1D model of pulsating flows in pipes dynamics using MOC (ENG038-15)

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Abstract: This paper presents a model of 1D pulsating flows in partially opened pipes. This is typical of inlet and exhaust pipes of internal combustion engines and piston compressors. The introduction describes the current state of affairs at one dimensional modeling pulsations according to the Method of Characteristics (MOC). Subsequently, the main idea of MOC is presented. In the fourth chapter general guidelines at initial and boundary conditions were presented. The parameters were analyzed at space times domain as 3D diagrams. Next, two crucial resonant frequencies are presented: first and second waveform which can be defined as a quarter and half-length waves. Firstly, partially open end phenomena at space and stage plane were presented. Secondly, the theory of acoustic wave propagation with description of stage waves at analyzed ends of pipe was shown. Finally, existing test rig dedicated to research the pulsating flow in pipes was shortly presented. This test rig was built at Flow Metrology Division of Lodz University of Technology. In the second part of this chapter the main assumption of an elaborated simulation algorithm at Matlab Simulink environment was shown. Lastly, simulation results compared with presented conceptions of modeling those systems with the emphasis into pulsation dynamics according to resonant frequencies were discussed.

1. Introduction

The dynamics of pulsating flows in pipes is a very interesting case at various pipelines designs which enables more efficient and silent fluid transport. Sometimes resonance effect is used to improve a significant parameter of chosen process. The method of characteristics is a mathematical technique for solving a hyperbolic type of partial differential equations. This method reduces partial differential equations (PDE) into the family of ordinary equations enabling to integrate a solution from initial data [12]. This is a type of finite difference methods which can be divided into implicit and explicit. A first order of accuracy explicit method was introduced in 1952 by Courant for use with the Characteristics form of the hyperbolic type PDE [12]. In 1952 Hartee proposed schemes with second order accuracy and in 1964 Benson proposed a method based on Courant schemes dedicated for simulating internal combustion engines and reciprocating compressors. The method compares well with other first order of accuracy explicit methods and has advantages over them dealing with boundary conditions.

In 1960 Lax and Wendorff introduced a second order of accuracy explicit method a Two Step version developed later by Richmyer. In the 80's and 90's the development of Total Variation Dimensioning schemes and Flux Corrected Transport has allowed for the practical use of conservations scheme such as two step Lax Wendorff LW2 or MacCormack methods for intra pipe modeling. The association of TVD and LW2 or MC method is nowadays a recognized solution for efficient 1D manifold modelling, and is embedded into research (GASDYN) or commercial (GT-POWER) codes. Polish authors have also consequently used MOC according to fluid dynamics: Jungowski [2], Wisłocki [3], Prosnak [4], Puzyrewski and Sawicki [5] and at manifold modeling: Jungowski, Kordziński [6] and Sobieszczański [7]. Guderlay constructed general theory of characteristics in which variable specific heat capacities where introduced into Riemann invariants. Those methods were applied graphically by Benson and Winterbone and Pearson.

2. Governing equations

Flows are assumed to be one dimensional, ideal compressive gas is assumed to be inviscid.

Continuity equation: the rate of change of mass within the control volume is equal to net mass flow rate through the element:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\rho u}{F}\frac{dF}{dx} = 0 \tag{1}$$

a) Momentum equation: the rate of change momentum within the control volume is equal to the sum of forces applied to the control volume. This covers friction between the flow and the wall, a shear stress opposed to the flow.

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2 + p}{\partial x} + \frac{\rho u^2}{F}\frac{dF}{dx} + \rho G = 0 \tag{2}$$

$$G = \frac{1}{2}u|u|f\frac{4}{D} \tag{3}$$

b) Energy equation is derived from the first law of thermodynamics applied to control volume:

$$\frac{\partial \rho e_0}{\partial t} + \frac{\partial \rho u h_0}{\partial x} + \frac{\rho u h_0}{F}\frac{dF}{dx} - \rho q = 0 \tag{4}$$

The governing equations can be rewritten at vectorial form, where there are particular vectors: state vector of conservative variables (mass, momentum and total energy), flux vector and correction.

3. Principles of M.O.C.

Benson's [12] non dimensional notation was used in figure 1. For first step approximation, where heat transfers and friction in pipes are often omitted. Analyzed flow can be defined as a homentropic. Additionally, there is no area section change. According to that condition two non-dimensional Rieman invariants (α and β) are defined along the characteristics lines $C^+$and C^-:

$$\beta = A_i^{n+1} - \frac{\kappa-1}{2} * U_i^{n+1} = A_R^n - \frac{\kappa-1}{2} U_R^n \tag{5}$$

$$\beta = A_i^{n+1} - \frac{\kappa-1}{2} * U_i^{n+1} = A_L^n + \frac{\kappa-1}{2} U_L^n \tag{6}$$

For the homentropic flow contraction of the gas between S and mesh node $(i, n+1)$ can be defined:

$$\frac{p_S^n}{(\rho_S^n)^\kappa} = \frac{p_i^{n+1}}{(\rho_i^{n+1})^\kappa} \tag{7}$$

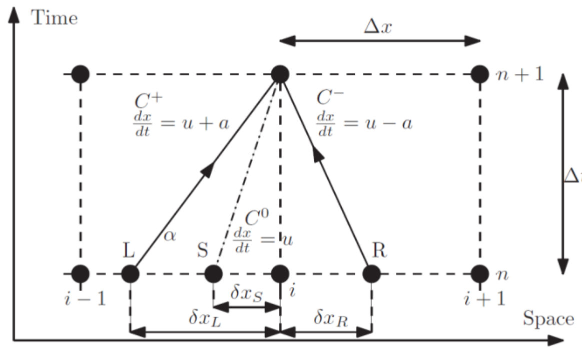

Figure 1. Method of characteristics on space time diagram [11]

Having found δx_L, δx_R, δx_S thermodynamic states can be determined at nodes L, R and S by linear interpolation. After that, there should be calculated: ρ_i^{n+1} , u_i^{n+1}, and p_i^{n+1}.

Gas parameters are presented at two planes: space $t = f(x)$ and stage $a = f(u)$; where: a – speed of sound [m/s] and u - velocity of flow at x direction [m/s]. In figure 1 we can find, three areas at space plane (1, 2, and 3) which are also represented at stage plane, according to the two basic phases of valve at A section. Upper index ‘ – represents small opening of valve and “ – represents small closing of valve.

For subsonic flows the pressure at pipe should be equal to external one, initially, while and after wave reflection. A nozzle at the end of pipe is added. This nozzle can be described as cross section area change coefficient as a relation: $\varphi = \frac{F_n}{F_p}$, where $F_{n,p}$ –area of nozzle or pipe cross section $[m^2]$.

According to boundary conditions and gas state for each value of mentioned coefficient the line is possible at state plane. It can be proven additionally, the $\varphi = const$ curve is straight line for sonic flows and their extension crosses the centre of coordinate systems of state plane. Limiting values of φ corresponds to opened end ($\varphi = 1$ from figure 1) or completely closed ($\varphi = 0$). Solution: initial and reflected state must be on the $\varphi = 0$ curve. It is worth to be mentioned, that abundance wave can be reflected as rarity, abundance or without reflection.

4. Boundary conditions for pressure and velocity at opened, closed and partially opened end [3,6]

The acoustic phenomenon is very useful at investigations of pulsating flows in pipes dynamics. Pulsating pipe flow propagates along acoustic wave, it means that the elements move in parallel to wave direction. This way, it can be said that we consider pulsations at plane wave with time-like variable. According to this, the argument of acoustic wave is$(\omega t + kx)$, so called wave phase, where: ω - wave frequency, k - wave number [1]. Mentioned variables are connected as follows:

$$a = \frac{\omega}{k} = \frac{\lambda}{T} \tag{8}$$

where:

λ – wave length [m]

T – wave period [s]

Thanks to this equation, speed of sound is also called phase wave speed. As it was shown in figure 5 there are following rules for pressure and speed values according to the boundary conditions:

a) "*open end*" reflection of pressure wave is within change of sign. Speed and displacement reflect with phase change about $\lambda/2$. Arising standing wave has characteristic arrows and nodes at boundaries. For speed standing wave, there is an "arrow" and there is node for pressure standing wave.
b) "c*losed end*" reflected pressure wave has the same sign as falling ones. Reflection of speed and displacement waves is within change of sign. Arising standing wave has characteristic arrows and nodes at boundaries. For speed standing wave there is node and there is arrow for pressure standing wave.

4. Experimental background and measurement data.

According to the main presented idea of MOC and their boundary conditions a simulation model of pipe with pulsating flow was prepared from test rig at Institute of Turbomachinery (Lodz University of Technology) presented in figure 2. The extensive research on dynamic phenomena in a straight pipe supplied with a pulsating flow of gas, shows, that the character of variation of flow parameters (pressure, temperature, velocity) depends on the nature of the pipe outlet. Based on research of Olczyk [9] experimental data could be implemented into simulation model elaborated at Matlab Simulink. The main idea of this model was presented at [15,16]. Simulation and experiment parameters of the dynamic states of pulsating flow parameters were compared and results of their synthesis is presented below.

The main parameters of simulated flow [9]:

a) Range of desired values of frequency of pulse generator $f = (20 \div 200)$ [Hz]
b) Pipe diameter $D_p = 42\ 10^{-3}$[m]
c) Pipe length $L_p = 0{,}544$ [m], determined with resonance at 70 Hz and 140 Hz
d) Nozzle diameter $D_n = 10\ 10^{-3}$ [m]. Nozzle is mounted at the end of pipe, at (3) cross section.
e) Desired flow temperature $T = 313{,}15$ [K]
f) Mean Flow speed $u = 20$[m/s] (mean value)
g) Mean Pressure $p = 115000$ [Pa]

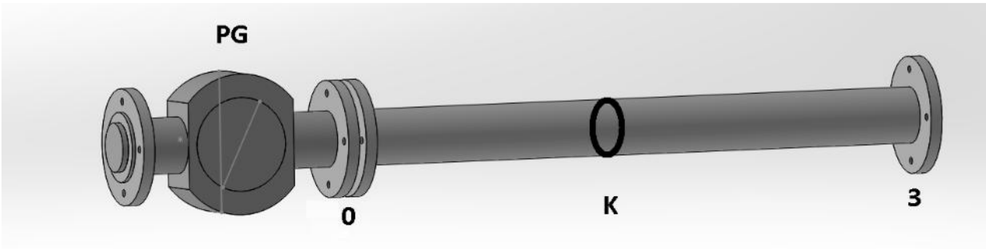

Figure 2. The main elements of the test rig with test pipe

Measurement of transient and mean values of pressure, temperature and specific mas flow rate were conducted at control sections (0) and (3) shown in figure 4. Additionally, transient pressure was measured in section (K) placed in the pipe in the mi length.

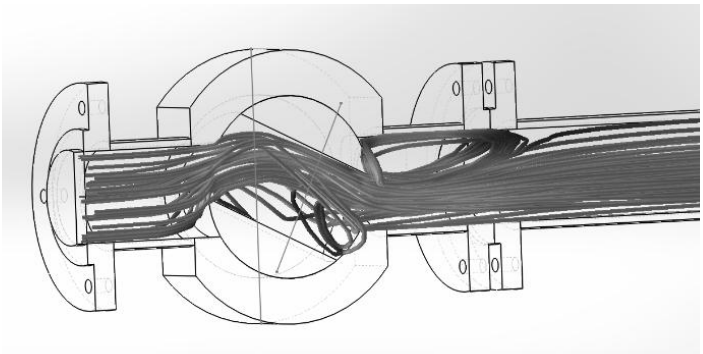

Figure 3. Flows visualization at pulse generator

Pulse generator enables measurements of variable reliable and repeatable flow pulsations as it was shown in figure 3. Example stream line was prepared using SOLIDWORKS Flow Simulation. This unit works almost contactless and is driven by electric motor with inverter controlled speed.

As far as measurement devices are concerned, the following equipment was used:

– piesoresistive transducers for pressure measurements: Endevco 8510C-15 and 8510C-50 (Endevco 2003);

– constant current thermometers (CCT) for temperature measurements;

– constant temperature anemometers (CTA) for specific mass flow rate measurements.

Both CCT and CTA probes incorporated a 5 lm tungsten wire.

The simulation algorithm was prepared according to listed assumption:

a) The heat transfer and friction phenomena were not taken into consideration. Adding successive parameters to complete the solution with mentioned phenomena is possible.

b) Two direction iteration process is provided, first at space field (i in figure 1), there is fifteenth steps, second at time (n in figure 1) 200 iterations. This is enforced by Courant Lewy condition connected with demanded calculation resolution. This is needed because of the angle between space axis and the $u - a$ characteristic line.
When the maximum space iteration "i" is reached, the time "n" is switched to next value.

c) Implemented algorithm enables easy switching between boundary conditions presented at paragraph 3. Thanks to that, acquisition of simulation data for clear comparison is possible.

d) The initial conditions were implemented from measurement data, to make calculation more adequate with the experiment, that data were processed by FFT.

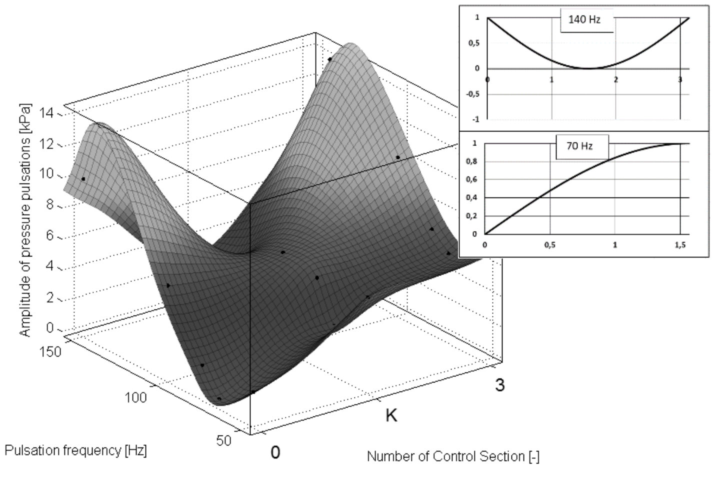

Figure 4. Estimation of pressure pulsations amplitude *(on the right side: quarter sinus wave for 70 Hz resonance and the half sinus wave for 140 Hz resonance)*

According to the experiment estimation of amplitude of pressure pulsations was established at the pulsation frequency and control section domain, as it was shown in figure 4. Estimation was based on Curve Fitting tool at Matlab R2015a software, figure 4. This estimation assumes whole spectrum of measured frequencies according to the three defined cross sections. Two forms of resonant wave were identified, for 70 Hz – explicit increase of pulsations amplitude along the pipe length (fig. 5), 140 Hz decrease of pulsations amplitude in the middle of pipe length (fig. 6).

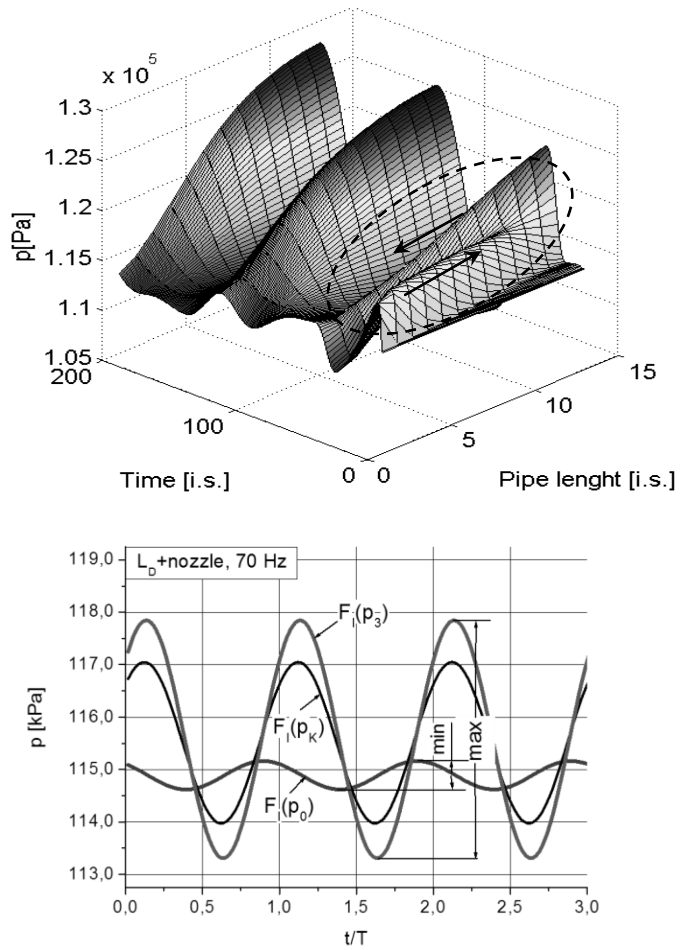

Figure 5. Pressure scope for resonant frequency 70Hz *(simulation-upper, measurements – down [10])*

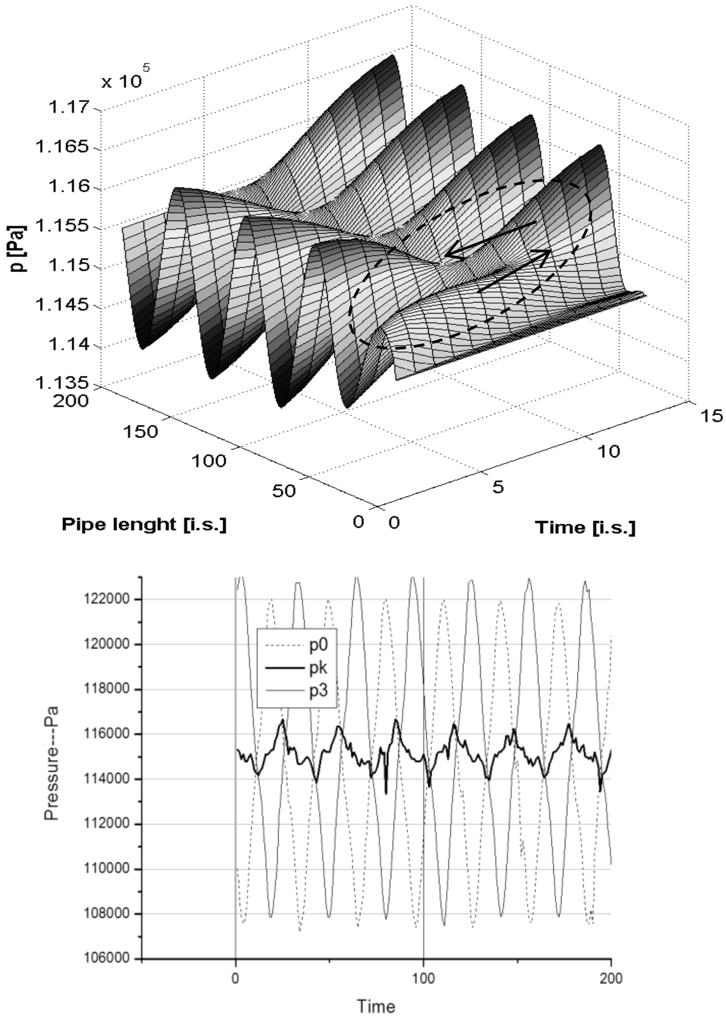

Figure 6. Pressure scope for resonant frequency 140Hz *(simulation- top, measurements – bottom [10])*.

Next, numerical simulations of the analysed pipe were provided and results were compared with experimental data. These cases can be also identified as a first and second harmonic figure of oscillations what can be defined as a 1/4 sinusoid and the half sinusoid. Figures 5 and 6 are presented at time-space domain using iteration step unit. It means, that there are 15 steps along pipe length ($L_p = 0{,}544$ [m]) and 200 steps at time domain (0,01 [s]).

Acoustic phenomena of simulation model can be easily found for the initial time steps pointed in figures 5 and 6 by dashed lines. Propagating wave is divided into two parts proportionally to cross section area change coefficient defined at 3. Reflected part of propagating wave will have boundary conditions from closed end type. Rest of the falling wave will propagate outside of the nozzle.
Qualitative comparison results of numerical experiment with measurements can be found quite satisfying. According to the mentioned above similarities, in author's opinion, quite good qualitative compliance between measurements and numerical experiment results was reached.

Similarly, figures 5 and 6, waveforms of instantaneous variations of fluid velocity, temperature and the local speed of sound are calculated.

The proposed numerical model is very simplified as it was presented in previous articles [15, 16], it does not take into consideration the heat transfer and friction phenomena. This simplification causes that the results obtained are mostly qualitative. Thanks to that, the model is simple and needs a little time for solving particular cases (about 15 seconds).

Comparing simulation results from figures 5 and 6 with measurement approximated in figure 4 it can be concluded that the proposed approximation method in figure 4 is acceptable. The proposed method enables to predict results for the length of the analyzed pipe to be estimated on the basis of measurements of only three control sections.

5. Conclusions

In this paper a simple and fast model of one dimensional pulsating flow in partially opened pipes was presented. This is a typical case for inlet and exhaust pipes of internal combustion engines and piston compressors. Proposed numerical model enables easy analysis of dynamics for pipes with pulsating flows especially resonances and their influence into crucial parameters such as pressure, temperature along pipe length.

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