INTERACTIVE BUCKLING IN THIN-WALLED BEAM-COLUMNS WITH WIDTHWISE VARYING ORTHOTROPY

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An analysis of local buckling of thin-walled beam-columns, taking account global precritical bending within the first order approximation, is presented in the paper. The problem of interactive buckling of the structure is solved by means of Byskov and Hutchinson’s (1977) or Koiter’s (1976) approximation theory. Beam-columns made from orthotropic plates with the main directions of orthotropy parallel to the wall edges characterised by a widthwise varying orthotropy coefficient \( \eta_i = \frac{E_{yi}}{E_{xi}} \) are investigated. Beam-columns with open sections (i.e. channel sections), simply supported on the loaded edges, are analysed. The girders are subjected to loads which cause a uniform or linearly variable shortening of the edges.

Key words: interactive buckling, thin-walled structures, orthotropy

Notation

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<td>cosinusoid amplitude</td>
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<td>( a_g, a_{gL}, a_L, a_{ggg} )</td>
<td>coefficients of the non-linear equilibrium equation, where the subscript ( g ) denotes the global mode ((n = 1)), ( L ) refers to the local buckling mode ((n = 2))</td>
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<td>( b_i )</td>
<td>( i )th band (narrow plate) width</td>
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<td>widthwise Young’s modulus for the ( i )th band</td>
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$G_i$ – modulus of elasticity (Kirchhoff’s modulus) for the $i$th band

$h_i$ – thickness of the $i$th band

$i$ – number of the band, wall (subscript $i = 1, 2, \ldots$)

$l$ – girder length

$M_{ix}, M_{iy}, M_{ixy}$ – sectional bending moment of the $i$th band

$N$ – force field

$N_i^{(0)}$ – force field of the zero state (prebuckling state)

$N_i^{(n)}$ – force field of the $n$th buckling mode

$N_{ix}, N_{iy}, N_{ixy}$ – sectional membrane forces for the $i$th band

$\mathbf{U}$ – displacement field

$\mathbf{U}_i^{(0)}$ – displacement field of the zero state (prebuckling state)

$\mathbf{U}_i^{(n)}$ – displacement field of the $n$th buckling mode

$u_i, v_i, w_i$ – middle surface displacements for the $i$th band

$u_i^{(0)}, v_i^{(0)}, w_i^{(0)}$ – prebuckling displacement field for the $i$th band (zero state)

$u_i^{(1)}, v_i^{(1)}, w_i^{(1)}$ – critical displacement field for the $i$th band (for the first order)

$x_i, y_i, z_i$ – local Cartesian co-ordinate system for the $i$th band

$\beta_0 = 3.2292$ – assumed constant value for the inversed coefficient of orthotropy

$\beta_i = 1/\eta_i$ – inverse of the assumed coefficient of orthotropy, assumed in order to facilitate the analysis of data

$\varepsilon_{ix}, \varepsilon_{iy}$ – relative strain along $x_i, y_i$

$\gamma_{ixy} = 2\varepsilon_{ixy}$ – non-dilatational strain angle

$\kappa$ – parameter of the external load distribution (ratio of the displacement of the upper part of the girder with respect to the bottom part)

$\eta_i = E_{iy}/E_{ix}$ – coefficient of orthotropy of the $i$th plate (band)

$\lambda$ – scalar load parameter

$\lambda_g$ – critical value of $\lambda$ (critical value of buckling) of the global mode

$\lambda_L$ – critical value of $\lambda$ (critical value of buckling) of the first local mode

$\lambda_{\text{min}}$ – minimal critical value of $\lambda$ (critical value of buckling)

$\lambda_k$ – critical value of local buckling with global pre-critical bending taken into account
\[ \nu_i = \nu_{ixy} \] – Poisson’s ratio for the \( i \)th band in the \( x \) direction (the first subscript denotes a transverse direction, whereas the second one – the load direction)

\[ \nu_{iyx} \] – Poisson’s ratio for the \( i \)th band in the \( y \) direction (the first subscript denotes a transverse direction, whereas the second one – the load direction)

\[ \varphi \] – angle enclosed between the wall \( i \) and \( i + 1 \)

\[ \xi \] – amplitude of the linear eigenvector of buckling (normalised with the equality condition between the maximum deflection and the thickness of the first plate \( h_1 \))

\[ \xi_n \] – amplitude of the linear eigenvector of buckling for the \( n \)th buckling mode

\[ \xi_L \] – amplitude of the linear eigenvector of buckling for the first local buckling mode

\[ \xi_g \] – amplitude of the linear eigenvector of buckling for the global buckling mode

\[ \xi^*_L \] – initial imperfection consistent with the first local buckling mode

\[ \xi^*_g \] – initial imperfection consistent with the global buckling mode.

Moreover, the following notation has been used

\[ (\cdot)_x = \frac{\partial(\cdot)}{\partial x} \quad (\cdot)_y = \frac{\partial(\cdot)}{\partial y} \]

1. Introduction

Interactive buckling of isotropic and orthotropic thin-walled structures has been investigated in many works (e.g. Luongo and Pignataro, 1988; Maniewicz and Kolakowski, 1997). Results of these investigations show a possibility of building thin-walled structures that are light, safe and reliable.

As far as composites are concerned, their material properties can be freely modelled in selected directions or regions, thus it is possible to manufacture plates or girders with variable strength properties. An example of materials characterised by such properties are fibrous composites with properly distributed (concentrated or dispersed) fibres. Composite materials are most often
modelled as orthotropic materials. In the wide literature devoted to stability problems, there is a lack of analysis of the influence of plate widthwise varying orthotropy on values of critical loads of coupled buckling of girders built of such plates.

In the present paper, the problem of local loss of stability, accounting for global pre-critical bending in the elastic range, is discussed. Thin-walled beam-columns with open sections built of homogeneous orthotropic plates with widthwise varying orthotropy are considered. The consideration of a particular variation in the orthotropy is carried out simply to demonstrate that such variations can be dealt with analytically, and to illustrate the influence of material properties.

2. Problem under consideration

Beam-columns with open sections and made of plates with widthwise varying orthotropy (Fig. 1) have been analysed. For discretized material properties varying widthwise, a model built of narrow longitudinal orthotropic bands has been assumed. Each band has constant material properties. The coefficient of orthotropy for individual bands (narrow plates) of the model varies according to the formula

$$\beta_i = \beta_0 + A \cos \frac{2\pi y_i}{b_i}$$

(2.1)

where $\beta_0 = 3.2292$, $A \in (-2, 2)$ is the amplitude of the cosine wave, $y_i$ is a coordinate defining the distance of the band from the one of the longitudinal edges; $b_i$ is the plate width.

It has been assumed that the main axes of wall orthotropy are parallel with respect to the wall edges. For the $i$th orthotropic band, a complete strain tensor for thin plates has been assumed in the form

$$\varepsilon_{ix} = u_{ix} + \frac{1}{2}(w_{ix}^2 + u_{ix}^2 + v_{ix}^2)$$
$$\varepsilon_{iy} = v_{iy} + \frac{1}{2}(w_{iy}^2 + u_{iy}^2 + v_{iy}^2)$$
$$2\varepsilon_{ixy} = \gamma_{ixy} = u_{iy} + v_{ix} + w_{ix}w_{iy} + u_{ix}u_{iy} + v_{ix}v_{iy}$$

(2.2)

where: $u_i$, $v_i$, $w_i$ are displacements parallel to the respective axes $x_i$, $y_i$, $z_i$ of the local Cartesian system of co-ordinates, whose plane $(x_i, y_i)$ coincides with the central area of the $i$th plate (ith band) before its buckling (Fig. 2).
Well known in the theory of orthotropic plates relations (e.g. Chandra and Raju, 1973; Królak, 1995) describe sectional forces and moments reduced to the middle surface of the \( i \)th plate (\( i \)th band)

\[
\begin{align*}
N_{ix} &= \frac{E_i h_i}{1 - \eta_i \nu_i^2} (\xi_{ix} + \eta_i \nu_i \xi_{iy}) \\
N_{iy} &= \frac{E_i h_i}{1 - \eta_i \nu_i^2} (\eta_i \nu_i \xi_{ix} + \xi_{iy}) \\
N_{ixy} &= N_{iyx} = G_i h_i \gamma_{ixy} = 2G_i h_i \xi_{ixy} \\
M_{ix} &= D_i (\kappa_{ix} + \eta_i \nu_i \kappa_{iy}) \\
M_{iy} &= \eta_i D_i (\nu_i \kappa_{ix} + \kappa_{iy}) \\
M_{ixy} &= D_{i1} \kappa_{ixy}
\end{align*}
\] (2.3)
Fig. 3. Local co-ordinate systems of interactive walls (bands)

where

\[ E_i = E_{ix} \quad \nu_i = \nu_{ixy} \quad \eta_i = \frac{E_{iy}}{E_{ix}} \]  \hspace{1cm} (2.4)

According to the Maxwell-Betti theorem, Young’s moduli and Poisson’s ratios occurring in equations (2.3) have to fulfil the following relation

\[ E_i \nu_{ixy} = E_{iy} \nu_i \]  \hspace{1cm} (2.5)

The equation of equilibrium of thin-walled structures has been derived using a variational method (Kołakowski, 1993; Królak, 1995). The total potential energy variation for the \( i \)th plate (\( i \)th band) can be written as

\[
\delta \Pi_i = \delta \left( \int \left( N_{ix} \varepsilon_{ix} + N_{iy} \varepsilon_{iy} + N_{ixy} \gamma_{ixy} \right) dS_i + \right.
\]

\[ -\delta \int \left( M_{ix} w_{i,xx} + M_{iy} w_{i,yy} + 2M_{ixy} w_{i,xy} \right) dS_i - \int p_i(y_i) h_i \delta u_i \ dy_i + \]

\[ - \int p_i(x_i) h_i \delta v_i \ dx_i - \int \tau_{ixy} h_i \delta u_i \ dy_i - \int \tau_{ixy} h_i \delta v_i \ dx_i - \int q_i \delta w_i \ dS_i \]

where: \( p_i(y), p_i(x), \tau_{ixy} \) are the external pre-critical loads of the plate, and \( q_i \) – transverse load.

The present paper deals with the stability problem, hence in the further part of the present analysis, the transverse load \( q_i \) will be neglected (\( q_i = 0 \)).

Equation (2.6) indicates that the potential energy \( \Pi_i \) of the \( i \)th plate sector in the equilibrium state has a non-varying value in the class of permissible
equations. It means that equation (2.6) for all permissible virtual displacements complying with the imposed constrains has to be satisfied.

In order to obtain the variation of potential energy for an orthotropic plate, strain tensor relation (2.2) has been substituted into equation (2.6). After grouping the components at respective variations, variational equations of equilibrium and boundary conditions have been obtained. Variational equations of equilibrium corresponding to equations (2.2) take the form

\[
\int_S \left[ N_{ix,x} + N_{ixy,y} + (N_{ix} u_{i,x})_x + (N_{iy} u_{i,y})_y + (N_{iz} u_{i,z})_z \right] \delta u_i \ dS = 0
\]

\[
\int_S \left[ N_{ixy,x} + N_{izy,y} + (N_{ix} v_{i,x})_x + (N_{iy} v_{i,y})_y + (N_{iz} v_{i,z})_z \right] \delta v_i \ dS = 0
\]

\[
\int_S \left[ (N_{ix} w_{i,x})_x + (N_{iy} w_{i,y})_y + (N_{iz} w_{i,z})_z \right] \delta w_i \ dS = 0
\]

(2.7)

and the boundary conditions are as follows

\[
\int_y M_{ix} \delta w_{i,x} \ dy_i \bigg|_{x_i = \text{const}} = 0
\]

\[
\int_x M_{iy} \delta w_{i,y} \ dx_i \bigg|_{y_i = \text{const}} = 0
\]

\[
2M_{ixy} \delta w_i = 0
\]

\[
\int_x (N_{ix} + N_{iy} u_{i,y} + N_{ixy} u_{i,x} - h_i \tau_{i,xy}) \delta u_i \ dx_i \bigg|_{y_i = \text{const}} = 0
\]

\[
\int_y (N_{ix} + N_{iy} u_{i,x} + N_{ixy} u_{i,y} - h_i \tau_{i,xy}) \delta u_i \ dy_i \bigg|_{x_i = \text{const}} = 0
\]

\[
\int_x (N_{iy} + N_{ix} v_{i,y} + N_{ixy} v_{i,x} - h_i \tau_{i,xy}) \delta v_i \ dx_i \bigg|_{y_i = \text{const}} = 0
\]

\[
\int_y (N_{ix} + N_{iy} v_{i,x} + N_{ixy} v_{i,y} - h_i \tau_{i,xy}) \delta v_i \ dy_i \bigg|_{x_i = \text{const}} = 0
\]

(2.8)
\[
\int_x (M_{iy,y} + 2M_{ixy,x} + N_{iy}w_{i,y} + N_{ixy}w_{i,x}) \delta w_i \, dx \bigg|_{y_i = \text{const}} = 0
\]
\[
\int_y (M_{ix,x} + 2M_{ixy,y} + N_{ix}w_{i,x} + N_{ixy}W_{i,y}) \delta w_i \, dy \bigg|_{x_i = \text{const}} = 0
\]

Static interaction conditions at the longitudinal edges of neighbouring plates, which follow from (2.8) for \( y = \text{const} \), can be written as

\[
\begin{align*}
&u_{i+1} \bigg|_{y_{i+1} = 0} = u_i \bigg|_{y_i = b_i} \\
&w_{i+1} \bigg|_{y_{i+1} = 0} = w_i \bigg|_{y_i = b_i} \cos(\varphi_{i;i+1}) - v_i \bigg|_{y_i = b_i} \sin(\varphi_{i;i+1}) \\
&v_{i+1} \bigg|_{y_{i+1} = 0} = w_i \bigg|_{y_i = b_i} \sin(\varphi_{i;i+1}) + v_i \bigg|_{y_i = b_i} \cos(\varphi_{i;i+1}) \\
&w_{i+1,y} \bigg|_{y_{i+1} = 0} = w_i,y \bigg|_{y_i = b_i}
\end{align*}
\]

(2.9)

\[
\begin{align*}
&M_{(i+1)y} \bigg|_{y_{i+1} = 0} = M_{iy} \bigg|_{y_i = b_i} \\
&N_{(i+1)y} \bigg|_{y_{i+1} = 0} - N_{iy}^* \bigg|_{y_i = b_i} \cos(\varphi_{i;i+1}) - Q_{iy}^* \bigg|_{y_i = b_i} \sin(\varphi_{i;i+1}) = 0 \\
&Q_{(i+1)y} \bigg|_{y_{i+1} = 0} + N_{iy}^* \bigg|_{y_i = b_i} \sin(\varphi_{i;i+1}) - Q_{iy}^* \bigg|_{y_i = b_i} \cos(\varphi_{i;i+1}) = 0 \\
&N_{(i+1)xy} \bigg|_{y_{i+1} = 0} = N_{ixy}^* \bigg|_{y_i = b_i}
\end{align*}
\]

where

\[
\begin{align*}
N_{iy}^* &= N_{iy} + N_{iy}v_{i,y} + N_{ixy}v_{i,x} \\
N_{ixy}^* &= N_{ixy} + N_{ixy}u_{i,x} + N_{iy}u_{i,y}
\end{align*}
\]

(2.10)

\[
\begin{align*}
M_{iy} &= -\eta_i D_i (w_{i,yy} + \nu_i w_{i,xx}) \\
Q_{iy}^* &= -\eta_i D_i w_{i,yyy} - (\nu_i \eta_i D_i + 2D_{11}) w_{i,xy} + N_{iy}w_{i,y} + N_{ixy}w_{i,x}
\end{align*}
\]

The interactive (coupled) stability problem has been solved by means of Koiter’s asymptotic method (Koiter, 1976). The fields of displacements \( \tilde{U}_i \) and the sectional forces \( \tilde{N}_i \) have been expanded into power series with respect to the parameter \( \xi_n \), i.e. the linear eigenvector amplitude of buckling (normalised
with the equality condition between the maximum deflection and the thickness of the first plate \( h_1 \)

\[
\begin{align*}
\mathbf{U}_i &= \lambda \mathbf{U}_i^{(0)} + \xi_n \mathbf{U}_i^{(n)} + \ldots \\
\mathbf{N}_i &= \lambda \mathbf{N}_i^{(0)} + \xi_n \mathbf{N}_i^{(n)} + \ldots
\end{align*}
\]

(2.11)

for \( n = 1, 2, \ldots, N \), where \( N \) is the number of coupled buckling modes, and \( \lambda \) is the load parameter.

In the investigations, only the first non-linear approximation, in which system characteristics depend only on eigenvectors, is taken into consideration. According to (2.11) and the number of coupled buckling modes \( N = 2 \), the displacement of the \( i \)th wall (band) has been assumed in the form

\[
\begin{align*}
\mathbf{U}_i &= \lambda \mathbf{U}_i^{(0)} + \xi_1 \mathbf{U}_i^{(1)} + \xi_2 \mathbf{U}_i^{(2)} + \ldots \\
\mathbf{V}_i &= \lambda \mathbf{V}_i^{(0)} + \xi_1 \mathbf{V}_i^{(1)} + \xi_2 \mathbf{V}_i^{(2)} + \ldots \\
\mathbf{W}_i &= \xi_1 \mathbf{W}_i^{(1)} + \xi_2 \mathbf{W}_i^{(2)} + \ldots
\end{align*}
\]

(2.12)

At the point where the load parameter \( \lambda \) reaches its maximum value for the imperfect structure with regard to the imperfection of the buckling mode with the amplitude \( \xi_n^* \) (secondary bifurcation or limit points), the Jacobian of the non-linear system of the equilibrium equation

\[
\begin{align*}
a_n \left( 1 - \frac{\lambda}{\lambda_n} \right) \xi_r + a_{jkn} \xi_j \xi_k + \ldots &= \frac{\lambda}{\lambda_n} a_n \xi_n^* \\
&\quad n = 1, 2, \ldots, N
\end{align*}
\]

(2.13)

is equal to zero. The expressions \( a_n \) and \( a_{jkn} \) in Eq. (2.13) are calculated by know from literature (Byskov and Hutchinson, 1977; Królak, 1995) formulas which only depend on the buckling modes. This fact is worth noticing as it reduces solutions to problems in the case when considerations can be limited to the first order non-linear approximation.

The non-linear equations of equilibrium are simplified within the first order approximation to a large extent in the case of interactions between two modes of buckling only. In a further part of this paper, \( N = 2 \) is assumed and equilibrium equations (2.13) have the form

\[
\begin{align*}
a_g \left( 1 - \frac{\lambda}{\lambda_g} \right) \xi_g + a_{grr} \xi_g^2 + a_{gLL} \xi_L^2 &= a_g \xi_g^* \frac{\lambda}{\lambda_g} \\
a_L \left( 1 - \frac{\lambda}{\lambda_L} \right) \xi_L + 2a_{gLL} \xi_g \xi_L &= a_L \xi_L^* \frac{\lambda}{\lambda_L}
\end{align*}
\]

(2.14)
where the subscript \( g \) denotes the global mode \( (n = 1) \), \( L \) refers to the local buckling mode \( (n = 2) \), \( \xi^* \) stands for the initial imperfection of the global character, and \( \xi^*_L \) indicates the initial imperfection consistent with the first local buckling mode.

If we assume that there are no local initial deflections \( (\xi^*_L = 0) \), \( \xi^*_g \neq 0 \) and that the minimum value of the critical stress corresponds to the local buckling mode \( (\lambda_{\min} = \lambda_L) \), then equations (2.14) assume the form

\[
a_g \left(1 - \frac{\lambda}{\lambda_g}\right)\xi_g + a_{gggL} \xi^2_g + a_{gLL} \xi^2_L = a_g \xi^*_g \frac{\lambda}{\lambda_g}
\]

\[
\xi_g \left[a_L \left(1 - \frac{\lambda}{\lambda_{\min}}\right)\frac{\xi_L}{\xi_g} + 2a_{gLL} \xi_L \right] = 0
\]

The following notation has been introduced

\[
\left(1 - \frac{\lambda}{\lambda_{\min}}\right) \frac{1}{\xi_g} = \psi
\]

where \( \psi \) denotes the slope of straight line (2.16) being the post-critical equilibrium path that lies in the plane \( (\lambda, \xi_g) \).

In the pre-critical state \( \xi_L = 0 \), the global deflections, according to (2.13), are described by the relation

\[
\xi_g = \xi^*_g \frac{\lambda}{\lambda_g} - \lambda
\]

Then, equation (2.15) takes the form corresponding to the eigenvalue problem

\[
\left(\frac{2a_{gLL}}{a_L} + \psi\right)\xi_L = 0
\]

For the eigenvalue determined from (2.17), \( \psi \) can be obtained from the following equation

\[
\psi = -\frac{2a_{gLL}}{a_L}
\]

The coupled (interactive) buckling between the global and local mode starts when we obtain a non-zero solution \( \xi_L \neq 0 \). For \( \psi \) from equation (2.19), the eigenvector has been calculated with the accuracy up to the constant \( \rho \) which has been normalised with the condition \( \sqrt{(\xi^0_L)^2} = 1 \). Hence, equation (2.14) can be written as

\[
\rho^2 = \frac{a_g}{a_{gLL} \xi^0_L} \left[ \xi^*_g \frac{\lambda}{\lambda_g} - \left(1 - \frac{\lambda}{\lambda_g}\right) \left(1 - \frac{\lambda}{\lambda_L}\right) \frac{1}{\psi} - \frac{a_{gggL}}{a_g} \left(1 - \frac{\lambda}{\lambda_L}\right)^2 \frac{1}{\psi^2} \right]
\]
The maximum value (the so called limit load carrying capacity) $\lambda_s$ obtained within the first order non-linear approximation and corresponding to $\rho = 0$ (the intersection point of pre-critical path (2.17) with post-critical one (2.16)) can be called the critical value of the local buckling mode that accounts for global pre-critical bending (as $\xi^*_g = 0$, $\xi^*_h \neq 0$). This approach is similar to that found in Pignataro and Luongo (1987), Luongo and Pignataro (1988) and Manievicz and Kolaõowski (1997), where, however, $a_{ggg} = 0$ was assumed.

For $\rho = 0$, equation (2.20) takes the form of a quadratic equation

$$\xi_g \lambda_s \frac{\lambda_s}{\lambda_g} + \left(1 - \frac{\lambda_s}{\lambda_g}\right) \left(1 - \frac{\lambda_s}{\lambda_L}\right) \frac{a_L}{2a_{LL}} - \frac{a_{ggg}}{a_g} \left(1 - \frac{\lambda_s}{\lambda_L}\right) \frac{a_L^2}{4a_{LL}^2} = 0$$

or

$$\lambda_s^2 \left[\frac{a_{ggg}}{a_g} \frac{1}{a_L} - \frac{2a_{LL}}{a_L} \frac{1}{\lambda_L \lambda_g}\right] + \lambda_s \left[\frac{\lambda_L + \lambda_g}{\lambda_L \lambda_g} \frac{2a_{LL}}{a_L} - \frac{a_{ggg}}{a_g} \frac{1}{\lambda_L} - \xi_g \frac{4a_{LL}^2}{a_L^2} \frac{1}{\lambda_g}\right] +$$

$$\frac{2a_{LL}}{a_L} + \frac{a_{ggg}}{a_g} = 0$$

The maximum value of the load $\lambda_s$ determined on the basis of equation (2.21) is smaller than the critical value $\lambda_L = \lambda_{min}$. Thus the load $\lambda_s$ can be interpreted as such that accounts for the influence of the load corresponding to global buckling ($\xi^*_g \neq 0$, $\xi^*_h \neq 0$) on the local load value. The usage of the term "critical value of local buckling load with global pre-critical bending taken into account" is probably too long, but it renders the main idea of the problem. The author suggests here making use of the term "reduced critical value of local buckling load" instead.

### 3. Results of calculations

The analysis presented is an expansion of the analysis taking into account the global pre-critical bending presented by Roorda (1988).

A beam-column with an open channel section and a channel section with edge stiffened flanges characterised by the following dimensions:

- section width $a = 50$ mm
- section height $b = 25$ mm
- edge stiffener width $c = 12.5$ mm
- wall thickness $h_a = h_b = h_c = 1$ mm
- length $l = 650$ mm

has been analysed.
In order to characterise the way in which the load is applied, a coefficient of edge shortening \( \kappa = u_1/u_2 \), where \( u_1, u_2 \) (Fig. 4) are values of displacements of the lower and upper plate of the girder under consideration for \( x = 0, l \), has been introduced.

Some sample results of numerical calculations obtained on the basis of the analysis of interactive buckling of thin-walled beam-columns with channel cross-sections with boundary reinforcement (Fig. 1) within the first order non-linear approximation are presented below.

![Cross-sections of the considered beam-columns](image)

Fig. 4. Cross-sections of the considered beam-columns

As it is known, some initial global deflection \( \xi_g^* \), which should not exceed 0.001 of the girder length, i.e. \( \xi_g^* = 0.001/h \) according to European standards, is admissible for long girders.

Figures 5-7 present the influence of the parameter \( A \) on the limit load carrying capacity within the first order approximation \( \lambda_* \) (2.21) with respect to the critical value of the local symmetrical buckling mode \( \lambda_L \) of the beam-columns. The results are presented for beam-columns with channel sections and channel sections with reinforcement subjected to a load causing non-uniform shortening of the loaded edges \( (\kappa = 0) \).

Two curves are shown on diagrams representing the ratio \( \lambda/\lambda_L \) vs. \( A \), namely: a dashed curve for the admissible initial deflection equal to 0.65 mm \( (\xi_g^* = 0.65) \) and a continuous one for the initial deflection higher than the admissible one and equal to 1 mm \( (\xi_g^* = 1) \).
Fig. 5. Influence of the parameter $A$ on the reduced critical value of local buckling of the beam-column with a channel section

Fig. 6. Influence of the parameter $A$ on the reduced critical value of local buckling of the beam-column with a channel section with inner reinforcement

Fig. 7. Influence of the parameter $A$ on the reduced critical value of local buckling of the beam-column with a channel section with outer reinforcement

It has been assumed that there are no local imperfections $\xi^i_0 = 0$.

For a beam-column with a plain channel section at the assumed initial deflection $\xi^i_0 = 0.65$ (Fig. 5), the reduced critical value of local buckling decreases with an increase in the parameter $A$, and it is lower than the local critical
value by approx. 3% for $A = -2$ and by approx. 22% for $A = 2$, respectively.
A sharp decrease in the value of $\lambda_s/\lambda_L$ is caused by the fact that the coefficient $a_{gg}$ in (2.20), which increases abruptly with an increase in the parameter $A$ (Fig. 8), is taken into consideration in the analysis of interactive buckling. It results from the fact that if there are no boundary reinforcements of the channel, the outer plates (flanges) have dominant impact on the stability, and the buckling mode corresponds to flexural-distorsional buckling.

![Graph](image)

**Fig. 8.** Influence of the parameter $A$ on the value of the coefficient $a_{gg}$ for thin-walled beam-columns with open sections characterised by the loading coefficient $\kappa = 0$

For the same initial deflection $\xi^*_g = 0.65$ in beams-columns with channel sections with inner (Fig. 6) and outer (Fig. 7) reinforcements, i.e. lipped channels with inwardly and outwardly turned lips, the reduced critical value of local buckling slightly increases with the parameter $A$ in the range $-2$ to $2$, and it is lower than the local critical value by approximately 3%.

### 4. Conclusions

The results of numerical calculations presented here show that variable material properties (the coefficient of orthotropy) exert an influence only in the case of a beam-column with a plain channel section. However, in the case of a beam-column with a lipped channel section, the variability in the coefficient of orthotropy along the wall width does not affect the reduced critical value of local buckling in the analysed structures by a great amount.
The presented method of modelling material properties allows one to select such a function describing the widthwise varying orthotropy that the local critical load of the structure, accounting for global imperfections, reaches its required value for a given structure.

It should also be noted that the presented method of investigating the interactive buckling of thin-walled structures is faster than the very popular recently finite element method. However, it has some disadvantage: in the presented method, the material properties and cross-section must be constant along the longitudinal axes of beam-columns; professional FEM software has better postprocessor.

References


Wyboczenie interakcyjne cienkościennych belek-słupów o zmiennym współczynniku ortotropii wzdłuż szerokości ścian

Streszczenie

W pracy analizowano wyboczenie lokalne cienkościennych belek-słupów z uwzględnieniem globalnego dokrytycznego zginania w ramach przybliżenia pierwszego rzędu. Wyboczenie interakcyjne konstrukcji rozwiązano stosując aproksymacyjną teorię Byskova i Hutchinsona (1977) lub Koitera (1976). Badano belki-słupy zbudowane z płyt ortotropowych o głównych kierunkach ortotropii równoległych do krawędzi ścian charakteryzujących się zmiennym wzdułż szerokości współczynnikiem ortotropii \( \eta = E_n/E_{xx} \). Analizowano belki-słupy o przekrojach otwartych (ceowym i ceowym ze wzmocnieniem), przegubowo podparte na obciążonych brzegach. Dźwigary poddano obciążeniom powodującym równomiernie i liniowo zmienne zbliżenie brzegów.

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