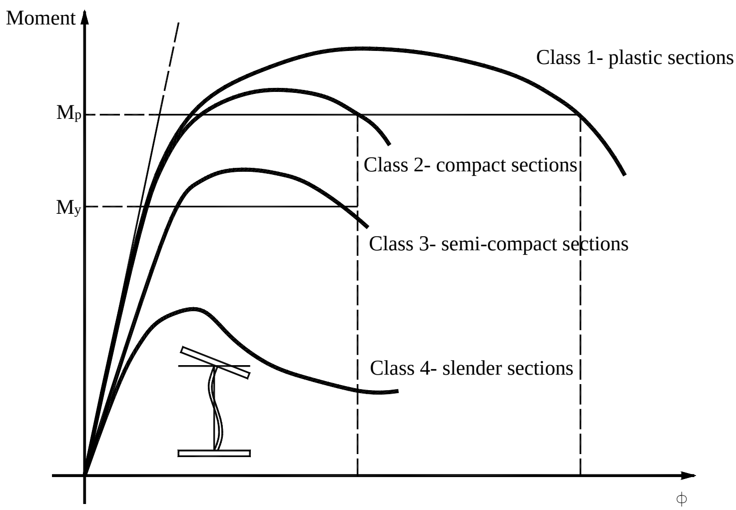
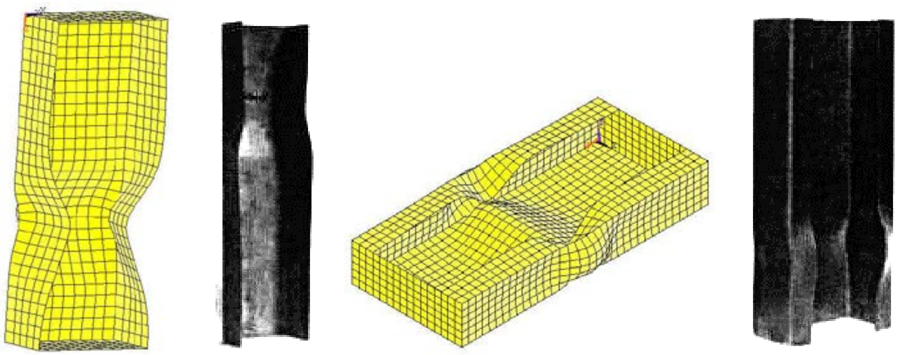
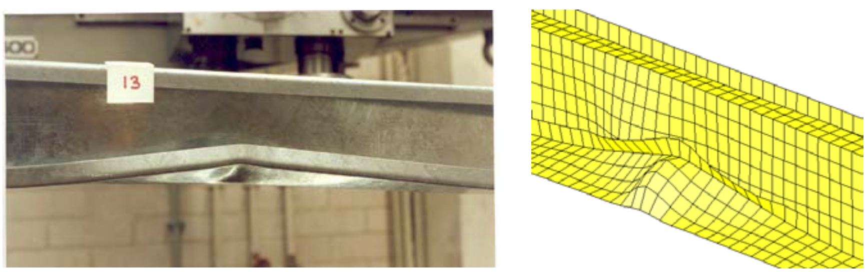


Fig. 9.1. Cross-section behaviour classes

Class 1 cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of the resistance. Class 2 cross-sections are those which can develop their plastic moment resistance, but have limited rotation capacity because of local buckling, while Class 3 cross-sections are those in which the stress in the extreme compression fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength, but local buckling is liable to prevent development of the plastic moment resistance. Class 4 cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross-section. Consequently, for Class 4 cross-sections effective widths may be used to make the necessary allowances for reductions in resistance due to the effects of local buckling.



a) members in compression



b) members in bending

Fig. 9.2. Numerical and experimental evidences of plastic mechanism failure [9.2]

Thin-walled cold-formed steel (TWCFS) structures are usually made of thin-walled members of class 4 sections. They are traditionally considered with no plastic capacity, and consequently non-ductile, mainly due to wall slenderness involving local instability phenomena. Since these sections are prematurely prone to local or distortional buckling, they do not have a real post-elastic capacity and do not have sufficient plastic rotation capacity to form plastic hinges, the failure at ultimate stage of those members, either in compression or bending, always occurs by forming a local plastic mechanism as shown in Fig. 9.2.

Due to the local buckling, its behaviour in the post-buckling range displays large local elasto-plastic and plastic deformations of the cross-section walls. Subsequently, yield lines occur and the plastic mechanism of failure is being formed. This fact suggests the possibility to use the local plastic mechanism analysis to characterize the ultimate strength of such members [9.2, 9.3].

For a slender member, when the localized buckling mode occurs prior to the overall one, the collapse, due to the interaction between these two modes, is always characterised by local plastic mechanism failure. In fact, the interaction occurs between the overall mode, which corresponds to an elastic non-linear behaviour of the TWCFS members and the local plastic buckling of the component walls [9.2, 9.3]. This kind of behaviour is confirmed both by tests and numerical simulations (see Fig. 9.3).

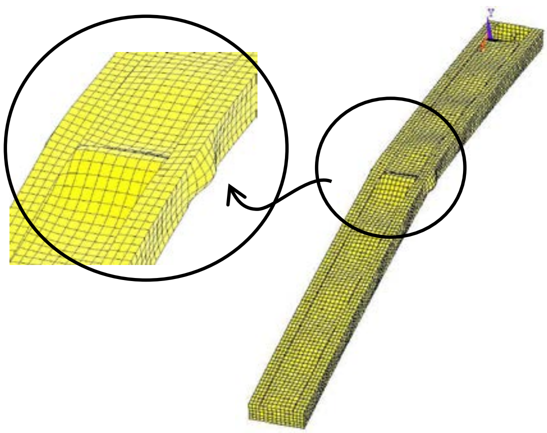


Fig. 9.3. FEM simulation of plastic-elastic interaction between the local plastic mechanism and global buckling for a lipped channel section in compression [9.3, 9.4]

Starting from this real behaviour of thin-walled stub columns and short beams, Ungureanu & Dubina [9.3, 9.4] used the ECBL approach in order to express the plastic-elastic interactive buckling of thin-walled members. The main problem of this approach is to evaluate properly the plastic strength of thin-walled members, via the local plastic mechanism theory and after, the erosion of critical load into the *'plastic-elastic coupling range'*.

The local plastic mechanism depends on the type of the cross-section and slenderness of component walls, as well as the stress state, and can be significantly influenced by imperfections. A proper identification of the geometry of a local plastic mechanism is crucial for the correct evaluation of the post-buckling rigid-plastic curve (*failure curve*), which subsequently results in a correct estimation of the load-carrying capacity of the member, as well as of the energy absorption at collapse [9.4].

Load carrying capacity of such members subjected to simple states of loading (pure bending or pure axial compression) is relatively well determined both on the basis of the theory of thin-walled structures and by the code specifications. However, determination of the load-capacity of TWCFS members subjected to combined loads, particularly eccentric compression, is still an open question and the code specifications for that case should be improved. A first attempt into the identification of plastic mechanisms of short TWCFS members subjected to eccentric compression has been published in [9.5].

9.2. Fundamentals of yield-line analysis

The rigid-plastic theory related to steel structures, based on the rigid-plastic material characteristics, was initially applied to the analysis of simple beams and frames [9.6]. In the 70's the research has been extended to investigate and characterize the plastic mechanisms of steel members. The term 'yield line' was initially used by Jones & Wood [9.7].

The yield line theory applied to thin-walled steel structures allows one to perform an analysis of structural behaviour in the vicinity of ultimate load and in the post-failure stage. The basic assumption is that the plastic mechanism is fully developed and the plastic zones developed in the walls of thin-walled steel member are concentrated at yield lines. At the level of yield lines the material is considered fully plastic. Initially, the strain hardening was neglected in the analysis. Later, the modified yield line theory [9.8] takes into account this phenomenon.

If a plastic mechanism is fully developed, we can assume that the flat parts of the walls (created by yield lines) are non-deformed (there are no membrane deformations) and continuous. In that case it is so-called 'true mechanism' [9.9]. However, in many cases (determined by the geometry of TWS member and loading conditions) true mechanisms cannot be developed. The so-called 'quasi-mechanism' is developed [9.9], where the flat parts of the walls are limited by yield lines but the walls undergo membrane deformation.

The plastic mechanism approach is based on two basic methods, namely the *energy method (work method)* and the *equilibrium strip method*. Using the energy method we apply the Principle of Virtual Velocities of the general following form

$$P \cdot \dot{\delta} = \int_V \sigma_{ij} \dot{\varepsilon}_{ij}^p \ (\beta, \chi) dV \tag{9.1}$$

where

P is generalized load,

δ is the global generalized displacement,

$\dot{\delta}$ is the rate of change of the global generalized displacement,

β is the vector of kinematical parameters of the plastic mechanisms (kinematical admissible displacements),

χ is vector of geometrical parameters of the plastic mechanisms,

σ_{ij} is the stress tensor and,

$\dot{\varepsilon}_{ij}^p$ is the strain rate tensor.

Usually, in the analysis concerning thin-walled members subjected to compression, Eq. (9.1) is rearranged into the following form

$$\delta W_{ext} = \delta W_b + \delta W_m \tag{9.2}$$

where δW_{ext} is the variation of work of external forces, δW_b is the variation of the energy of bending plastic deformation, while δW_m is the variation of the energy of membrane plastic deformation. Eq. (9.2) provides a relation of generalized load (e.g. compressive load, bending moment) in terms of general displacement (e.g. shortening, angle of rotation). The graphical representation of this relation will be termed in the present paper as post-failure curve. Alternatively, Eq. (9.2) may be rearranged into the following form

$$P = \frac{\partial (W_b + W_m)}{\partial \delta} \tag{9.3}$$

The equilibrium strip method treats the plastic mechanism as a compatible collection of strips of infinite small or unit width parallel to the direction of applied force. On the basis of free-body diagram of a separated strip an equilibrium equation is formulated and then, those equations are integrated across the walls of the plastic mechanism, in order to obtain simultaneous equilibrium equation for the mechanism as a whole [9.10]. An application of this method is restricted to the analysis of local plastic mechanisms build of stationary yield lines only. It is widely used in investigations of plated columns under compression and delivers a direct relation between an applied compressive force and the deflection of the column.

A comparative study has been already carried out by Flockhart et al. [9.11], who analysed a thin-walled spot welded box-section beam. He has come to the conclusion that for large rotations of the mechanism (global plastic hinge), the energy absorption determined by the energy method is higher up to 30% than that determined by the equilibrium strip method. Further investigation related to the comparison of those two approaches has been made by Kotełko et al. [9.12] and Kotełko & Mania [9.13]. A discussion of different solutions based on both methods was carried out by Zhao [9.14].

9.3. Plastic mechanisms identified in TWCFS members under compression

Experiments carried out by many researchers on beams or columns built from plate strips, subjected to uniform compression, show, that in such members some simple plastic mechanisms can be distinguished, which have been termed as basic mechanisms. Simultaneously, results of experiments performed, among others, by Murray & Khoo [9.10] confirmed, that even a very complex mechanisms can be described as superposition of some simple basic mechanisms. Murray & Khoo developed and classified 8 basic plastic mechanisms in plate strips under uniform compression [9.10]. They also derived for them relations, determining the failure equilibrium path (load versus deflection).

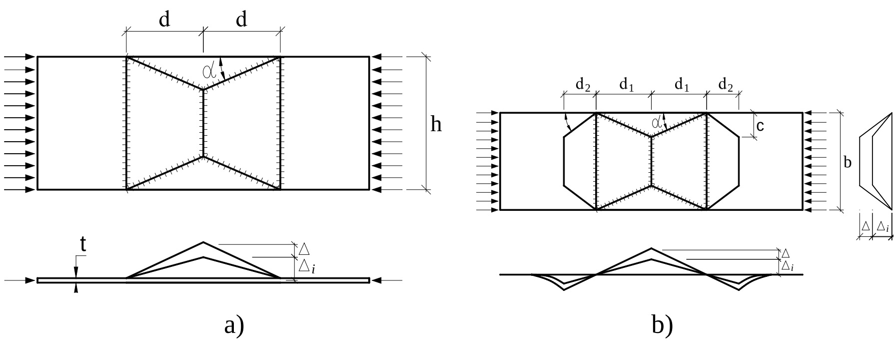
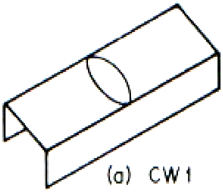
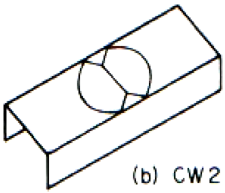
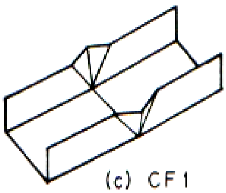
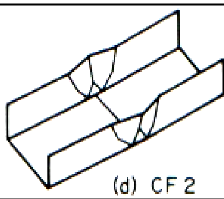
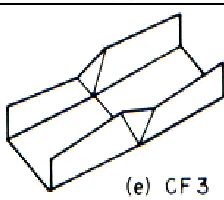


Fig. 9.4. Plastic mechanisms in thin plates under compression: a) pitched-roof, b) roof

In the case of plates subjected to uniform compression, with symmetrical boundary conditions, which corresponds to the case of column web in compression, a pitched-roof mechanism (Fig. 4a) or roof mechanism (Fig. 4b) can develop. The pitched-roof mechanism has been described by some researchers, i.e. Kato [9.15], Korol & Sherbourne [9.16], as well as Sin [9.17] and Mahendran [9.18]. Modifications of pitched-roof mechanisms (mainly roof mechanism) were developed by Rondal & Maquoi [9.19] and Kragerup [9.20].

The plastic mechanism approach of the above mentioned mechanisms, using the *energy method* or the *equilibrium strip method*, were described in details together with corresponding failure equilibrium path equations (load vs. deflection) by Kotełko [9.21].

Table 9.1. Plastic mechanisms of plain channel section members subjected to compression or bending

No.	Mechanism type	Post-failure load-deformation relations - references
1	 (a) CW1	Królak [9.23] Dubina & Ungureanu [9.2,9.3,9.4] Mahendran [9.18]
2	 (b) CW2	Mahendran [9.18] Dubina & Ungureanu [9.2,9.3,9.4]
3	 (c) CF1	Murray & Khoo [9.10], Dubina & Ungureanu [9.2,9.3,9.4]
4	 (d) CF2	Rasmussen [9.24,9.25]
5	 (e) CF3	Rasmussen [9.24,9.25]

A database for plastic mechanisms for thin plates, thin-walled cold-formed steel members in compression and bending has been presented in details by Ungureanu et al. [9.22].

Five plastic mechanisms of failure in channel section columns subjected to axial compression were originally developed by Murray & Khoo [9.10], as shown in Table 9.1.

Mechanisms CW1 and CW2 correspond to the case of the web in compression, while mechanisms CF1, CF2 and CF3 to the case of the flanges in compression. They were also analysed by Dubina & Ungureanu [9.2, 9.3, 9.4], who described the mechanism CW2, and by Rasmussen & Hancock [9.24, 9.25], who analysed the mechanisms CF2 and CF3. In the papers mentioned above the equilibrium strip method has been applied.

9.3.1. Plastic mechanisms of plain channel sections

As mentioned, theoretical models of 3D plastic mechanisms in plain channel section columns were originally elaborated by Murray & Khoo [9.10].

Among them a true mechanism (three-hinge flange mechanism), which develops due to axial compression and/or bending (flanges deflect laterally towards the free edge - CF1, no. 3 in Table 9.1) has been developed.

If a channel is subject to bending or eccentric compression, the web being in compression, a web mechanism can develop. Murray and Khoo distinguished two quasi-mechanisms of that kind, namely flip disc (CW1, no. 1 in Table 9.1) and roof (CW2, no. 2 in Table 9.1). The web mechanism is in fact that one, which develops in the plate subjected to uniform compression, with symmetric boundary conditions. Basic plastic mechanisms in such plates are described in details by Królak [9.23] and Ungureanu et al. [9.22]. Two of those mechanisms, namely the ‘pitched-roof’ and ‘roof’ one, are shown in details in Fig. 9.4.

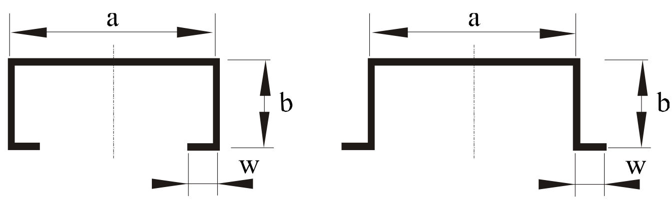
Also in the case of compression and/or bending, the flange being in compression, Murray & Khoo developed two other mechanisms, i.e. a true mechanisms CF3 (no. 5 in Table 9.1) and a quasi-mechanism CF2 (no. 4 in Table 9.1).

9.3.2. Plastic mechanisms in lipped channel sections

Some specific problems appear in the buckling and post-buckling analysis of TWCFS open section mono-symmetric members with edge stiffeners, because of rapid transition from symmetric to anti-symmetric buckling mode for certain buckling lengths, for which local-distortional buckling modes may take place.

These buckling modes influence significantly the member's post-buckling and failure behaviour. Detailed elastic buckling and post-buckling analysis of such members was carried out by Kotełko et al. [9.26, 9.27].

Let us consider two channel sections with edge stiffeners in and out, i.e. lipped channel (Fig. 9.5a) and top hat (Fig. 9.5b), and subjected to uniform compression.



a) b)

Fig. 9.5. Members with edge stiffeners: a) lipped channel section, b) top hat section

The problem of plastic mechanism in lipped channel section column subjected to uniform compression was proposed by Dubina & Ungureanu [9.2, 9.3, 9.4]. The mechanism shown in Fig. 9.6 consists of local roof mechanisms (CW2) in the web and flanges. In some solutions local true mechanisms CF1 in lips are taken into account.

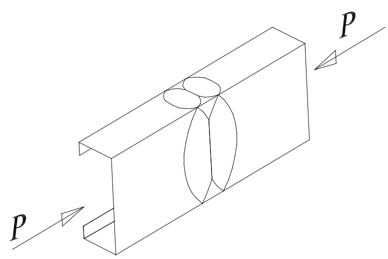


Fig. 9.6. Plastic mechanism of failure for lipped channel section in compression [9.4]

Two plastic mechanisms for lipped channel sections under uniform compression were developed by Morino et al. [9.28], as shown in Fig. 9.7.

The "triple-roof" mechanism (see Fig. 9.7a) is in fact a combination of pitched-roof mechanisms (Fig.9.4a) in the web and flanges and local CF1 (see Table 9.1) mechanisms in lips. The CF-quasi-mechanism (see Fig. 9.7b) consists of the local "pitched-roof" mechanism in the web and two local mechanisms similar to CF2 in the flanges, as well as local CF1 mechanisms in lips (see

Table 9.1). What kind of mechanism develops depends mainly on the buckling mode, which is induced by length to width ratio.

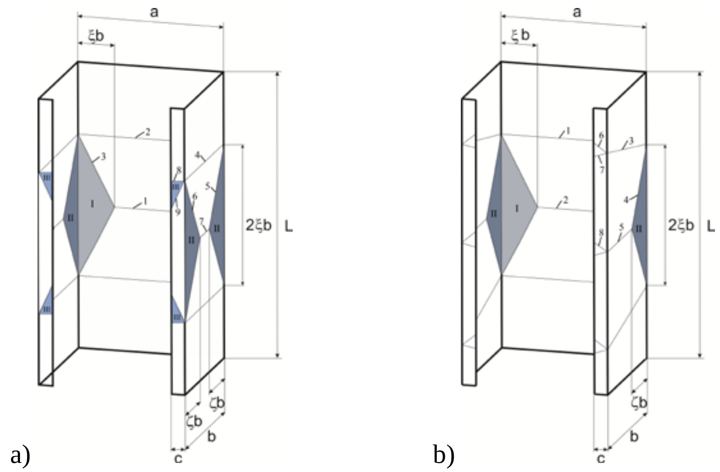


Fig. 9.7. Theoretical models of: a) "triple-roof" mechanisms, b) CF-quasi-mechanism

In the case of short top hat columns, for certain range of web/flange ratio, subjected to uniform compression or bending, numerical FE calculations and experimental tests results indicate different types of plastic mechanisms (see Fig. 9.8a). Its theoretical model is shown in Fig. 9.8b [9.26].

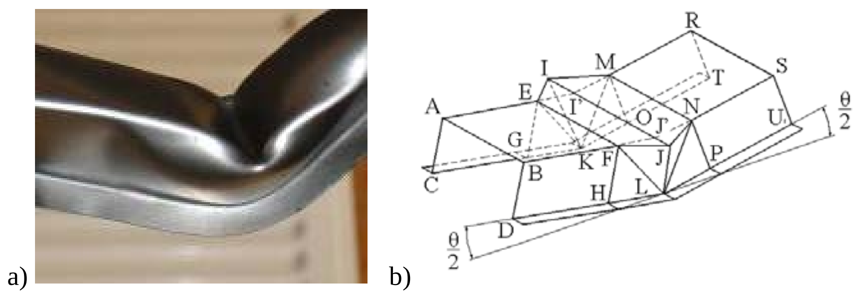


Fig. 9.8. Plastic mechanism of failure for a top hat sections [9.26]:
a) real mechanism of failure (quasi-static test), b) theoretical model

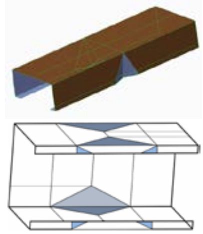
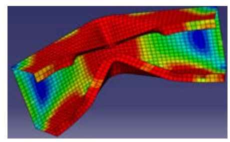
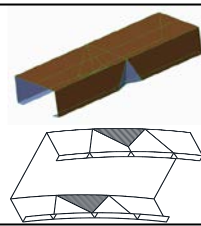
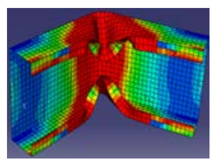
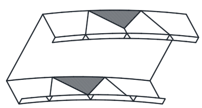
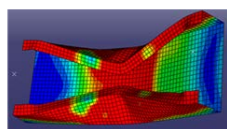
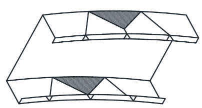
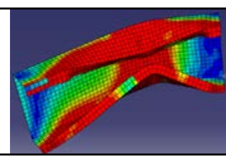
9.4. Plastic mechanisms identified in TWCFS members under eccentric compression

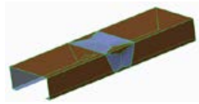
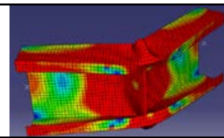
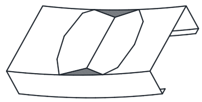
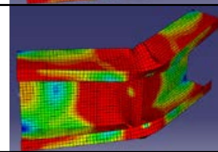
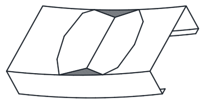
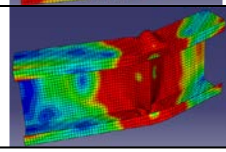
On the basis of FE numerical experiments, several plastic mechanisms of failure, shown in Table 9.2, were identified by Ungureanu et al. [9.29].

For small positive eccentricities the true mechanism CF1, initiated by distortional buckling, originally developed by Murray & Khoo [9.10] was observed. In the initial stage of failure, for the smallest eccentricities, a mechanism similar to ‘triple roof’, originally developed by Morino et al. [9.28] was identified. For larger positive eccentricities (*e* = 20 - 100 mm) the CF quasi-mechanism, similar to that originally developed by Morino et al. [9.28] and to mechanism CF2, developed by Murray & Khoo [9.10] has been observed.

For negative eccentricities, for which local-distortional or local buckling in the web takes place, the pitched-roof mechanism or roof mechanism was identified.

Table 9.2. Plastic mechanisms of lipped channel section members subjected to eccentric compression

Ecc. *e* [mm]	Buckling mode	Failure mode (mechanism)	Mechanism model	FE pattern
5 10	distortional distortional	CF1/triple roof CF1		
20	distortional	CF1/CF-quasi-mechanism		
30 40 50	distortional distortional distortional	CF-quasi-mechanism CF-quasi-mechanism CF-quasi-mechanism		
60 100	distortional distortional	CF-quasi-mechanism CF-quasi-mechanism		

-5	Local-distortional	Pitched-roof		
-10	Local-distortional	Roof		
-30 -60	Local Local	Roof		

9.5. Methods for ultimate strength estimation of TWCFS members under compression and/or bending

The methods allowing the estimation of ultimate strength of thin-walled steel plated structures, including TWCFS members, can be classified into four categories:

1. Purely analytical methods based on the concept of effective width or the interactive buckling approach. Solutions obtained within the second approach are mainly based on the asymptotic Koiter's theory.
2. Numerical methods, namely the Finite Element Method (FEM) and the Finite Strip Method (FSM). Linear and non-linear General Beam Theory (GBT) should be also mentioned within this group.
3. Analytical-numerical or semi-empirical methods. The example of the first category is the Direct Strength Method (DSM) introduced by Schaffer [9.30], while the representative of the second one is ECBL plastic-elastic method proposed by Dubina & Ungureanu [9.4], which uses the rigid-plastic theory and consists in the introduction of a reduction factor, that takes into account the interaction of local rigid-plastic failure mode with the global elastic buckling. This method requires an analysis of local plastic mechanisms.
4. Kinematical methods based on principle of virtual velocities (yield-line analysis), leading to the upper-bound estimation of the ultimate load [9.8]. In that case an upper-bound load capacity estimation is performed as intersection point of post-buckling path and post-failure curve, obtained from yield line analysis.

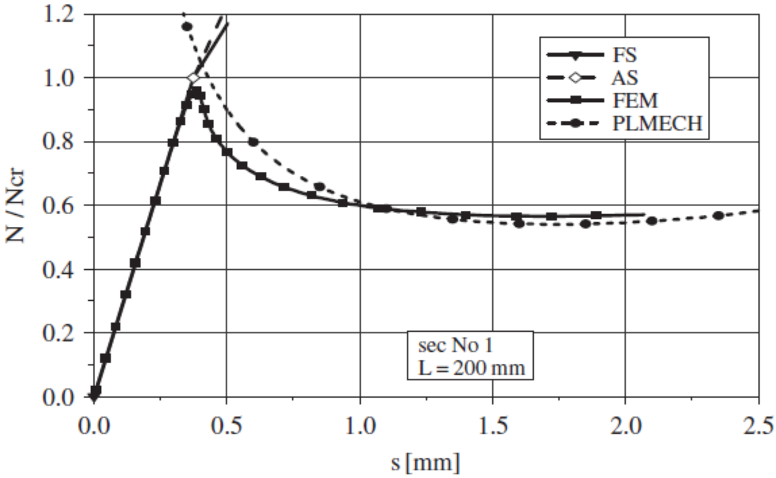


Fig. 9.9. Structural behaviour of the top hat section: a = 50, b = 33.5, w = 12.5, t = 1 [mm], σ_Y = 480 Mpa [9.31]

The exemplary results of load-capacity analysis using some of the methods mentioned above, for top hat-section column subjected to uniform compression are shown in Fig. 9.9 [9.31]. The diagram of the structural behaviour of the top hat section (load normalized with respect to the buckling load in terms of total shortening) is presented. The post-buckling path obtained from FE calculations was compared with the elastic post-buckling path (AS) determined by means of the asymptotic method [9.32] and Finite Strip method (FS). Intersection points of those paths with the post-failure path (PLMECH- mechanism shown in Fig.9.8b) indicate an upper bound estimation of the load-carrying capacity.

9.6. Load carrying capacity and post-failure behaviour of TWCFS members under eccentric compression

The subjects of investigation are TWCFS members subjected to eccentric compression about minor axis, namely lipped channel section columns. Positive and negative eccentricities along the symmetry axis are investigated, as shown in Fig. 9.10.

The dimensions of the cross-section investigated in the present work are $h \times b \times c \times t$ = 150×60×20×2 mm with and an internal radius r = 3 mm. The columns of length L = 450 mm were investigated, made of structural steel of yield stress f_y = 355 N/mm^2. Since the buckling mode influences to large extent the failure mode of compressed bar, the buckling behaviour of examined columns was investigated.

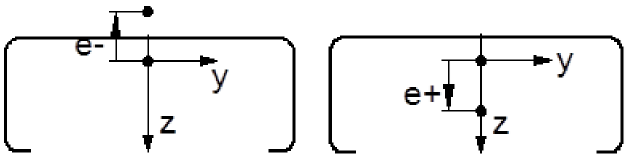


Fig. 9.10. Lipped channel sections with negative and positive eccentricities

Buckling loads and modes were determined for the large range of eccentricities, from: *e* = -60 mm to *e* = +100 mm. The analysis was carried out using CUFSM code [9.33] based on Finite Strip Method. For two cases (positive and negative eccentricities) the FSM results were verified using analytical-numerical method based on the asymptotic approach [9.32].

As shown in Table 9.2, for positive eccentricities (*e* = 10, 30, 60, 100 mm) and *e* = -5 mm a typical distortional buckling takes place. For the negative eccentricity *e* = -10 mm a local-distortional buckling mode was observed, while for eccentricities *e* = -30 mm and *e* = -60 mm the local buckling takes place.

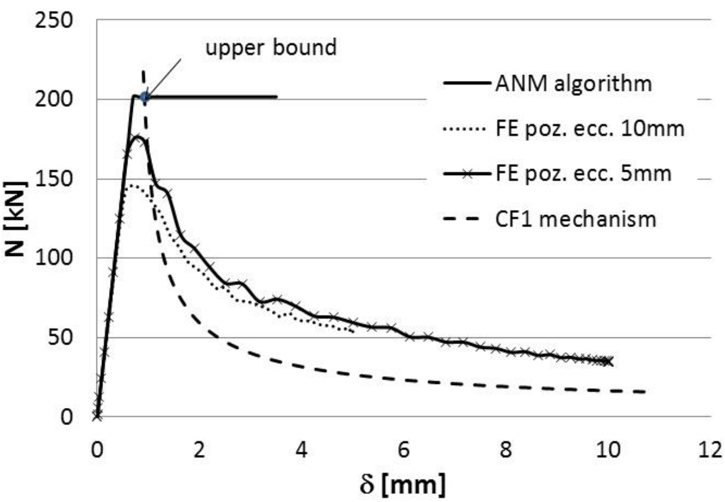


Fig. 9.11. Load-shortening diagrams for small positive eccentricities

Fig. 9.11 shows the comparison of load-shortening diagrams for the bar subjected to small positive eccentricities (*e* = 5, 10 mm) obtained from FE calculations, together with pre- and post-buckling paths derived from analytical-numerical (ANM) algorithm based on the asymptotic method [9.32] (for the eccentricity *e* = 10 mm only) and the failure curve, corresponding to CF1 mechanism, derived from energy method.

For small positive eccentricities the mechanism CF1 gives a relatively good prediction of post-ultimate column behaviour. Also an upper-bound estimation of load-carrying capacity obtained from the intersection of post-buckling path and failure curve is in good agreement with FE prediction for the smallest eccentricity, i.e. *e* = 5 mm.

As was mentioned, for small positive eccentricities the 'triple-roof' mechanism (Fig. 9.7a) was considered. Post-failure curves based on this mechanism (evaluated using the energy method) overestimate post-failure capacity obtained from FE simulations. However, a relatively good agreement of these two simulations was observed in initial stage of failure (Fig. 9.12).

For larger positive eccentricities, as shown in Table 9.2, the identified mechanism was the CF-quasi-mechanism. The post-failure paths for this mechanism were evaluated on the basis of the theoretical model of this mechanism (see Fig. 9.7b) using the energy method. Geometrical parameters of this mechanism, particularly ξ and ς have to be calibrated for each eccentricity. After the calibration, the agreement of YLA results and FE results is very good, as shown in Fig. 9.13.

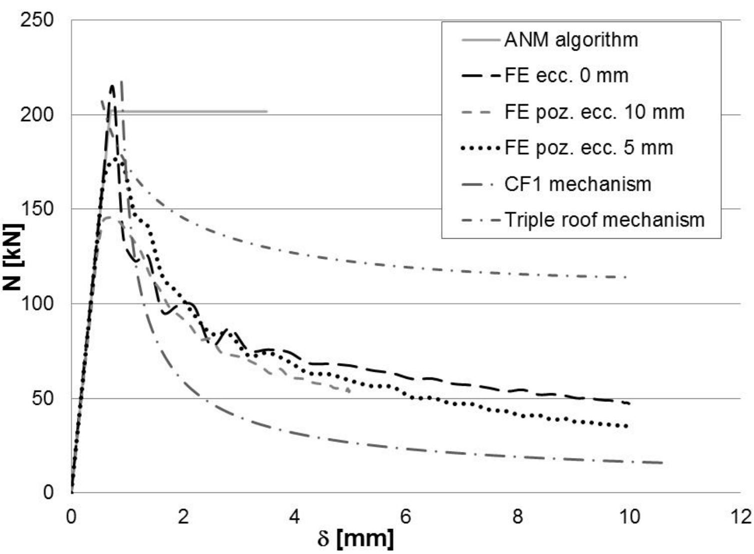


Fig. 9.12. Load-shortening diagrams for small positive eccentricities - comparison for two mechanisms

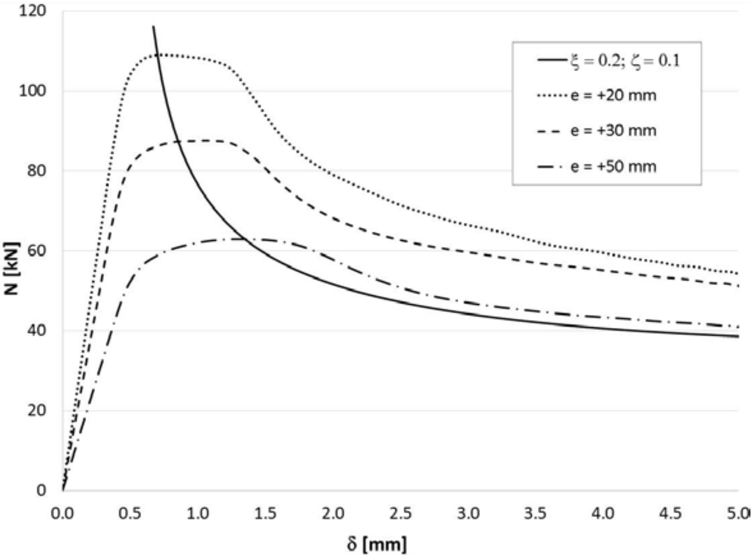


Fig. 9.13. Load-shortening diagrams for large positive eccentricities

Comparative diagram for large negative eccentricities (e = -60 mm) is shown in Fig. 9.14. FE results are compared with the results of ANM calculations for e = -60 mm and two failure curves, corresponding to ‘roof’ mechanism and ‘pitched-roof’ mechanism - energy method, respectively. In both diagrams upper-bound load capacity estimation is indicated as intersection point of post-buckling path and failure curve.

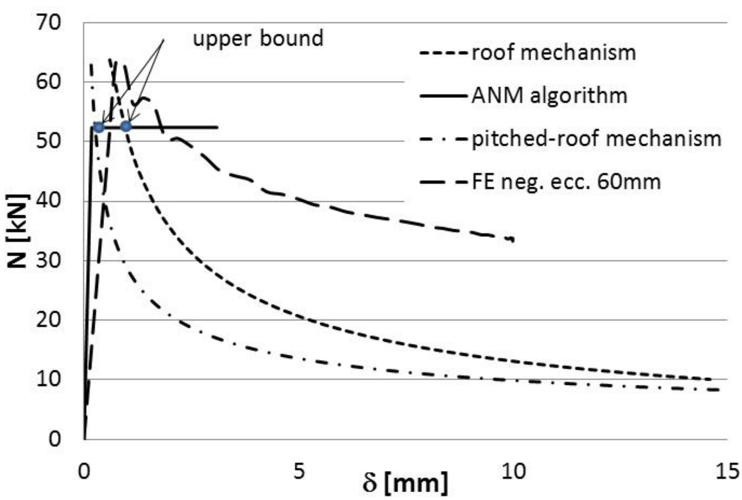


Fig. 9.14. Load-shortening diagrams for large negative eccentricities

For large negative eccentricity the post-ultimate capacity obtained for both mechanisms: ‘pitched-roof’ and ‘roof’ is underestimated. It indicates that additional yield lines and tension fields in the flanges cannot be entirely neglected. Nevertheless, an upper-bound estimation of load-carrying capacity (intersection of post-buckling path and failure curve) is in relatively good agreement with FE prediction.

9.7. Final remarks

The main aim of this chapter was to confirm the possibility to use the local plastic mechanism analysis to characterize the ultimate strength of short columns under compression and/or bending.

The local rigid-plastic model, describes properly the behaviour of thin-walled stub columns. This model is consistent with the real phenomenon of stub columns and short beams failure and is confirmed by test results and advanced elastic-plastic FEM analysis.

Even if plastic mechanisms are identified for compression and bending separately, in the case of eccentric compression, this is far from a linear superposition of basic mechanisms. The obtained results confirm the possibility to use the local plastic mechanism analysis to characterise the ultimate strength of short columns under eccentric compression, although the plastic mechanisms models should be improved.

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9.8. References

9.1 EN1993-1-1, Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings, European Committee for Standardization, Brussels, Belgium, 2005 (including EN1993-1-1:2005/AC, 2009).

9.2 Ungureanu V., Dubina D., Post-elastic strength and ductility of cold-formed steel sections, Proceedings of the Fourth International Conference on Thin-Walled Structures, Loughborough, UK, 22-24 June 2004, pp. 283-290.

9.3 Dubina D., Ungureanu V., Plastic strength of thin-walled members, Proceedings of the Sixteenth Int. Specialty Conference on Cold-Formed Steel Structures, Orlando, Florida 2002, pp. 324-332.
9.4 Ungureanu V., Dubina D., Recent research advances on ECBL approach. Part I: Plastic-elastic interactive buckling of cold-formed steel sections, Thin-Walled Structures, Vol. 42, No. 2, 2004, pp. 177-194.
9.5 Ungureanu V., Kotełko M., Grudziecki J., Plastic mechanisms for thin-walled cold-formed steel members in eccentric compression, Acta Mechanica et Automatica, 2016, Vol. 10, No. 1, pp. 33-37.
9.6 Horne M.R., Plastic Theory of Structures, Thomas Nelson &Sons, Ltd, 1971.
9.7 Jones, L.L., Wood R.H., Yield Line Analysis of Slabs, American Elsevier, New York 1967.
9.8 Kotełko M., Load-capacity estimation and collapse analysis of thin-walled beams and columns - Recent advances, Thin-Walled Structures, 2004, Vol. 42, No. 2, pp. 153-175.
9.9 Murray N.W., Introduction to the theory of thin-walled structures, Clarendon Press, Oxford 1986.
9.10 Murray N.W., Khoo P.S., Some basic plastic mechanisms in thin-walled steel structures, Int. J. Mech. Sci., 1981, Vol. 23, No. 12, pp. 703-713.
9.11 Flockhart C.J., Murray N.W., Grzebieta R.H., Comparison of upper-bound rigid-plastic yield line mechanism analysis by the energy and equilibrium strip method, Proceedings of the 2nd Australasia Congress of Applied Mechanics, Canberra, Australia, Paper R053, 1999.
9.12 Kotełko M., Mania R.J., Alternative solution of the problem of load-capacity of thin-walled plated structures, Mechanics and Mechanical Eng., 2008, Vol. 12, No. 4, pp. 323-336.
9.13 Kotełko M., Ungureanu V., Dubina D., Macdonald M., Plastic strength of thin-walled plated members - Alternative solutions review, Thin-Walled Structures, 2011, Vol. 49, No. 5, pp. 636-644.
9.14 Zhao X-L., Yield line mechanism analysis of steel members and connections, Prog. Struct. Engng. Mater., Vol. 200, No. 5, pp. 252-262.
9.15 Kato B., Buckling strength of plates in the plastic range, International Association of Bridge and Structural Engineering, 1965, Vol. 25, pp. 127-141.
9.16 Korol R.M., Sherbourne A.N., Strength prediction of plates in uniaxial compression, Journal of Structural Division, 1962, Vol. 98, 1965-1986.
9.17 Sin K.W., The collapse behaviour of thin-walled sections, PhD Thesis, Dept. of Mechanical Engineering, University of Strathclyde, Glasgow 1985.
9.18 Mahendran M., Local plastic mechanisms in thin steel plates under in-plane compression, Thin-Walled Structures, 1997, Vol. 27, No. 3, pp. 245-261.
9.19 Rondal J., Maquoi R., Stub-column strength of thin-walled square and rectangular hollow sections, Thin-Walled Structures, 1985, Vol. 3, No. 1, pp. 15-34.
9.20 Kragerup J., Five notes on plate buckling, Technical University of Denmark, Department of Structural Engineering, Series R, No. 143, 1982.

9.21 Kotełko M., Load-carrying capacity and mechanisms of failure of thin-walled structures, WNT Warszawa 2010.
9.22 Ungureanu V., Kotełko M., Mania R.J., Dubina D., Plastic mechanisms database for thin-walled cold-formed steel members in compression and bending, Thin-Walled Structures, 2010, Vol. 48, Nos. 10-11, pp. 818-826.
9.23 Królak M. [ed.], Post-buckling and load-capacity of thin-walled girders of flat walls (in Polish), PWN, Warszawa-Łódź 1990
9.24 Rasmussen K.J.R, The behaviour of thin-walled channel-section columns, PhD thesis, School of Civil and Mining Engineering, University of Sydney, 1988.
9.25 Rasmussen K.J.R., Hancock G.J., Nonlinear analyses of thin-walled channel-section columns, Thin-Walled Structures, 1991, Vol. 13, Nos. 1-2, pp. 145-176.
9.26 Kotełko M., Kołakowski Z., Mania R., Ultimate loads and collapse behaviour of thin-walled hat section columns under compression, Proceedings of the International Colloquium on Stability and Ductility of Steel Structures (SDSS'06), Lisbon, September 6-8, IST Press, 2006, pp. 267-273.
9.27 Kotełko M., Kolakowski Z., Rhodes J., Comparative investigation into buckling loads and load-bearing capacity of thin-walled columns under compression, Lightweight Structures in Civil Engineering, Warsaw 2005, pp. 206-210.
9.28 Morino S., Kawaguchi J., Mizuno Y., Hanya K., Distortional buckling of light-gauge lipped channel short columns, Steel Structures, 2003, Vol. 3, 203-217.
9.29 Ungureanu V., Kotełko M., Karmazyn A., Dubina D., Plastic mechanisms of thin-walled cold-formed steel members in eccentric compression, Proceedings of the 7th International Conference on Coupled Instabilities in Metal Structures, Baltimore, Maryland, November 7-8, 2016 (in print).
9.30 Schafer B.W., Designing Cold-Formed Steel Using the Direct Strength Method, Proceeding of the 18th International Specialty Conference on Cold-Formed Steel Structures, October 26-27, Orlando, Florida, USA, 2006.
9.31 Kotełko M., Thin-walled profiles with edge stiffeners as energy absorbers, Thin-Walled Structures, Vol. 45, Nos. 10-11, pp. 872-876, 2007.
9.32 Kolakowski Z., A semi-analytical method for the analysis of the interactive buckling of thin-walled elastic structures in the second order approximation, Int. J. Solids Structures, 1996, Vol. 33, No. 25, pp. 3779-3090.
9.33 Li Z., Schafer B.W., Buckling analysis of cold-formed steel members with general boundary conditions using CUFSM: conventional and constrained finite strip methods, Proceedings of the 20th Int. Spec. Conf. on Cold-Formed Steel Structures, St. Louis, MO., USA, November, 2010.