Dynamics of two coupled 4-DOF mechanical linear sliding systems with dry friction

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Abstract: The paper introduces a model of two identical coupled 4-DOF mechanical linear sliding systems with dry friction coupled with each other by a linear torsional spring. The appropriate components (bodies) of the coupled systems are riding on two separated driving belts, which are driven at constant velocities, and stick-slip vibrations can be observed. In this case the physical interpretation of the considered model could be two rows of carriages laying on the guideways and coupled by an elastic shaft, which are moving at constant velocity with respect to the guideways as a foundation. From a mathematical point of view the analyzed problem is governed by eight nonlinear ordinary second order differential equations of motion yielded by the second kind Lagrange equations. Numerical analysis is performed in Mathematica software using the qualitative and quantitative theories of differential equations. Some interesting non-linear system dynamics are detected and reported using the phase portraits and the Poincaré maps. Next, power spectra obtained by the FFT technique are reported. The presented results show periodic, quasi-periodic, chaotic and hyper-chaotic orbits. Moreover, synchronization effects between the coupled systems are also detected and studied.

1. Introduction

The question of stick-slip vibrations caused by dry friction is still opened. The fundamental laws of stick-slip phenomena based on dry friction dynamics have been promulgated in the pioneering experiments of Rabinovicz and in the works of Baumberger et al [7]. Firstly, a concept of nonlinear dry friction should be explained. The force, which is required to start the movement of an object, is called the static friction force, but the kinetic force is essential to maintain a constant velocity during the movement of the body. A sufficient condition for stick-slip is that the static coefficient of friction is higher than the kinetic coefficient of friction [10]. Stick-slip phenomena are expected during contact interaction at low-velocity friction. The considered stick-slip phenomenon depends on frequency of vibrations, a relative humidity and load. Stick-slip phenomena occur in everyday life, for instance, from earthquakes, through brake systems (when car is started to move from stationary state) [11], to nano-devices showing up in the scale above several microns. Examples of scientific literature devoted to sticks-slip vibrations in system can be found in the references [1, 3, 5, 6, 8, 9].

Different models in micro- and macro-scale are used for description of stick- slip phenomena. In this work an 8 degree-of-freedom model is used. The body consists of two identical subsystems coupled by torsion spring. Every subsystem rides on two separated belts which are driven at constant velocity. Bearing in mind principles of relativity one can say that the bodies are moving because of immovable belts. In this case the real interpretation of model may take place in a mine, where two rows of carriages fixed to guideways are moving at constant velocity. As a nonlinear (in stick–slip regime) system, the spring–slider model is very sensitive to weak external impacts, which on a large scale manifests itself in phenomena of induced seismicity, triggering and synchronization effects [2]. The considered in this work mechanical system can be treated as an extension of the mechanical model presented in the paper [4].

2. Mechanical Model

The considered 8-DOF model (two coupled by torsional spring 4-DOF mechanical linear sliding systems with dry friction) is shown in Fig. 1.

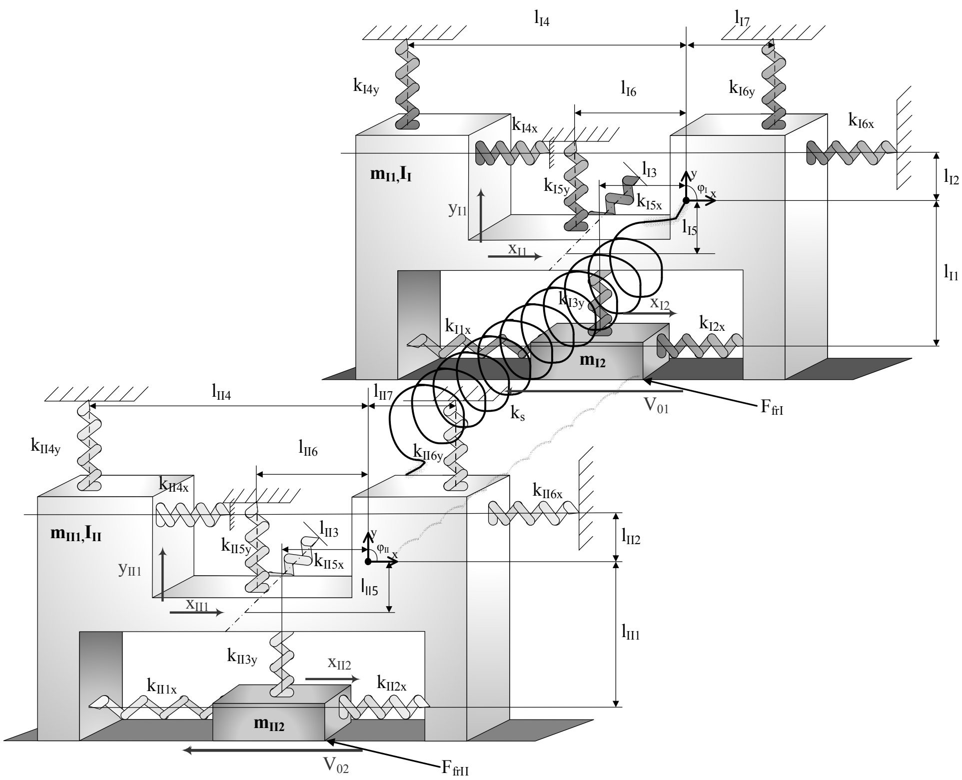

Figure 1. The 8-DOF model with dry friction.

The presented system can be considered as a planar system in the Cartesian coordinate system (in the Earth's gravitational field with the gravity coefficient g) with horizontal axis x and vertical axis y. Dynamics of the considered system can be described by the following variables: x_{I1}, $v_{I1} = \dot{x}_{I1}$, y_{I1}, $z_{I1} = \dot{y}_{I1}$, φ_I, $\omega_I = \dot{\varphi}_I$, x_{I2}, $v_{I2} = \dot{x}_{I2}$, x_{II1}, $v_{II1} = \dot{x}_{II1}$, y_{II1}, $z_{II1} = \dot{y}_{II1}$, φ_{II}, $\omega_{II} = \dot{\varphi}_{II}$, x_{II2}, $v_{II2} = \dot{x}_{II2}$. The masses m_{I1}, m_{II1} can rotate about the pivot axes S (moments of inertia about the pivot point S of the mentioned masses are I_I, I_{II}). The entire system is characterized by lengths l_{Ii}, l_{IIi} ($i = 1,2,...,6$) and springs with stiffness coefficients k_{Iix}, k_{IIix}, k_{Ijy}, k_{IIjy} ($i = 1,2,4,5,6$; $j = 3,4,5,6$). Moreover, two additional masses m_{I2}, m_{II2} are laying on the appropriate belts as a foundation, which are moving with a constant velocities v_{I0} and v_{II0}, respectively. Between the mentioned masses m_{I2}, m_{II2} and appropriate belts dry friction forces occur as a functions of the relative sliding velocities $v_{I0} - \dot{x}_{I2}$, $v_{II0} - \dot{x}_{II2}$, respectively.

Equations of motion of the considered system have been derived using the Lagrangian method (the second kind Lagrange equations) [4] and they are as follows

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial T}{\partial \boldsymbol{q}} + \frac{\partial V}{\partial \boldsymbol{q}} = \boldsymbol{Q_n}, \tag{1}$$

where: $\boldsymbol{q}$ - vector of generalized coordinates, $\boldsymbol{Q_n}$ - vector of generalized non-conservative force acting in the system, T - total kinetic energy of the system, V - total potential energy of the system, t - time.

In this case dot means differentiation with respect to time t. For presented previously 8-DOF model with dry friction, vector $\boldsymbol{q}$ is reads:

$$\boldsymbol{q} = \left[x_{I1},\ \ y_{I1},\ \ \varphi_I,\ \ x_{I2},\ \ x_{II1},\ \ y_{II1},\ \ \varphi_{II},\ \ x_{II2}\right]^T. \tag{2}$$

Simultaneously, $\boldsymbol{Q_n}$ can be described by the following vector

$$\boldsymbol{Q_n} = \left[0,\ \ 0,\ \ 0,\ \ F_{frI},\ \ 0,\ \ 0,\ \ 0,\ \ F_{frII}\right]^T. \tag{3}$$

The friction forces F_{frI} and F_{frII} are equal to the product of nonlinear kinetic friction coefficients $\mu_k(v_{I0} - \dot{x}_{I2})$, $\mu_k(v_{II0} - \dot{x}_{I2})$ (associated with relative velocities of every subsystems) and the normal forces $N_I = m_{I2}g - (k_{I3y}y_{I1} - k_{I3y}l_{I3}\varphi_I)$, $N_{II} = m_{II2}g - (k_{II3y}y_{II1} - k_{II3y}l_{II3}\varphi_{II})$, which press the masses m_{I2} and m_{II2} to the first belt and to the second one, respectively. It should also be noted that in numerical calculations the values of the normal forces N_I and N_{II} can be less than zero, greater than zero or equal to zero. In the case of $N_I, N_{II} > 0$, the friction contact between masses m_{I2}, m_{II2} and the appropriate belts moving with velocities v_{I0}, v_{II0} occur. In turn, the case $N_I, N_{II} \le 0$ means a loss of friction contact between the masses m_{I2}, m_{II2} and the appropriate belts. This is why in our mathematical model we use a discontinuous step functions describing these phenomena, and defined as follow

$$\boldsymbol{1}(N_I) = \begin{cases} 1 & \text{for} \quad N_I > 0, \\ 0 & \text{for} \quad N_I \le 0. \end{cases}, \quad \boldsymbol{1}(N_{II}) = \begin{cases} 1 & \text{for} \quad N_{II} > 0, \\ 0 & \text{for} \quad N_{II} \le 0. \end{cases} \tag{4}$$

Finally, forces F_{frI} and F_{frII} have the following form

$$\begin{aligned} &F_{frI}(v_{I0} - \dot{x}_{I2}, y_{I1}, \varphi_I) = \\ &= \mu_{Ik}(v_{I0} - \dot{x}_{I2}) \cdot [m_{I2}g - (k_{I3y}y_{I1} - k_{I3y}l_{I3}\varphi_I)] \cdot \boldsymbol{1}(m_{I2}g - (k_{I3y}y_{I1} - k_{I3y}l_{I3}\varphi_I), \end{aligned} \tag{5}$$

$$\begin{aligned} &F_{frII}(v_{II0} - \dot{x}_{II2}, y_{II1}, \varphi_{II}) = \\ &= \mu_{IIk}(v_{II0} - \dot{x}_{II2}) \cdot [m_{II2}g - (k_{II3y}y_{II1} - k_{II3y}l_{II3}\varphi_{II})] \cdot \boldsymbol{1}(m_{II2}g - (k_{II3y}y_{II1} - k_{II3y}l_{II3}\varphi_{II}). \end{aligned} \tag{6}$$

Total kinetic energy T of studied model has the following form:

$$T = \frac{1}{2}m_{I1}(\dot{x}_{I1}^2 + \dot{y}_{I1}^2) + \frac{1}{2}I_I\dot{\varphi}_I^2 + \frac{1}{2}m_{I2}\dot{x}_{I2}^2 + \frac{1}{2}m_{II1}(\dot{x}_{II1}^2 + \dot{y}_{II1}^2) + \frac{1}{2}I_{II}\dot{\varphi}_{II}^2 + \frac{1}{2}m_{II2}\dot{x}_{II2}^2 . \tag{7}$$

Since small values of angles φ_I and φ_{II} are taken into consideration, the total potential energy V has the following form

$$\begin{aligned} V &= \frac{1}{2}k_{I1x}(x_{I1} + l_{I1}\varphi_I - x_{I2})^2 + \frac{1}{2}k_{I2x}(x_{I1} + l_{I1}\varphi_I - x_{I2})^2 + \\ &+ \frac{1}{2}k_{I3y}(y_{I1} - l_{I3}\varphi_I)^2 + \frac{1}{2}k_{I4x}(x_{I1} - l_{I2}\varphi_I)^2 + \frac{1}{2}k_{I4y}(y_{I1} - l_{I4}\varphi_I)^2 + \\ &+ \frac{1}{2}k_{I5x}(x_{I1} + l_{I5}\varphi_I)^2 + \frac{1}{2}k_{I5y}(y_{I1} - l_{I6}\varphi_I)^2 + \frac{1}{2}k_{I6x}(x_{I1} - l_{I2}\varphi_I)^2 + \\ &+ \frac{1}{2}k_{I6y}(y_{I1} + l_{I7}\varphi_I)^2 + m_{I1}gy_{I1} + \\ &+ \frac{1}{2}k_{II1x}(x_{II1} + l_{II1}\varphi_I - x_{II2})^2 + \frac{1}{2}k_{II2x}(x_{II1} + l_{II1}\varphi_{II} - x_{II2})^2 + \\ &+ \frac{1}{2}k_{II3y}(y_{II1} - l_{II3}\varphi_{II})^2 + \frac{1}{2}k_{II4x}(x_{II1} - l_{II2}\varphi_{II})^2 + \frac{1}{2}k_{II4y}(y_{II1} - l_{II4}\varphi_{II})^2 + \\ &+ \frac{1}{2}k_{II5x}(x_{II1} + l_{II5}\varphi_{II})^2 + \frac{1}{2}k_{II5y}(y_{II1} - l_{II6}\varphi_{II})^2 + \frac{1}{2}k_{II6x}(x_{II1} - l_{II2}\varphi_{II})^2 + \\ &+ \frac{1}{2}k_{II6y}(y_{II1} + l_{II7}\varphi_{II})^2 + m_{II1}gy_{II1} + \frac{1}{2}k_s(\varphi_I - \varphi_{II})^2. \end{aligned} \tag{8}$$

Computing the partial derivatives $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\boldsymbol{q}}}\right), \frac{\partial T}{\partial \boldsymbol{q}}, \frac{\partial V}{\partial \dot{\boldsymbol{q}}}$, based on (1), we obtain

$$\begin{cases} m_{I1}\ddot{x}_{I1} + (k_{I1x} + k_{I2x} + k_{I4x} + k_{I5x} + k_{I6x})x_{I1} + \\ + (k_{I1x}l_{I1} + k_{I2x}l_{I1} - k_{I4x}l_{I2} + k_{I5x}l_{I5} - k_{I6x}l_{I2})\varphi_I - (k_{I1x} + k_{I2x})x_{I2} = 0, \\ m_{I1}\ddot{y}_{I1} + (k_{I3y} + k_{I4y} + k_{I5y} + k_{I6y})y_{I1} + \\ + (-k_{I3y}l_{I3} - k_{I4y}l_{I4} - k_{I5y}l_{I6} + k_{I6y}l_{I7})\varphi_I + m_{I1}g = 0, \\ I_I\ddot{\varphi}_I + (k_{I1x}l_{I1} + k_{I2x}l_{I1} - k_{I4x}l_{I2} + k_{I5x}l_{I5} - k_{I6x}l_{I2})x_{I1} + \\ + (-k_{I3y}l_{I3} - k_{I4y}l_{I4} - k_{I5y}l_{I6} + k_{I6y}l_{I7})y_{I1} + \\ + (k_{I1x}l_{I1}^2 + k_{I2x}l_{I1}^2 + k_{I3y}l_{I3}^2 + k_{I4x}l_{I2}^2 + k_{I4y}l_{I4}^2 + k_{I5x}l_{I5}^2 + k_{I5y}l_{I6}^2 + k_{I6x}l_{I2}^2 + k_{I6y}l_{I7}^2)\varphi_I + \\ - (k_{I1x} + k_{I2x})l_{I1}x_{I2} + k_s(\varphi_I - \varphi_{II}) = 0, \\ m_{I2}\ddot{x}_{I2} - (k_{I1x} + k_{I2x})x_{I1} - (k_{I1x} + k_{I2x})l_{I1}\varphi_I + (k_{I1x} + k_{I2x})x_{I2} = \\ = \mu_k(v_{I0} - \dot{x}_{I2}) \cdot [m_{I2}g - (k_{I3y}y_{I1} - k_{I3y}l_{I3}\varphi_I)] \cdot \boldsymbol{1}(m_{I2}g - (k_{I3y}y_{I1} - k_{I3y}l_{I3}\varphi_I), \\ m_{II1}\ddot{x}_{II1} + (k_{II1x} + k_{II2x} + k_{II4x} + k_{II5x} + k_{II6x})x_{II1} + \\ + (k_{II1x}l_{II1} + k_{II2x}l_{II1} - k_{II4x}l_{II2} + k_{II5x}l_{II5} - k_{II6x}l_{II2})\varphi_{II} - (k_{II1x} + k_{II2x})x_{II2} = 0, \\ m_{II1}\ddot{y}_{II1} + (k_{II3y} + k_{II4y} + k_{II5y} + k_{II6y})y_{II1} + \\ + (-k_{II3y}l_{II3} - k_{II4y}l_{II4} - k_{II5y}l_{II6} + k_{II6y}l_{II7})\varphi_{II} + m_{II1}g = 0, \\ I_{II}\ddot{\varphi}_{II} + (k_{II1x}l_{II1} + k_{II2x}l_{II1} - k_{II4x}l_{II2} + k_{II5x}l_{II5} - k_{II6x}l_{II2})x_{II1} + \\ + (-k_{II3y}l_{II3} - k_{II4y}l_{II4} - k_{II5y}l_{II6} + k_{II6y}l_{II7})y_{II1} + \\ + (k_{II1x}l_{II1}^2 + k_{II2x}l_{II1}^2 + k_{II3y}l_{II3}^2 + k_{II4x}l_{II2}^2 + k_{II4y}l_{II4}^2 + k_{II5x}l_{II5}^2 + k_{II5y}l_{II6}^2)\varphi_{II} + \\ + k_{II6x}l_{II2}^2 + k_{II6y}l_{II7}^2)\varphi_{II} - (k_{II1x} + k_{II2x})l_{II1}x_{II2} + k_s(\varphi_{II} - \varphi_I) = 0, \\ m_{II2}\ddot{x}_{II2} - (k_{II1x} + k_{II2x})x_{II1} - (k_{II1x} + k_{II2x})l_{II1}\varphi_{II} + (k_{II1x} + k_{II2x})x_{II2} = \\ = \mu_k(v_{II0} - \dot{x}_{II2}) \cdot [m_{II2}g - (k_{II3y}y_{II1} - k_{II3y}l_{II3}\varphi_{II})] \cdot \boldsymbol{1}(m_{II2}g - (k_{II3y}y_{II1} - k_{II3y}l_{II3}\varphi_{II}). \end{cases} \tag{9}$$

3. Non-dimensional form

We introduce non-dimensional time $\tau = t\Big/\sqrt{m_{I2}/(k_{I1x} + k_{I2x})}$, non-dimensional coordinates $X_{I1} = x_{I1}/l_{I1}$, $Y_{I1} = y_{I1}/l_{I1}$, $X_{I2} = x_{I2}/l_{I1}$, $X_{II1} = x_{II1}/l_{I1}$, $Y_{II1} = y_{II1}/l_{I1}$, $X_{II2} = x_{II2}/l_{I1}$ and the following non-dimensional parameters:

$$a_{I1} = \frac{m_{I2}}{m_{I1}}\left(\frac{k_{I1x} + k_{I2x} + k_{I4x} + k_{I5x} + k_{I6x}}{k_{I1x} + k_{I2x}}\right), \tag{10}$$

$$a_{II1} = \frac{m_{I2}}{m_{II1}}\left(\frac{k_{II1x} + k_{II2x} + k_{II4x} + k_{II5x} + k_{II6x}}{k_{I1x} + k_{I2x}}\right), \tag{11}$$

$$a_{I2} = \frac{m_{I2}}{m_{I1}}\left(\frac{k_{I1x}l_{I1} + k_{I2x}l_{I1} - k_{I4x}l_{I2} + k_{I5x}l_{I5} - k_{I6x}l_{I2}}{(k_{I1x} + k_{I2x})l_{I1}}\right), \tag{12}$$

$$a_{II2} = \frac{m_{I2}}{m_{II1}} \left(\frac{k_{II1x} l_{II1} + k_{II2x} l_{II1} - k_{II4x} l_{II2} + k_{II5x} l_{II5} - k_{II6x} l_{II2}}{(k_{I1x} + k_{I2x}) l_{I1}} \right), \tag{13}$$

$$a_{I3} = \frac{m_{I2}}{m_{I1}}, \; a_{II3} = \frac{m_{I2}}{m_{II1}} \left(\frac{k_{II1x} + k_{II2x}}{k_{I1x} + k_{I2x}} \right), \tag{14}$$

$$b_{I1} = \frac{m_{I2}}{m_{I1}} \left(\frac{k_{I3y} + k_{I4y} + k_{I5y} + k_{I6y}}{k_{I1x} + k_{I2x}} \right), \; b_{II1} = \frac{m_{I2}}{m_{II1}} \left(\frac{k_{II3y} + k_{II4y} + k_{II5y} + k_{II6y}}{k_{I1x} + k_{I2x}} \right), \tag{15}$$

$$b_{I2} = \frac{m_{I2}}{m_{I1}} \left(\frac{k_{I3y} l_{I3} + k_{I4y} l_{I4} + k_{I5y} l_{I6} - k_{I6y} l_{I7}}{(k_{I1x} + k_{I2x}) l_{I1}} \right), \tag{16}$$

$$b_{II2} = \frac{m_{I2}}{m_{II1}} \left(\frac{k_{II3y} l_{II3} + k_{II4y} l_{II4} + k_{II5y} l_{II6} - k_{II6y} l_{II7}}{(k_{I1x} + k_{I2x}) l_{I1}} \right), \tag{17}$$

$$f_g = \frac{m_{I2} g}{(k_{I1x} + k_{I2x}) l_{I1}}, \tag{18}$$

$$c_{I1} = \frac{m_{I2} l_{I1} (k_{I1x} l_{I1} + k_{I2x} l_{I1} - k_{I4x} l_{I2} + k_{I5x} l_{I5} - k_{I6x} l_{I2})}{(k_{I1x} + k_{I2x}) I_I}, \tag{19}$$

$$c_{II1} = \frac{m_{I2} l_{I1} (k_{II1x} l_{II1} + k_{II2x} l_{II1} - k_{II4x} l_{II2} + k_{II5x} l_{II5} - k_{II6x} l_{II2})}{(k_{I1x} + k_{I2x}) I_{II}}, \tag{20}$$

$$c_{I2} = \frac{m_{I2} l_{I1} (k_{I3y} l_{I3} + k_{I4y} l_{I4} + k_{I5y} l_{I6} - k_{I6y} l_{I7})}{(k_{I1x} + k_{I2x}) I_I}, \tag{21}$$

$$c_{II2} = \frac{m_{I2} l_{I1} (k_{II3y} l_{II3} + k_{II4y} l_{II4} + k_{II5y} l_{II6} - k_{II6y} l_{II7})}{(k_{I1x} + k_{I2x}) I_{II}}, \tag{22}$$

$$c_{I3} = \frac{m_{I2} (k_{I1x} l_{I1}^2 + k_{I2x} l_{I1}^2 + k_{I3y} l_{I3}^2 + k_{I4x} l_{I2}^2 + k_{I4y} l_{I4}^2 + k_{I5x} l_{I5}^2 + k_{I5y} l_{I6}^2 + k_{I6x} l_{I2}^2 + k_{I6y} l_{I7}^2)}{(k_{I1x} + k_{I2x}) I_I} \tag{23}$$

$$c_{II3} = \frac{m_{I2} (k_{II1x} l_{II1}^2 + k_{II2x} l_{II1}^2 + k_{II3y} l_{II3}^2 + k_{II4x} l_{II2}^2 + k_{II4y} l_{II4}^2 + k_{II5x} l_{II5}^2 + k_{II5y} l_{II6}^2 + k_{II6x} l_{II2}^2 + k_{II6y} l_{II7}^2)}{(k_{I1x} + k_{I2x}) I_{II}} \tag{24}$$

$$c_{I4} = \frac{m_{I2} l_{I1}^2}{I_I}, \; c_{II4} = \frac{m_{I2} l_{I1} l_{II1} (k_{II1x} + k_{II2x})}{(k_{I1x} + k_{I2x}) I_{II}}, \tag{25}$$

$$d_{II1} = \frac{m_{I2}(k_{II1x} + k_{II2x})}{m_{II2}(k_{I1x} + k_{I2x})}, \; d_{II2} = \frac{m_{I2} l_{II1}(k_{II1x} + k_{II2x})}{m_{II2} l_{I1}(k_{I1x} + k_{I2x})}, \tag{26}$$

$$k_I = \frac{m_{I2} k_s}{(k_{I1x} + k_{I2x}) I_I}, \; k_{II} = \frac{m_{I2} k_s}{(k_{I1x} + k_{I2x}) I_{II}}, \tag{27}$$

$$e_{I1} = \frac{k_{I3y}}{(k_{I1x} + k_{I2x})}, \; e_{II1} = \frac{m_{I2} k_{II3y}}{m_{II2}(k_{I1x} + k_{I2x})}, \tag{28}$$

$$e_{I2} = \frac{l_{I3}}{l_{I1}} \frac{k_{I3y}}{(k_{I1x} + k_{I2x})}, \; e_{II2} = \frac{l_{II3}}{l_{I1}} \frac{m_{I2} k_{II3y}}{m_{II2}(k_{I1x} + k_{I2x})}, \tag{29}$$

and the following non-dimensional functions

$$\mu_{Ik}\left(\frac{l_{I1}}{\sqrt{m_{I2}/(k_{I1x} + k_{I2x})}} V_{I0} - \frac{l_{I1}}{\sqrt{m_{I2}/(k_{I1x} + k_{I2x})}} \dot{X}_{I2} \right) = f_{Ik}(V_{I0} - \dot{X}_{I2}), \tag{30}$$

$$\mu_{IIk}\left(\frac{l_{I1}}{\sqrt{m_{I2}/(k_{I1x} + k_{I2x})}} V_{II0} - \frac{l_{I1}}{\sqrt{m_{I2}/(k_{I1x} + k_{I2x})}} \dot{X}_{II2} \right) = f_{IIk}(V_{II0} - \dot{X}_{II2}), \tag{31}$$

$$\boldsymbol{1}(m_{I2} g - (k_{I3y} y_{I1} - k_{I3y} l_{I3} \varphi_I)) = \boldsymbol{1}(f_g - (e_{I1} Y_{I1} - e_{I2} \varphi_I)), \tag{32}$$

$$\boldsymbol{1}(m_{II2} g - (k_{II3y} y_{II1} - k_{II3y} l_{II3} \varphi_{II})) = \boldsymbol{1}(f_g - (e_{II1} Y_{II1} - e_{II2} \varphi_{II})). \tag{33}$$

In result, equations of motion in the counter part non-dimensional form are as follows

$$\begin{cases} \ddot{X}_{I1} + a_{I1} X_{I1} + a_{I2} \varphi_I - a_{I3} X_{I2} = 0, \\ \ddot{Y}_{I1} + b_{I1} Y_{I1} - b_{I2} \varphi_I + f_g = 0, \\ \ddot{\varphi}_I + c_{I1} X_{I1} - c_{I2} Y_{I1} + c_{I3} \varphi_I - c_{I4} X_{I2} + k_I(\varphi_I - \varphi_{II}) = 0, \\ \ddot{X}_{I2} - X_{I1} - \varphi_I + X_{I2} = f_{Ik}(V_{I0} - \dot{X}_{I2}) \cdot [f_g - (e_{I1} Y_{I1} - e_{I2} \varphi_I)] \cdot \boldsymbol{1}(f_g - (e_{I1} Y_{I1} - e_{I2} \varphi_I)) \\ \ddot{X}_{II1} + a_{II1} X_{II1} + a_{II2} \varphi_{II} - a_{II3} X_{II2} = 0, \\ \ddot{Y}_{II1} + b_{II1} Y_{II1} - b_{II2} \varphi_{II} + f_g = 0, \\ \ddot{\varphi}_{II} + c_{II1} X_{II1} - c_{II2} Y_{II1} + c_{II3} \varphi_{II} - c_{II4} X_{II2} + k_{II}(\varphi_{II} - \varphi_I) = 0, \\ \ddot{X}_{II2} - d_{II1} X_{II1} - d_{II2} \varphi_{II} + d_{II1} X_{II2} = \\ = f_{IIk}(V_{II0} - \dot{X}_{II2}) \cdot [f_g - (e_{II1} Y_{II1} - e_{II2} \varphi_{II})] \cdot \boldsymbol{1}(f_g - (e_{II1} Y_{II1} - e_{II2} \varphi_{II})). \end{cases} \tag{34}$$

4. Numerical computations

Our numerical computations have been performed via the fourth order Runge-Kutta method with constant time step $h = 0.001$ and zero initial conditions. We consider symmetric system with the values of non-dimensional parameters and non-dimensional functions taken from the previous paper [4], namely:

$a_{I1} = a_{II1} = a_1 = 0{,}07836$, $a_{I2} = a_{II2} = a_2 = 0{,}03344$, $a_{I3} = a_{II3} = a_3 = 0{,}04058$,

$b_{I1} = b_{II1} = b_1 = 0{,}09375$, $b_{I2} = b_{II2} = b_2 = 0{,}03314$, $c_{I1} = c_{II1} = c_1 = 0{,}02689$,

$c_{I2} = c_{II2} = c_2 = 0{,}02666$, $c_{I3} = c_{II3} = c_3 = 0{,}06181$, $c_{I4} = c_{II4} = c_3 = 0{,}03264$,

$d_{II1} = d_{II2} = 1$, $f_g = 0{,}00529$, $e_{I1} = e_{II1} = e_1 = 1{,}37931$, $e_{I2} = e_{II2} = e_2 = 0{,}47237$.

Kinetic friction functions $f_{Ik}(V_{I0} - \dot{X}_{I2})$ and $f_{IIk}(V_{II0} - \dot{X}_{II2})$ in our model are described by the Stribeck functions. Because classical signum function is discontinuous, we decide to approximate the mentioned functions by hyperbolic function with numerical control parameter ε and $V_{I0} = V_{II0} = V_0$ in the form

$$f_{Ik}(V_0 - \dot{X}_{I2}) = \mu_0 \tanh\left(\frac{V_0 - \dot{X}_{I2}}{\varepsilon}\right) - \alpha(V_0 - \dot{X}_{I2}) + \beta(V_0 - \dot{X}_{I2})^3 , \tag{35}$$

$$f_{IIk}(V_0 - \dot{X}_{II2}) = \mu_0 \tanh\left(\frac{V_0 - \dot{X}_{II2}}{\varepsilon}\right) - \alpha(V_0 - \dot{X}_{II2}) + \beta(V_0 - \dot{X}_{II2})^3 , \tag{36}$$

with fixed $\mu_0 = 0{,}8$, $\alpha = 15{,}59$, $\beta = 4252{,}12$ and $\varepsilon = 0{,}0001$.

Moreover, because functions $\boldsymbol{1}(f_g - (e_1 Y_{I1} - e_2\varphi_I))$, $\boldsymbol{1}(f_g - (e_1 Y_{II1} - e_2\varphi_{II}))$ are also discontinuous, in our computations we use the following approximations

$$f_{In}(f_g - (e_1 Y_{I1} - e_2\varphi_I)) = \tanh^3\left(\frac{f_g - (e_1 Y_{I1} - e_2\varphi_I)}{\varepsilon}\right) \cdot \boldsymbol{1}(f_g - (e_1 Y_{I1} - e_2\varphi_I)) , \tag{37}$$

$$f_{IIn}(f_g - (e_1 Y_{II1} - e_2\varphi_{II})) = \tanh^3\left(\frac{f_g - (e_1 Y_{II1} - e_2\varphi_{II})}{\varepsilon}\right) \cdot \boldsymbol{1}(f_g - (e_1 Y_{II1} - e_2\varphi_{II})) . \tag{38}$$

In result, in our numerical simulation we consider the following equations of motion

$$\begin{cases} \ddot{X}_{I1} + a_1 X_{I1} + a_2\varphi_I - a_3 X_{I2} = 0, \\ \ddot{Y}_{I1} + b_1 Y_{I1} - b_2\varphi_I + f_g = 0, \\ \ddot{\varphi}_I + c_1 X_{I1} - c_2 Y_{I1} + c_3\varphi_I - c_4 X_{I2} + k(\varphi_I - \varphi_{II}) = 0, \\ \ddot{X}_{I2} - X_{I1} - \varphi_I + X_{I2} = f_{Ik}(V_0 - \dot{X}_{I2})\cdot[f_g - (e_1 Y_{I1} - e_2\varphi_I)]\cdot \boldsymbol{1}(f_g - (e_1 Y_{I1} - e_2\varphi_I)), \\ \ddot{X}_{II1} + a_1 X_{II1} + a_2\varphi_{II} - a_3 X_{II2} = 0, \\ \ddot{Y}_{II1} + b_1 Y_{II1} - b_2\varphi_{II} + f_g = 0, \\ \ddot{\varphi}_{II} + c_1 X_{II1} - c_2 Y_{II1} + c_3\varphi_{II} - c_4 X_{II2} + k(\varphi_{II} - \varphi_I) = 0, \\ \ddot{X}_{II2} - X_{II1} - \varphi_{II} + X_{II2} = f_{IIk}(V_0 - \dot{X}_{II2})\cdot[f_g - (e_1 Y_{II1} - e_2\varphi_{II})]\cdot \boldsymbol{1}(f_g - (e_1 Y_{II1} - e_2\varphi_{II})). \end{cases} \tag{39}$$

5. Numerical results

Fig. 2 shows the phase trajecories of the system for the velocity of driving belt $V_0 = 0.002$ and zero initial conditions in time interval $\tau \in [10000,12000]$. The time interval was chosen to avoid the transition state.

Obtained results and detect an irregular dynamics of the considered 8 –DOF system. The phase trajectories, Poincaré maps (Fig. 3) as well as power spectral densities (Fig.4) indicate that the character of motion is chaotic. If we increase the value of V_0 then the character of motion changes. This situation is presented in the Fig. 5, Fig.6 and Fig.7. When the dimensionless velocity of driving belts reaches the value of 0.05, the motion exhibit a periodic character.

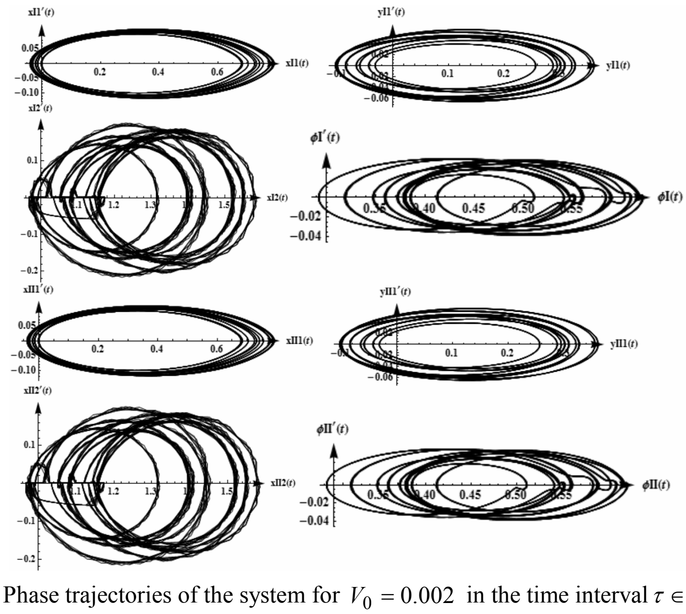

Figure 2. Phase trajectories of the system for $V_0 = 0.002$ in the time interval $\tau \in [10000,12000]$.

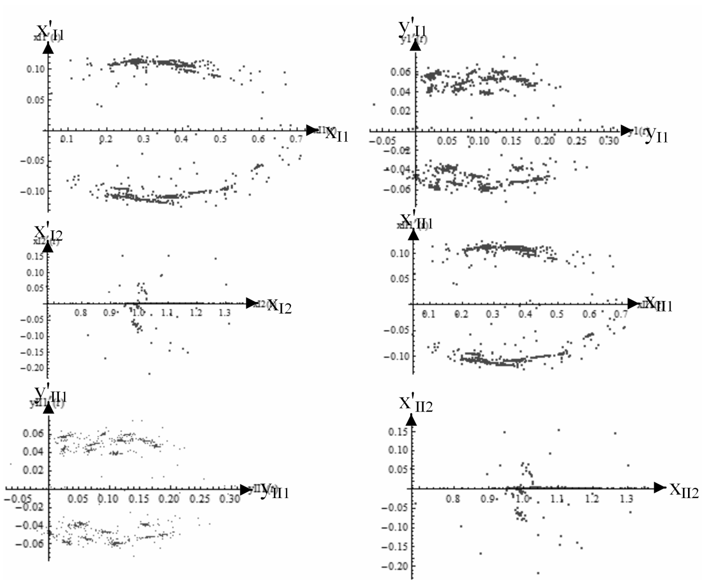

Figure 3. Poincaré map of the system for $V_0 = 0.002$ in the time interval $\tau \in [10000,12000]$.

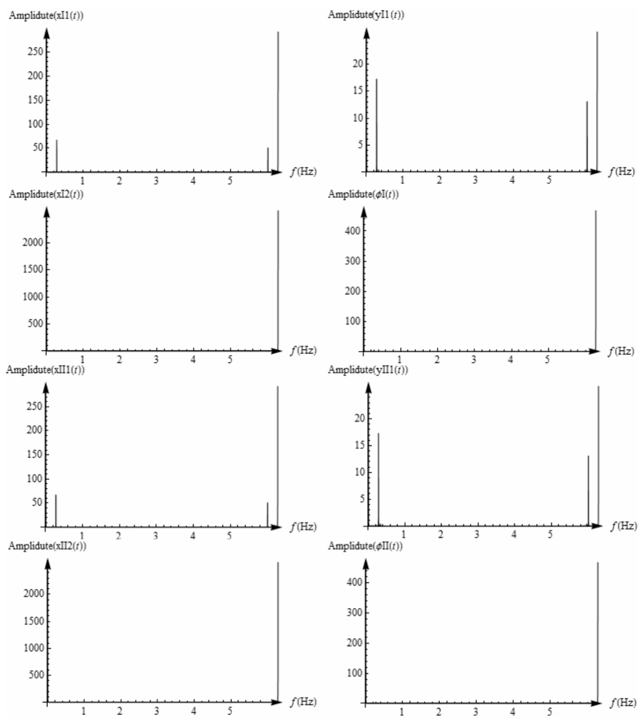

Figure 4. Power spectral of the system for $V_0 = 0.002$ in the time interval $\tau \in [10000,12000]$.

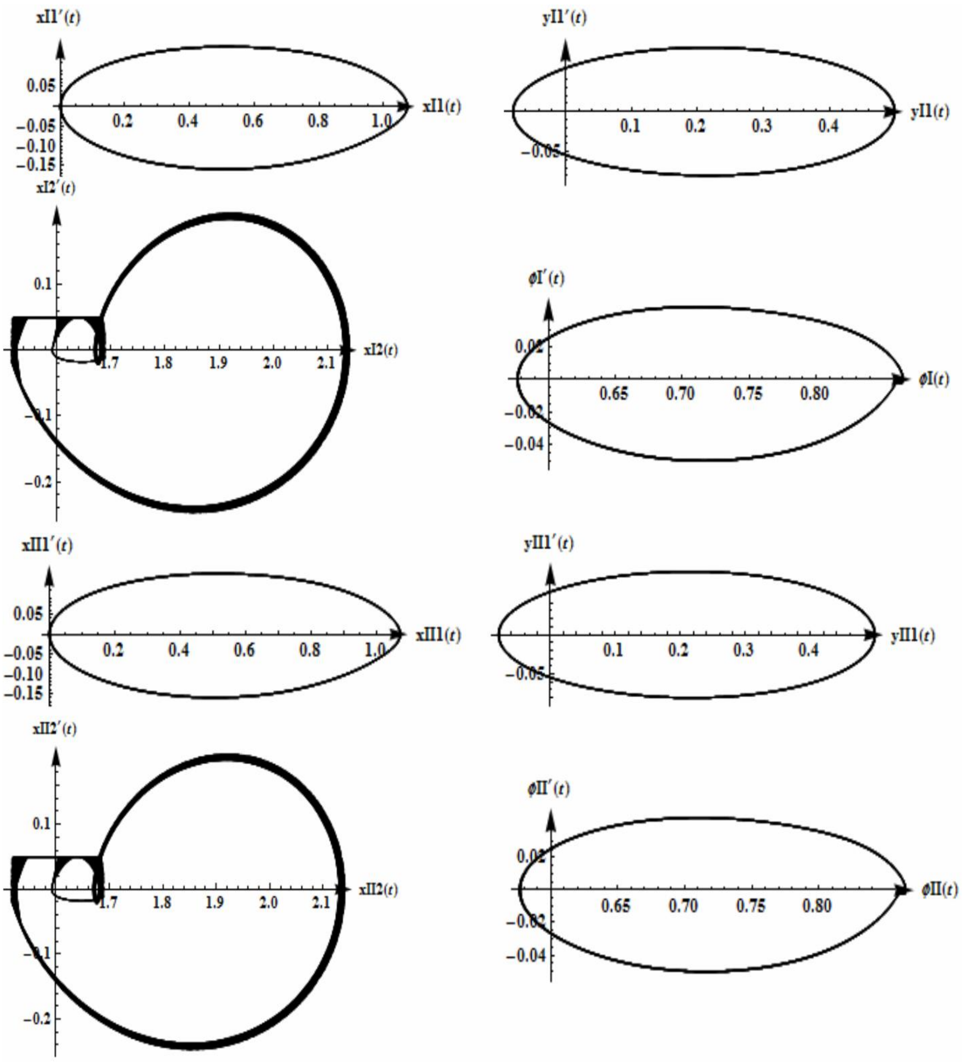

Figure 5. Phase trajectories of the system for $V_0 = 0.05$ in the time interval $\tau \in [10000, 12000]$.

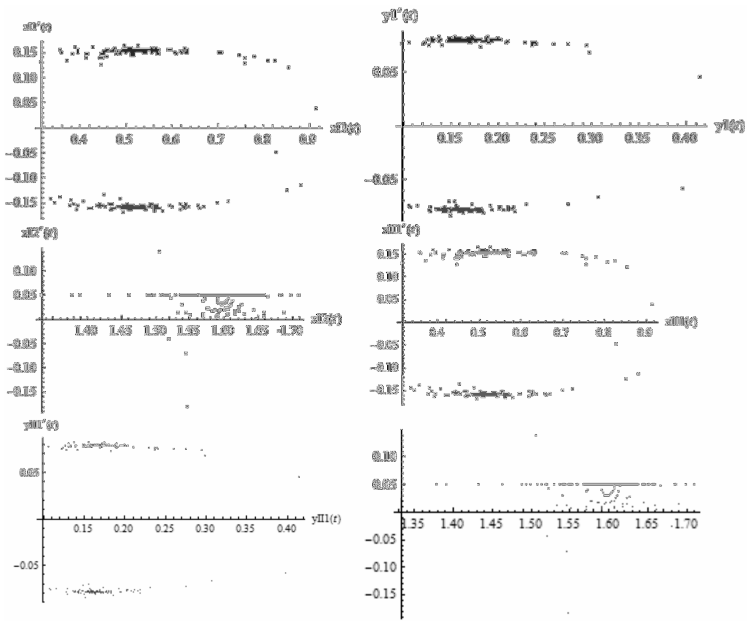

Figure 6. Poincaré map of the system for $V_0 = 0.002$ in the time interval $\tau \in [10000,12000]$.

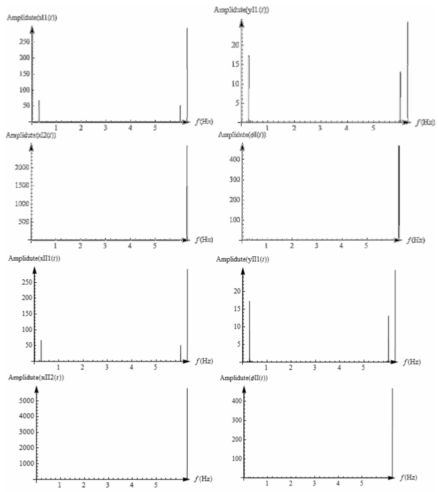

Figure 7. Power spectral of the system for $V_0 = 0.05$ in the time interval $\tau \in [10000,12000]$

6. Conclusions

In the paper mathematical model of two coupled 4-DOF mechanical linear sliding systems with dry friction is considered. The considered system can be treated as a system of two identical 4-DOF systems presented earlier in [4] and coupled by torsional spring. In this case the physical interpretation of the considered model could be two rows of carriages laying on the guideways and coupled by an elastic shaft, which moves at constant velocity with respect to the guideways as a foundation. From a mathematical viewpoint the mentioned system is presented as a nonlinear equations of motion, which are obtained using second kind Lagrange's equations. Dynamics of the analyzed system is carried out for one set of system parameters and various non-dimensional V_0. Interesting dynamics behaviors of the considered system are reported using time series and phase trajectories. The obtained results indicate, that the analyzed system possesses periodic, quasi-periodic or chaotic orbits, as well as fixed points. Moreover, the mentioned results show that synchronization effects between the coupled systems are possible.

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