SOME CONTRIBUTIONS INTO A SOLUTION FOR PLASTIC MECHANISMS IN THIN-WALLED BEAMS

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Two contributions into the problem of plastic mechanisms in thin-walled box-section beams are presented in the paper. The first solution concernes a beam built of strain-hardening izotropic material and incorporates membrane strain energy in the total energy of plastic deformation. The second solution takes into account orthotropic properties of a beam material while a rigid-perfectly plastic behaviour of the material is considered. Numerical results are presented in diagrams showing an energy of plastic deformation and bending moment at the global plastic hinge in terms of rotation angle at that hinge.

1. Introduction

A problem of the load-carrying capacity at collapse of a thin-walled structure which is of significant importance according to some aspects (1), can be solved on the basis of of the rigid-plastic theory and accomplished using the energy method.
The method was applied by several authors who investigated plastic mechanisms in thin-walled beams (2) and columns (4). Solutions comprised in those papers are based upon some simplifications which allow to formulate a relatively simple theoretical model of the problem. These assumptions are :– a rigid-perfectly plastic behaviour of the structure material, – neglecting of membrane strains in mechanism walls (*true mechanism*), – limitation of plastic zones to concentarted yield lines of infinitesimal width..

Some authors (3),(4) have recently taken up research into an incorporation of the strain-hardening characteristic of a structure material in the plastic mechanism analysis. A different problem which has not been investigated so far is an influence of orthotropic properties of the material upon a collapse of the structure. One can face this problem not only in the case of steel structures but also of strucures built of composites and stiffened structures which are of structural orthotropy. Considering the last case a paper (5) should be mentioned.

A present paper is an contribution into of the solution of the plastic mechanism problem and also is a continuation of the work (3). A new factor incorporated is a preliminary study of the plastic mechanism in orthotropic structure and of an influence of orthotropic properties on an energy absorbed and load-carrying capacity at collapse.

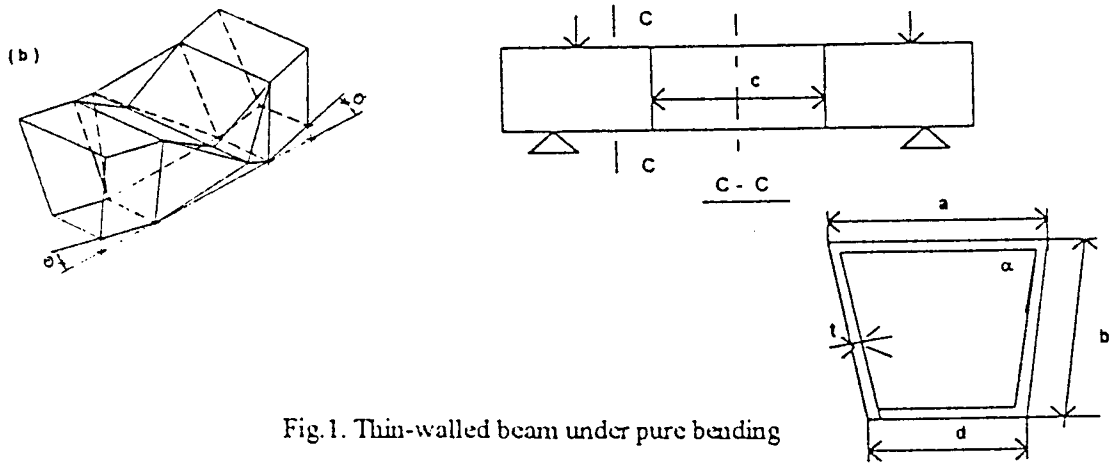

Fig. 1. Thin-walled beam under pure bending

2. Subject of the analysis

A subject of the analysis was a thin-walled beam of trapezoidal cross-section (Fig. 1). The beam was stiffened by diaphragms at a distance c from each other. It was assumed to be subject to a bending moment constant between two adjacent diaphragms. A plastic mechanism of failure taken into consideration is shown in Fig. 1b. This mode of failure was confirmed in several experimental tests (1) and also by means of FEM analysis. It can be observed in both trapezoidal and rectangular cross-section beams of an outline close to the square.

3. Energy of plastic deformation for a beam built from strain-hardening material

An energy of plastic deformation absorbed during collapse of a thin-walled beam is evaluated below under assumptions given in (2), (3). While taking into account a strain hardening behaviour of the beam material, the following stress-strain relation is assumed

$$\sigma = \sigma_y + E_t \cdot \varepsilon \qquad (3.1)$$

where E_t - tangent modulus, σ_y -yield stress, ε - linear strain at a yield line cross-section. The strain is regarded to be a sum of two components $\varepsilon = \varepsilon_b + \varepsilon_m$ which are a bending strain resulting from the Bernoulli hypothesis and a membrane strain in a yield strip, coming from compression or tension at a limited yield area - respectively. It should be emphasized here that a considered mechanism is a *true mechanism* i.e. it can be developed without inducing any membrane strain in its walls. However, due to the stress and strain distribution assumed (Fig.2.) a membrane strain appears at a yield strip.

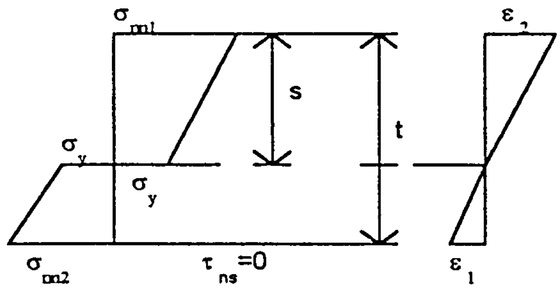

Fig.2. Stress and strain distribution at a yield line

Two following postulates (..), (..) are taken into the analysis

$$\rho = nt/\beta \qquad \text{and} \qquad \varepsilon_b = \beta/2n$$

where β is an angle of relative rotation between two adjacent walls of the mechanism along a yield line, ρ is an approximated *rolling radius* while n is a multiple of the beam wall thickness, determined either experimentally or in minimalization procedure. Thus, the principle of virtual velocities takes form

$$M\cdot\theta = \sum_i t_1 l_i \int_0^{\beta_i} (\sigma_{nn}\, \dot{\varepsilon}_m^{nn}\, \rho)\, d\beta + \sum_i l_i \int_0^{\beta_i} (m_{i\beta}\dot{\beta}_i)\, d\beta +$$

$$\sum_j \dot{\Gamma}_j\, (m_{j\beta}, \rho_j, \beta_j)\, d\beta \tag{3.2}$$

where

M - bending moment at the global plastic hinge,

θ - angle of relative rotation of two beam parts in the global plastic hinge,

l_i - length of a stationary yield line,

$m_{i\beta}$ - plastic moment at a yield line,

t_1 - thickness of the compressed core of thge wall cross-section,

Γ - energy absorbed at a local plastic hinge (1), (2).

The total energy of plastic deformation is expressed as a sum of two components $\Pi = W + E_m$, where W is a plastic energy of bending deformation and E_m is a membrane strain energy. The second component E_m was neglected in hitherto existing solutions (3) without any quantitative estimation of its contribution into the total energy of plastic deformation. Under assumptions taken above an energy E_{mi} absorbed during relative rotation of two walls of the plastic mechanism along i^{th} stationary yield line takes form

$$E_{mi} = \iint_A N_i \varepsilon_m dA$$

where N_i is an axial load per unit width in a plate member of the beam. Taking the assumptions given in (2), (3) , the energy E_{mi} can be evaluated as follows

$$E_{mi} = \frac{\sigma_{cr}^2 t^2}{E_t} \cos^4 \gamma \cdot \alpha_n \phi_{n\beta} \cdot \Psi_e \cdot l_i \tag{3.3}$$

where $\phi_{n\beta} = \ln(2n\sigma_y + E_t\beta) - \ln(2n\sigma_y)$, Ψ_e and α_n are coefficients depending upon a situation of a yield line (flange/web) and a geometry of the beam cross-section, γ is an angle of inclination of a yield line with respect to the normal load direction (2) while σ_{cr} and σ_y are buckling stress in the compressed flange and a yield stress of the beam material respectively.
An energy of membrane deformation absorbed at j^{th} traveling yield line or local plastic hinge (3) is evaluated as

$$\Xi_{mj} = \int_0^{\beta_j} F(\rho_j, \phi_{n\beta}^j, l_j) d\beta \tag{3.4}$$

The plastic energy W of bending deformation has been formulated in (3) as

$$W = \sum_i l_i \int_0^{\beta_i} m_{i\beta} d\beta + \sum_j \int_0^{\beta_j} \Gamma_j(m_{j\beta}, \rho_j, \beta_j) d\beta \tag{3.5}$$

Thus, the total energy of plastic deformation is as follows

$$\Pi = W + \sum_i E_{mi} + \sum_j \Xi_{mj} \tag{3.6}$$

4. Energy of plastic deformation for orthotropic beam material

In this paragraph an orthotropic plasticity theory is introduced in the plastic mechanism analysis. For simplicity a rigid - perfectly plastic behaviour of a beam material is taken into consideration. A system of coordinates is chosen so that x is parallel to the beam longitudinal axis and y is perpendicular to this direction.

Furthermore these two directions are principal direction of orthotropy. When the mechanism shown in Fig. 1 is reanalyzed one can notice that plastic deformations take place in direction which may be specified by an angle γ measured from x treated as a reference coordinate (2).

When applying a Hill's yield criterion for an orthotropic material , a yield stress corresponding to any direction γ is evaluated as

$$\sigma^2_{\gamma,y} = \frac{\sigma^2_{1y}}{a_{10}\cos^4\gamma + a_{20}\sin^4\gamma - a_{120}\sin^2\gamma\cos^2\gamma + 0.75a_{30}\sin^2 2\gamma} \tag{4.1}$$

where $a_{10}=1$, $a_{20} = \sigma^2_{2y}/\sigma^2_{1y}$, $a_{330} = \sigma^2_{1y}/\sigma^2_{\theta y}$, $a_{30} = (1/3)\cdot\sigma^2_{10}/\tau^2_{12_y}$. $\sigma_{1y}, \sigma_{2y}, \tau_{12y}$ are yield stresses in two orthogonal directions (one of them is x) and shear yield stress respectively, $\sigma_{\theta y}$ is a yield stress in direction inclined at θ with respect to the reference coordinate x.

For $\theta=45^o$ a coefficient a_{120} takes form $a_{120}=1+ a_{20} + 3a_{30} - 4a_{330}$. The criterion (4.1) is then introduced into a fully plastic moment at a yield line (2)

$$m_{p\gamma} = \sigma_{\gamma y}t^2/4 \tag{4.2}$$

and subsequently to the energy of plastic deformation limited to an energy of bending deformation only

$$W(\theta) = \sum_{j=1}^{m} m_{p\gamma j}l_j\beta_j + \sum_{i=1}^{k}\Gamma_i(m_{p\gamma i},\rho_i,\beta_i) \tag{4.3}$$

Thus the energy of plastic deformation for an orthotropic rigid-perfectly plastic material is approximated as $\Pi = W$.

5. Numerical results and comments

A contribution of an investigated membrane strain deformation into the total energy absorbed at collapse of the beam built of izotropic strain hardening material is shown in Fig. 3. It has been noticed that the lower b/t ration the greater is the influence of this part of energy on the total energy of plastic deformation. The influence of this factor on the bending moment capacity at the global plastic hinge is significant for very low b/t ratio only.

An influence of orthotropic properties on both energy of plastic deformation and bending moment capacity for a beam built of rigid - perfectly plastic material is shown in Fig.4. The results are of preliminary character and the problem should be analyzed and discussed separately.

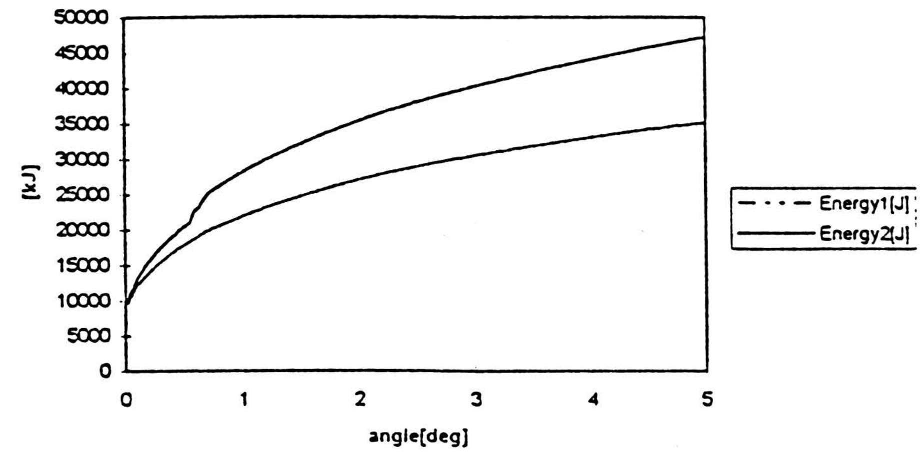

Fig.3. Energy of plastic deformation; 1 - bending strain energy, 2 - total.
a=d=60 mm, t = 0.75 mm, E = 2e5, E_t=2915 [MPa], σ_y = 168 , σ_{ult} = 327 [MPa]

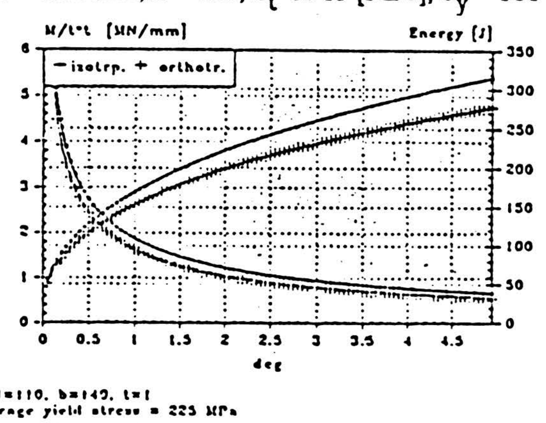

Fig.4. Comparison of isotropic (σ_y average) and orthotropic material:

$\sigma_{10} = 250, \sigma_{20} = 200, \sigma_{45} = 175, \tau_y = 150$ [MPa]

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