STATIC AND DYNAMIC INTERACTIVE BUCKLING REGARDING AXIAL EXTENSION MODE OF THIN-WALLED CHANNEL

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The present paper deals with an influence of the axial extension mode on static and dynamic interactive buckling of a thin-walled channel with imperfections subjected to uniform compression when the shear lag phenomenon and distortional deformations are taken into account. A plate model is adopted for the channel. The structure is assumed to be simply supported at the ends. Equations of motion of component plates were obtained from Hamilton's principle taking into account all components of inertia forces. In the frame of first order nonlinear approximation, the dynamic problem of modal interactive buckling is solved by the transition matrix using the perturbation method and Godunov's orthogonalization.

Key words: thin-walled structures, dynamic interactive buckling, axial mode

1. Introduction

Thin-walled structures composed of plate elements have many different buckling modes that vary in quantitative and qualitative aspects. In the case of finite displacements, different buckling modes are interrelated even with the loads close to their critical values. The postcritical behavior cannot be described any more by a single generalized displacement. When the postcritical behavior of each individual mode is stable, their interaction can lead to unstable behavior, and thus to an increase in the imperfection sensitivity. A nonlinear stability theory should describe all modes and interactive buckling, taking into consideration imperfections of the structure.

The theory of interactive buckling of thin-walled structures subjected to static and dynamic loading has been already widely developed for over forty years. Although the problem of static coupled buckling can be treated as

pretty well recognized, the analysis of dynamic interactive buckling is limited in practice to columns (adopting their beam model), single plates and shells. In the world literature, a substantial lack of the nonlinear analysis of dynamic stability of thin-walled structures with complex cross-sections can be felt.

In this study, a special attention is focused on the influence of the axial extension mode on the global flexural buckling mode of a thin-walled channel.

1.1. Static interactive buckling

When components of the displacement state for the first nonlinear order approximation are taken into account, it can be followed by a decrease in values of global loads. In the case when the critical values corresponding to global buckling modes are significantly lower than those for local modes, then their interaction can be considered within the first nonlinear approximation. It is possible as the post-buckling coefficient for uncoupled buckling is equal to zero for the second order global mode in the Euler column model, and in the case of an exact solution, it is very often of little significance. The theoretical static load-carrying capacity obtained within the frame of the asymptotic theory of the nonlinear first order approximation is always lower than the minimum value of critical load for the linear problem, and the imperfection sensitivity can only be obtained.

Since the late 1980's, the Generalized Beam Theory (GBT) (Basaglia *et al.*, 2008; Camotim *et al.*, 2008; Silva *et al.*, 2008) has been developed extensively. Recently, a new approach has been proposed, i.e., the constrained Finite Strip Method (cFSM) (Adany and Schafer, 2006a,b; Dinis *et al.*, 2007; Schafer, 2006). These two alternative modal approaches to analyze the elastic buckling behavior have been compared in papers.

In the current decade, in more and more numerous publications (Adany and Schafer, 2006a,b; Basaglia *et al.*, 2008; Camotim *et al.*, 2008; Dinis *et al.*, 2007; Schafer, 2006; Silva *et al.*, 2008), the attention has been paid to the global axial mode, which is considered only in the theoretical aspect in linear issues, that is to say, in critical loads. Adany and Schafer (2006a) said that "*it should be noted that this axial mode is a theoretically possible buckling mode, even though it has little practical importance*". In the axial extension mode, longitudinal displacements of the cross-section dominate and this mode can be referred to as the shortening one (Fig. 1). The axial mode is symmetric with respect to the cross-section axis of symmetry and it is symmetric with respect to the axis of overall bending (Kołakowski and Kowal-Michalska, 2010).

In the present study, a trial to dispute with the statement included in the paper by Adany and Schafer (2006a) has been undertaken and the attention
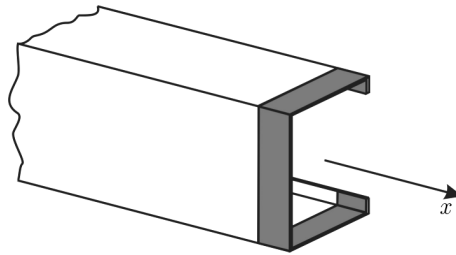
Fig. 1. Longitudinal displacement distributions of the axial mode of the channel

has been focused on an interaction of the global flexural mode of buckling with the global axial extension mode in the first order nonlinear approximation of the perturbation method (Kołakowski and Kowal-Michalska, 2010).

The present study is based on the numerical method of the transition matrix using Godunov's orthogonalization. Instead of the finite strip method, the exact transition matrix method is used in this case. A plate model of the column has been adopted in the study to describe global buckling, which leads to lowering the theoretical value of the limit load. In the solution obtained, the co-operation between all the walls of structures being taken into account, the effects of an interaction of certain modes having the same wavelength, the shear lag phenomenon and also the effect of cross-sectional distortions are included. The distortion instability of the channel is investigated using the nonlinear theory. The solution method was partially presented in paper Kołakowski and Królak (2006).

1.2. Dynamic interactive buckling

Dynamic pulse load of thin-walled structures can be divided into three categories, namely: impact with accompanying perturbation propagation (a phenomenon that occurs with the sound wave propagation speed in the structure), dynamic load of a mean amplitude and a pulse duration comparable to the fundamental flexural vibration period, and quasi-static load of a low amplitude and a load pulse duration approximately twice as long as the period of fundamental natural vibrations. As for dynamic load, effects of damping can be neglected in practice.

Dynamic buckling of a column can be treated as reinforcement of imperfections, initial displacements or stresses in the column through dynamic loading in such a manner that the level of dynamic response becomes very high. When the load is low, the column vibrates around the static equilibrium position. On the other hand, when the load is sufficiently high, then the column can

vibrate very strongly or can move divergently, which is caused by dynamic buckling (i.e. dynamic response).

In the literature on this problem, various criteria concerning dynamic stability have been adopted. The most widely used is the Budiansky-Hutchinson criterion (Budiansky and Hutchinson, 1966; Hutchinson and Budiansky, 1966), in which it is assumed that the loss of dynamic stability occurs when the velocity with which displacements grow is the highest for a certain force amplitude. Other criteria were discussed in Ari-Gur and Simonetta (1997), Huyan and Simitses (1977), Petry and Fahlbusch (2000), Volmir (1972), for instance.

The critical values of dynamic stresses, obtained for the first nonlinear approximation, may be higher than the minimum values of static critical stresses, respectively.

The dynamic response to the rectangular pulse load of the duration corresponding to the fundamental period of flexural free vibration has been analysed.

2. Formulation of the problem

The cross-section of the structure composed of a few plates is presented in Fig. 2 along with local Cartesian systems of co-ordinates. A long prismatic thin-walled channel built of panels connected along longitudinal edges has been considered. The channel is simply supported at its ends. In order to account for all modes and coupled buckling, a plate model of the thin-walled channel has been assumed. The material the channel is made of is subject to Hooke's law.

For each plate component, precise geometrical relationships are assumed in order to enable the consideration of both out-of-plane and in-plane bending of the i-th plate (Kołakowski and Królak, 2006)

$$\varepsilon_{xi} = u_{i,x} + \frac{1}{2}(w_{i,x}^2 + v_{i,x}^2 + u_{i,x}^2)$$

$$\varepsilon_{yi} = v_{i,y} + \frac{1}{2}(w_{i,y}^2 + u_{i,y}^2 + v_{i,y}^2) \tag{2.1}$$

$$2\varepsilon_{xyi} = \gamma_{xyi} = u_{i,y} + v_{i,x} + w_{i,x}w_{i,y} + u_{i,x}u_{i,y} + v_{i,x}v_{i,y}$$

and

$$\kappa_{xi} = -w_{i,xx} \qquad \kappa_{yi} = -w_{i,yy} \qquad \kappa_{xyi} = -2w_{i,xy} \tag{2.2}$$

where: u_i, v_i, w_i are components of the displacement vector of the i-th plate in the x_i, y_i, z_i axis direction, respectively, and the plane $x_i y_i$ overlaps the central plane before its buckling.
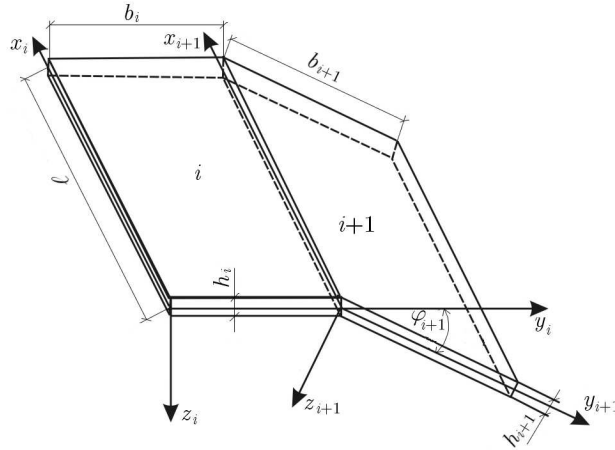
Fig. 2. Prismatic plate structure and the local co-ordinate system

When the full tensor of membrane strains (2.1) ε_{xi}, ε_{yi}, $\gamma_{xyi} = 2\varepsilon_{xyi}$ is taken into account, then a "full" analysis of all buckling modes, including the axial extension mode, can be carried out.

The main limitation of the assumed theory lies in an assumption of linear relationships between curvatures (2.2) and second derivatives of the displacement w. This is the most often applied limitation in the theory of thin-walled structures.

The attention has been drawn to the necessity of considering the full strain tensor and all the components of inertial forces in order to carry out a proper dynamic analysis in the whole range of length of the structures.

For thin-walled structures with initial deflections, Lagrange's equations of motion for the case of an interaction of N eigenmodes can be written as (Kowal-Michalska, 2007; Schokker $et\ al.$, 1996; Sridharan and Benito, 1984)

$$\frac{1}{\omega_r^2}\zeta_{r,tt} + \left(1 - \frac{\sigma}{\sigma_r}\right)\zeta_r + a_{ijr}\zeta_i\zeta_j - \zeta_r^*\frac{\sigma}{\sigma_r} + \ldots = 0 \qquad \text{for} \quad r = 1, \ldots, N \quad (2.3)$$

where: ζ_r is the dimensionless amplitude of the r-th buckling mode, σ_r, ω_r, ζ_r^* – critical stress, circular frequency of free vibrations and dimensionless amplitude of the initial deflection corresponding to the r-th buckling mode, respectively.

Due to the fact that the axial mode is taken into consideration, in the present paper it is assumed that the absolute maximum value of one of the components of the displacement field of the r-th mode is equal to the first plate thickness t_1. The expressions for a_{ijr} are to be found in Kołakowski

and Królak (2006), Kowal-Michalska (2007). In equations of motion (2.3), the inertia forces of the pre-buckling state and second order approximations have been neglected (Kowal-Michalska, 2007; Schokker *et al.*, 1996). The initial conditions have been assumed in the form

$$\zeta_r(t=0) = 0 \qquad\qquad \zeta_{r,t}(t=0) = 0 \tag{2.4}$$

The static problem of interactive buckling of the thin-walled channel (i.e. for $\zeta_{r,tt} = 0$ in (2.3)) has been solved with the method presented in Kołakowski and Królak (2006). The frequencies of free vibrations have been determined analogously as in Teter and Kołakowski (2003), whereas the problem of interactive dynamic buckling (2.3) has been solved by means of the Runge-Kutta numerical method modified by Hairer and Wanner.

At the point where the load parameter for static problems σ reaches its maximum value σ_s (the so-called theoretical load carrying capacity) for the imperfect structure with regard to the imperfection of the buckling mode with the amplitude ζ_r^*, the Jacobian of the nonlinear system of equations (2.3) is equal to zero.

3.   Analysis of the calculation results

3.1.   Eigenvalue problems

A detailed analysis of the calculations is conducted for the compressed channel with the following dimensions of its cross-section (Fig. 3) (Adany *et al.*, 2008)

$$b_1 = 150 \,\text{mm} \qquad\qquad b_2 = 60 \,\text{mm}$$
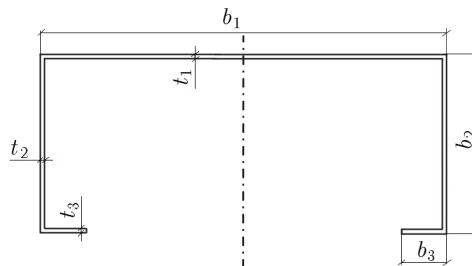$$b_3 = 15 \,\text{mm} \qquad\qquad t_1 = t_2 = t_3 = 2 \,\text{mm}$$

Fig. 3. Geometry of the thin-walled channel

Each plate is made of steel characterized by the following mechanical properties: $E = 210\,\text{GPa}$, $\nu = 0.3$, $\rho = 7850\,\text{kg/m}^3$.

The global flexural mode $(m = 1)$ and the axial extension mode (for $m = 1$) for the assumed length ℓ have been analyzed. The following index symbols have been introduced: 1 – flexural mode for $m = 1$; 2 – axial mode for $m = 1$.

Calculations have been conducted for four lengths of the column: $\ell = 10000, 7500, 5000, 2500\,\text{mm}$. The lengths ℓ have been selected as to make the values of global critical stresses (i.e., for $m = 1$) lower than local critical loads, which enables us to analyze the column buckling within the first order approximation.

In Table 1 values of critical loads σ_r for four lengths ℓ of the channel under investigation are shown. The critical loads σ_2 for the axial mode, which is identical for the lengths considered, are collected. For the channel whose length is $2500 \leqslant \ell \leqslant 10000\,\text{mm}$, the maximum displacements in the cross-section plane (i.e., $v^{(2)}$, $w^{(2)}$) are equal to approx. 2% of the longitudinal displacements (i.e., $u^{(2)}$) at most (Kołakowski and Kowal-Michalska, 2010). The displacements $u^{(2)}$ are practically constant for the cross-section for $x = \text{const}$. Thus, this mode can be called the 'pure' axial extension mode. For the axial mode, the displacements are equal to the thickness t_1 (that is to say, $u_{max}^{(2)} \cong u^{(2)} = t_1$).

Values of the natural frequencies ω_r of free vibrations corresponding to the two modes under analysis for different column lengths ℓ are presented in Table 1, too. Vibration frequencies were determined taking into account all components of the inertia forces (Kowal-Michalska, 2007; Teter and Kołakowski, 2003) (i.e. in-plane $\rho u_{,tt}$, $\rho v_{,tt}$ and out-of-plane $\rho w_{,tt}$). The same index symbols have been adopted as for the static problems.

Table 1. Critical stresses σ_r and natural frequencies ω_r of the channel

ℓ	σ_1 [MPa]	σ_2 [MPa]	ω_1 [s^{-1}]	ω_2 [s^{-1}]
10000	10.69	80212	11.595	1624.8
7500	19.00	80212	20.609	2166.4
5000	42.69	80211	46.332	3250.4
2500	167.88	80211	183.73	6498.6

3.2. Interactive buckling in the first order approximation

Static coupling buckling

Detailed results of the static interactive buckling analysis are presented in Table 2 for the channel. The following two variants of the imperfections are assumed:

1. $\zeta_1^* = |\ell/(1000t_1)|$, $\zeta_2^* = 0$,

2. $\zeta_1^* = |1.0|$, $\zeta_2^* = 0$.

In each case, the sign of the imperfection ζ_1^* has been selected in the most unfavorable way, that is to say, as to obtain the lowest theoretical load-carrying capability σ_S for the given level of imperfection when the interaction of buckling modes is accounted for.

In Table 2, the ratio of the theoretical load carrying capacity to the global flexural critical stress σ_s/σ_1 for the assumed variants of imperfections is given.

A comparison of the results presented in Table 2 allows us to state that when the axial mode is accounted for in the interaction, then the theoretical load carrying-capacity σ_s is considerably decreased. A decrease in the load-carrying capacity σ_s/σ_1 does not exceed 40%.

Table 2. Theoretical static load carrying capacity σ_s/σ_1

Imperfection variant	σ_s/σ_1			
	$\ell = 10000$	$\ell = 7500$	$\ell = 5000$	$\ell = 2500$
1	0.6369	0.6852	0.7455	0.8238
2	0.8495	0.8495	0.8492	0.8452

It follows from this comparison that the consideration of the axial mode in the interaction is necessary as it results in a visible decrease in the theoretical load-carrying capacity in the first order approximation.

The nonlinear coefficient of system of equations (2.3), namely a_{211}, exerts the main influence on the decrease in the load σ_s. The key role in the interaction of the buckling mode is played by the coefficient a_{211}, i.e., the term $a_{211}\zeta_2\zeta_1^2$ of the third order in the expression for potential energy (Kołakowski and Kowal-Michalska, 2010). It is related to the term $\sigma^{(2)}L_2(U^{(1)})$ (following the notation of papers by Kowal-Michalska (2007), Schokker *et al.* (1996), Sridharan and Benito (1984)). This term arises by the product stress $\sigma^{(2)}$ associated with the axial mode with the term representing the midsurface strain $L_2(U^{(1)})$ and integrating the same over the structure. The longitudinal displacements $u^{(2)}$ of the axial mode are of the same order as the displacements $w^{(1)}$

for the global flexural mode due to the assumed conditions for the mode normalization (i.e., $u_{max}^{(2)} = w_{max}^{(1)}$). In the theory of thin plates, we always have $w^{(i)} \gg u^{(j)}$ for $i \neq j$, apart from the interaction of the mode with the axial mode analyzed in this study. It makes the coefficient a_{211} play such a key role for the first order approximation.

Dynamic response

Further on, an analysis of dynamic interactive buckling (i.e. dynamic response) of the channel under consideration was conducted. Identically as in the static analysis, an interaction of the same modes was considered. A detailed analysis was conducted for: a rectangular pulse load $\sigma(t) = \sigma_D$ for $0 \leqslant t \leqslant T_1$ and $\sigma(t) = 0$ for $T < t$. This case corresponds to the pulse duration equal to the period of the fundamental flexural free vibration $T_1 = 2\pi/\omega_1$.

In Table 3, values of the critical dynamic load factors $DLF_{cr} = \sigma_D^{BH}/\sigma_1$ for various column lengths ℓ and for two imperfection variants under analysis are given, where σ_D^{BH} denotes the critical value of dynamic stress determined from the Budiansky-Hutchinson criterion (Budiansky and Hutchinson, 1966; Hutchinson and Budiansky, 1966).

The values of critical dynamic load factors $DLF_{cr} = \sigma_D^{BH}/\sigma_1$ presented in Table 3 correspond with some accuracy to the maximum values of deflections $\zeta_{r\,max}$ within the applicability of the assumed theory (i.e. the total maximum deflection of the column is at least sixty times as high as the cross-section wall thickness) (Kowal-Michalska, 2007), and not to asymptotic values ((Budiansky and Hutchinson, 1966; Hutchinson and Budiansky, 1966). The main limitation that results from the adopted theory is the assumption of a linear dependence between the curvatures and second order derivatives of the displacement.

Table 3. Critical dynamic load factors $DLF_{cr} = \sigma_D^{BH}/\sigma_1$

Imperfection variant	$DLF_{cr} = \sigma_D^{BH}/\sigma_1$			
	$\ell = 10000$	$\ell = 7500$	$\ell = 5000$	$\ell = 2500$
1	0.669	0.746	0.850	1.045
2	1.199	1.148	1.138	1.095

For the cases of column dimensions under analysis, the obtained values of DLF_{cr} are higher than the respective values of theoretical static load carrying capacity σ_s/σ_1. For the second variant of imperfections $DLF_{cr} > 1$, whereas for the first variant $DLF_{cr} < 1$ for the column length $10000 \leqslant \ell \leqslant 5000$ and $DLF_{cr} > 1$ for $\ell = 2500$, correspondingly.

4. Conclusion

In the study, special attention has been focused on the coupled buckling of the Euler global mode of buckling with the axial mode in the first nonlinear approximation of the perturbation method. In the world literature, it is probably the first study, to the author knowledge, devoted to the dynamic interaction of buckling with the axial mode. This problem may be of great significance and it requires further investigations. According to the author's opinion, a further analysis of the interactive buckling ought to include an interaction of the axial mode with global and local modes. Therefore, the interactive buckling should be further analyzed and comprehensively and thoroughly discussed.

References

1. ADANY S., SCHAFER B.W., 2006a, Buckling mode decomposition of single-branched open cross-section members via finite strip method: Derivation, *Thin-Walled Structures*, **44**, 563-584

2. ADANY S., SCHAFER B.W., 2006b, Buckling mode decomposition of single-branched open cross-section members via finite strip method: Application and examples, *Thin-Walled Structures*, **44**, 585-600

3. ADANY S., SILVESTRE N., SCHAFER B.W., CAMOTIM D., 2007, On the identification and characterization of local, distortional and global modes in thin-walled members using the cFSM and GBT approaches, *Proceeding of the Sixth International Conference on Steel and Aluminium Structures ICSAS'2007*, Beale R.G. (Ed.), Oxford Brookes University, 760-767

4. ADANY S., SILVESTRE N., SCHAFER B.W., CAMOTIM D., 2008, Buckling mode identification of thin-walled members: A comparison between cFSM and CBT approaches, *Proceedings of the fifth International Conference on Coupled Instabilities in Metal Structures CIMS2008*, Rasmussen K. and Wilkinson T. (Eds.), University Publishing Service, The University of Sydney, 249-256

5. ARI-GUR J., SIMONETTA S.R., 1997, Dynamic pulse buckling of rectangular composite plates, *Composites Part B*, **28B**, 301-308

6. BASAGLIA C., CAMOTIM D., SILVESTRE N., 2008, GBT-based post-buckling analysis of thin-walled lipped I-section steel columns, *Proceedings of the fifth International Conference on Coupled Instabilities in Metal Structures CIMS2008*, Rasmussen K. and Wilkinson T. (Eds.), University Publishing Service, The University of Sydney, 241-248

7. BUDIANSKY B., HUTCHINSON J.W., 1966, Dynamic buckling of imperfection-sensitive structures, *Proceedings of the Eleventh International Congress of Applied Mechanics*, Goetler H. (Ed.), Munich, 636-651

8. CAMOTIM D., BASAGLIA C., SILVESTRE N., 2008, GBT buckling analysis on thin-walled steel frames, *Proceedings of the fifth International Conference on Coupled Instabilities in Metal Structures CIMS2008*, Rasmussen K. and Wilkinson T. (Eds.), University Publishing Service, The University of Sydney, 1-18

9. DINIS P.B., CAMOTIM D., SILVESTRE N., 2007, FEM-based analysis of the local-plate/distortional mode interaction in cold-formed steel lipped channel columns, *Computers and Structures*, **85**, 1461-1474

10. HUTCHINSON J.W., BUDIANSKY B., 1966, Dynamic buckling estimates, *AIAA Journal*, **4-3**, 525-530

11. HUYAN X., SIMITSES G.J., 1977, Dynamic buckling of imperfect cylindrical shells under axial compression and bending moment, *AIAA Journal*, **35**, 8, 1404-1412

12. KOŁAKOWSKI Z., KOWAL-MICHALSKA K., 2010, Influence of the axial mode on interactive buckling of the thin-walled channels in the first nonlinear approximation, *Shell Structures: Theory and Applications*, **2**, Pietraszkiewicz W., Kreja I. (Eds.), Taylor and Francis Group, 125-128

13. KOŁAKOWSKI Z., KRÓLAK M., 2006, Modal coupled instabilities of thin-walled composite plate and shell structures, *Composite Structures*, **76**, 303-313

14. KOWAL-MICHALSKA K. (ED.), 2007, *Dynamic Stability of Composite Plated Structures*, WNT, Warszawa [in Polish]

15. PETRY D., FAHLBUSCH G., 2000, Dynamic buckling of thin isotropic plates subjected to in-plane impact, *Thin-Walled Structures*, **38**, 267-283

16. SCHAFER B.W., 2006, Review: The direct strength method of cold-formed member design, *Proceeding of International Colloquium on Stability and Ductility of Steel Structures SDSS 2006*, Camotim D., Silvestre N., Dinis P.B. (Eds.), Instituto Superior Tecnico, Lisbon, 49-65

17. SCHOKKER A., SRIDHARAN S., KASAGI A., 1996, Dynamic buckling of composite shells, *Computers and Structures*, **59**, 1, 43-55

18. SILVA N.M.F., CAMOTIM D., SILVESTRE N., 2008, GBT cross-section analysis of thin-walled members with arbitrary cross-sections: A novel approach, *Proceedings of the Fifth International Conference on Thin-Walled Structures*

   2008, Mahendran M. (Ed.), Queensland University of Technology, Brisbane, Australia, 1161-1171

19. SRIDHARAN S., BENITO R., 1984, Columns: static and dynamic interactive buckling, *J. Engineering Mechanics, ASCE*, **110**, 1, 49-65

20. TETER A., KOŁAKOWSKI Z., 2003, Natural frequencies of thin-walled structures with central intermediate stiffeners or/and variable thickness, *Thin-Walled Structures*, **41**, 291-316

21. VOLMIR S.A., 1972, *Nonlinear Dynamics of Plates and Shells*, Science, Moscow [in Russian]

Statyczne i dynamiczne interakcyjne wyboczenie cienkościennego ceownika uwzględniające osiową postać wzdłużną

Streszczenie

W prezentowanej pracy omówiono wpływ osiowej wzdłużnej postaci na statyczne i dynamiczne interakcyjne wyboczenie cienkościennego ceownika z niedokładnościami poddanego równomiernemu ściskaniu przy uwzględnieniu zjawiska *shear-lag* oraz dystorsyjnej deformacji. Przyjęto płytowy model ceownika. Konstrukcja jest przegubowo podparta na obu końcach. Równania ruchu płyt składowych otrzymano z zasady Hamiltona, biorąc pod uwagę wszystkie składowe sił bezwładności. Dynamiczne zagadnienie modalnego interakcyjnego wyboczenia w ramach pierwszego rzędu nieliniowej aproksymacji rozwiązano metodą macierzy przeniesienia i metodą ortogonalizacji Godunova.