

ON A MODELLING OF STABILITY OF ANNULAR PLATES WITH FUNCTIONALLY GRADED STRUCTURE INTERACTING WITH AN ELASTIC HETEROGENEOUS SUBSOIL

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The object of analysis is a composite annular plate with the apparent properties smoothly varying along a radial direction. The plate interacting with an elastic heterogeneous subsoil with two moduli. The aim of contribution is to formulate macroscopic mathematical model describing stability of this plate. The considerations are based on those summarized in monographs (Woźniak et al. 2008, 2010). Some applications of the tolerance averaging technique for the modelling of various stability problems for elastic microheterogeneous structures are presented in papers; Baron (2003), Jędrysiak (2007), Jędrysiak and Michalak (2011), Michalak (1998), Tomczyk (2010).

1. INTRODUCTION

The contribution is devoted to the determine of stability of microheterogeneous annular plates interacting with an elastic foundation with two foundation moduli. The assumed model of foundation is a generalization of the well known Winkler model. The introduction of an additional modulus of horizontal deformability of the foundation makes it possible to describe the stability of the plate resting on a sufficiently fine net of elastic point supports such as piles or columns. The object of the analysis is a composite thin plate with the apparent properties smoothly varying along a radial direction of the plate (Fig.1).

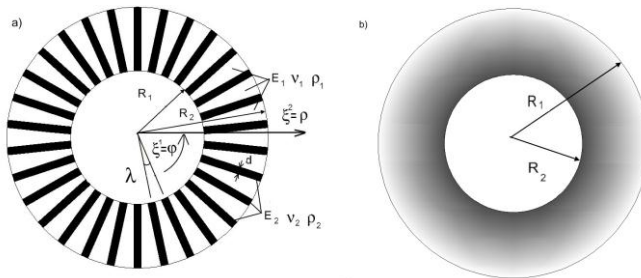


Fig. 1. A fragment of midplane of the plate with longitudinally graded microstructure: a) microscopic level, b) macroscopic level

Considerations are restricted to the two-phase of the functionally graded - type composites. The plate is made of isotropic homogeneous matrix and isotropic homogeneous beams which are situated along the radial direction. The plate is resting on foundation, which has different moduli under the matrix and the beams. The plate and the foundation have λ -periodic microstructure along the angular axis and smooth and slow gradation of effective properties in the radial direction. The generalized period λ of inhomogeneity is assumed to be sufficiently small when compared to the characteristic length dimension of the plate along the angular axis. Thus we deal with plate and foundation having space-varying periodic microstructure.

The aim of this contribution is to formulate macroscopic models of stability of the plate under consideration. There models will be referred to as tolerance and asymptotic, respectively.

2. PRELIMINARES

Introduce the orthogonal curvilinear coordinate system $O\xi^1\xi^2\xi^3$ in the physical space occupied by a plate under consideration. The time coordinate will be denoted by t . Sub- and super-scripts i, k, l run over 1, 2, 3 and α, β, δ run over 1, 2. Setting $\mathbf{x} \equiv (\xi^1, \xi^2)$ and $z = \xi^3$ it is assumed that the undeformed plate occupies the region $\Omega \equiv \{(\mathbf{x}, z) : -h/2 \leq z \leq h/2, \mathbf{x} \in \Pi\}$, where Π is the plate midplane and h is the plate thickness. We denote by $g_{\alpha\beta}$ a metric tensors and by $\epsilon_{\alpha\beta}$ a Ricci tensor. Here and in the sequel, a vertical line before the subscripts stands for the covariant derivative and $\partial_\alpha = \partial / \partial \xi^\alpha$. The plate rests on the generalized Winkler foundation whose properties are characterized by vertical k_z and horizontal k_t foundation moduli. The foundation reaction according to (Gomuliński (1967)) has three components acting in the direction of the coordinates (z, ρ, φ) :

$$R_z = k_z w \quad R_\rho = k_t \frac{h}{2} \partial_\rho w \quad R_\varphi = k_t \frac{h}{2} \partial_\varphi w, \quad (1)$$

The model equations for the stability of the considered plate will be obtained in the framework of the well-known second order non-linear theory for thin plates resting on elastic foundation (Woźniak et al. 2001). Denoting the displacement field of the plate midsurface by $w(\mathbf{x}, t)$, the external forces by $p(\mathbf{x}, t)$ and by μ the mass density related to this midsurface, we obtain strain-displacement and constitutive equations

$$\kappa_{\alpha\beta} = -w|_{\alpha\beta} \quad m^{\alpha\beta} = -D^{\alpha\beta\eta\mu} \kappa_{\eta\mu}, \quad (2)$$

where: $D^{\alpha\beta\eta\mu} = 0.5D(g^{\alpha\mu} g^{\beta\gamma} + g^{\alpha\gamma} g^{\beta\mu} + \nu(\epsilon^{\alpha\gamma} \epsilon^{\beta\mu} + \epsilon^{\alpha\mu} \epsilon^{\beta\gamma}))$, $D = Eh^3 / 12(1 - \nu^2)$.

The governing equations of the plate under consideration can be described by the well known principle of stationary action. We introduce action functional defined by

$$A(w(\cdot)) = \int_{\Pi} \int_{t_0}^{t_1} \Lambda(\mathbf{y}, w_{|\alpha\beta}(\mathbf{y}, t), w_{|\alpha}(\mathbf{y}, t), \dot{w}(\mathbf{y}, t), w(\mathbf{y}, t)) dt d\mathbf{y}, \quad (3)$$

with Lagrangian defined by

$$\Lambda = \frac{1}{2}(\mu\dot{w}\dot{w} - n^{\alpha\beta}w_{|\alpha}w_{|\beta} - D^{\alpha\beta\gamma\delta}w_{|\alpha\beta}w_{|\gamma\delta} - k_z w w - \frac{h^2}{4}k_t \delta^{\alpha\beta}w_{|\alpha}w_{|\beta}) + pw. \quad (4)$$

where $n^{\alpha\beta}$ are in-plane forces and Kronecker-deltas $\delta^{\alpha\beta}$ will be treated as a tensor; $\delta^{11} = 1/\rho^2$, $\delta^{22} = 1$.

For Lagrangian (4) we can write the Euler-Lagrange equation

$$\frac{\partial}{\partial t} \frac{\partial \Lambda}{\partial \dot{w}} - \frac{\partial \Lambda}{\partial w} + \left(\frac{\partial \Lambda}{\partial w_{|\alpha}} \right)_{|\alpha} - \left(\frac{\partial \Lambda}{\partial w_{|\alpha\beta}} \right)_{|\alpha\beta} = 0. \quad (5)$$

and the equilibrium equations

$$(D^{\alpha\beta\gamma\delta}w_{|\gamma\delta})_{|\alpha\beta} - (n^{\alpha\beta})_{|\beta} - \frac{h^2}{4}(k_t \delta^{\alpha\beta}w_{|\beta})_{|\alpha} + k_z w + \mu\dot{w} = p. \quad (6)$$

This direct description leads to plate equations with discontinuous and highly oscillating coefficients. These equations are too complicated to be used in the engineering analysis and will be used as starting point in the tolerance modelling procedure.

3. TOLERANCE MODELLING

Introduce the polar coordinates system $O\xi^1\xi^2$, $0 \leq \xi^1 \leq \varphi$, $R_1 \leq \xi^2 \leq R_2$ so that the undeformed midplane of annular plate occupies the region $\Pi \equiv [0, \varphi] \times [R_1, R_2]$. Let λ , $0 < \lambda \ll \varphi$, is the known microstructure parameter. Denote Π_{Δ} as a subset of Π of points with coordinates determined by conditions $(\xi^1, \xi^2) \in (\lambda/2, \varphi - \lambda/2) \times (R_1, R_2)$. An arbitrary cell with a centre at point with coordinates (ξ^1, ξ^2) in Π_{Δ} will be determined by $\Delta(\xi^1, \xi^2) = (\xi^1 - \lambda/2, \xi^1 + \lambda/2) \times \{\xi^2\}$. At the same time, the thickness h of the plate under consideration is supposed to be constant and small compared to the microstructure parameter λ .

In order to derive averaged model equations we applied tolerance averaging approach. This technique based on the concept of tolerance and indiscernibility relations and on the definition of slowly-varying functions. The general modelling procedures of this technique are given in books (Woźniak et al. 2008, 2010).

The fundamental concept of the modelling technique is the averaging an arbitrary integrable function $f(\cdot)$ over the cell $\Delta(\cdot)$.

$$\langle f \rangle (\xi^1, \xi^2) = \frac{1}{\lambda} \int_{\xi^{1-\lambda/2}}^{\xi^{1+\lambda/2}} f(\eta, \xi^2) d\eta. \quad (7)$$

for every $\xi^1 \in [\lambda/2, \varphi - \lambda/2]$, $\xi^2 \in [R_1, R_2]$.

The first assumption in the tolerance modelling is micro-macro decomposition of the displacement field

$$w(\xi^\alpha, t) = w^0(\xi^\alpha, t) + h^A(\xi^1) V_A(\xi^\alpha, t) \quad (8)$$

for $\xi^\alpha \in \Pi$ and $t \in (t_0, t_1)$.

The modelling assumption states that $w^0(\cdot, \xi^2)$, $V_A(\cdot, \xi^2)$ are slowly-varying functions with respect to the argument ξ^1 . Functions $w^0(\cdot, \xi^2, t) \in SV_\delta^2(\Pi, \Delta)$, $V_A(\cdot, \xi^2, t) \in SV_\delta^2(\Pi, \Delta)$ are the basic unknowns of the tolerance model. Function $h^A(\xi^1)$ are known, dependent on the microstructure length parameter λ , fluctuation shape functions.

Let $\tilde{h}^A(\cdot)$, $\partial_1 \tilde{h}^A(\cdot)$ stand for periodic approximation of $h^A(\cdot)$, $\partial_1 h^A(\cdot)$ in Δ , respectively. Due to the fact that $w(\cdot, \xi^2, t)$ are tolerance periodic functions, it can be observe that the periodic approximation of $w_h(\cdot, \xi^2, t)$ and $\partial_\alpha w_h(\cdot, \xi^2, t)$ in $\Delta(\cdot)$, have the form

$$\begin{aligned} w_h(y, \xi^2, t) &= w^0(\xi^\alpha, t) + h^A(y) V_A(\xi^\alpha, t), \\ \partial_\alpha w_h(y, \xi^2, t) &= \partial_\alpha w^0(\xi^\alpha, t) + \partial_1 h^A(y) V_A(\xi^\alpha, t) + h^A(y) \partial_2 V_A(\xi^\alpha, t), \\ \dot{w}_h(y, \xi^2, t) &= \dot{w}^0(\xi^\alpha, t) + h^A(y) \dot{V}_A(\xi^\alpha, t), \end{aligned} \quad (9)$$

for every $x^\alpha \in \Pi$, almost every $y \in \Delta(\xi^\alpha)$ and every $t \in (t_0, t_1)$.

The tolerance model equations will be obtained by the averaging of the lagrangian $\Lambda(\xi^\alpha, w, w|_{\alpha\beta}, w|_\alpha, \dot{w})$. Substituting the decomposition (8) of displacement field into Lagrangian Λ and using the tolerance averaging approach, we obtain

$$\begin{aligned}
 \langle \Lambda \rangle = & \frac{1}{2} \langle \mu \rangle \dot{w}^0 \dot{w}^0 + \langle \mu h^A \rangle \dot{w}^0 \dot{V}_A + \frac{1}{2} \langle \mu h^A \phi^B \rangle \dot{V}_A \dot{V}_B + \langle p \rangle w^0 + \langle ph^A \rangle V_A + \\
 & - \frac{1}{2} \langle D^{\alpha\beta\gamma\mu} \rangle w^0_{|\alpha\beta} w^0_{|\gamma\mu} - \langle D^{11\gamma\mu} h^A_{|11} \rangle w^0_{|\gamma\mu} V_A - \langle D^{22\gamma\mu} h^A \rangle V_{A|22} w^0_{|\gamma\mu} + \\
 & - 2 \langle D^{12\gamma\mu} h^A_{|1} \rangle w^0_{|\gamma\mu} V_{A|2} - \langle D^{1122} h^A_{|11} h^B \rangle V_A V_{B|22} - \frac{1}{2} \langle D^{1111} h^A_{|11} h^B_{|11} \rangle V_A V_B + \\
 & - 2 \langle D^{1212} h^A_{|1} h^B_{|1} \rangle V_{A|2} V_{B|2} - \frac{1}{2} \langle D^{2222} h^A h^B \rangle V_{A|22} V_{B|22} - \frac{1}{2} \langle k_z \rangle w^0 w^0 + \\
 & - \langle k_z h^A \rangle w^0 V_A - \frac{1}{2} \langle k_z h^A h^B \rangle V_B V_A - \frac{h^2}{8} \langle k_t g^A g^B \rangle + \delta^{22} V_{A|2} V_{B|2} + \\
 & - \frac{h^2}{4} \langle k_t h^A_{|1} \rangle + \delta^{1\beta} w^0_{|\beta} V_A - \frac{h^2}{4} \langle k_t h^A \rangle + \delta^{2\beta} w^0_{|\beta} V_{A|2} + \frac{h^2}{4} \langle k_t h^A_{|1} h^B_{|1} \rangle + \delta^{11} V_A V_B + \\
 & - \frac{h^2}{8} \langle k_t \rangle + \delta^{\alpha\beta} w^0_{|\alpha} w^0_{|\beta} - \frac{1}{2} \langle n^{\alpha\beta} \rangle w^0_{|\alpha} w^0_{|\beta} - \langle n^{1\beta} h^A_{|1} \rangle w^0_{|\beta} V_A - \langle n^{2\beta} h^A \rangle V_{A|2} w^0_{|\beta} + \\
 & - \frac{1}{2} \langle n^{11} h^A_{|1} h^B_{|1} \rangle V_A V_B - \langle n^{12} h^A_{|1} h^B \rangle V_A V_{B|2} - \frac{1}{2} \langle n^{22} h^A h^B \rangle V_{A|2} V_{B|2}
 \end{aligned} \tag{10}$$

Applying the principle of stationary action to averaged functional A_h the Euler-Lagrange equations take the form

$$\begin{aligned}
 \frac{\partial}{\partial t} \frac{\partial \langle L \rangle}{\partial \dot{w}^0} - \left(\frac{\partial \langle L \rangle}{\partial w^0_{|\alpha\beta}} \right)_{|\alpha\beta} + \left(\frac{\partial \langle L \rangle}{\partial w^0_{,\alpha}} \right)_{,\alpha} - \frac{\partial \langle L \rangle}{\partial w^0} &= 0 \\
 \frac{\partial}{\partial t} \frac{\partial \langle L \rangle}{\partial \dot{V}_A} - \left(\frac{\partial \langle L \rangle}{\partial V_{A|22}} \right)_{|22} + \left(\frac{\partial \langle L \rangle}{\partial V_{A,2}} \right)_{,2} - \frac{\partial \langle L \rangle}{\partial V_A} &= 0
 \end{aligned} \tag{11}$$

Using the averaged Lagrangian (10) we obtain the following system of equations describing stability of the plate resting on microheterogeneous foundation

$$\begin{aligned}
 & (\langle D^{\alpha\beta\gamma\mu} \rangle w^0_{|\gamma\mu})_{|\alpha\beta} + (\langle D^{\alpha\beta 11} h^A_{|11} \rangle V_A)_{|\alpha\beta} + (\langle D^{\alpha\beta 22} h^A \rangle V_{A|22})_{|\alpha\beta} + \\
 & + \langle k_z \rangle w^0 + \langle k_z h^A \rangle V_A - \frac{h^2}{4} (\langle k_t \rangle + \delta^{\alpha\beta} w^0_{|\beta})_{|\alpha} - \frac{h^2}{4} (\langle k_t h^A_{|1} \rangle + \delta^{1\beta} V_A)_{|\beta} + \\
 & - \frac{h^2}{4} (\langle k_t h^A \rangle + \delta^{2\beta} V_{A|2})_{|\beta} - (N^{\alpha\beta} w^0_{|\beta})_{|\alpha} + \langle \mu \rangle \dot{w}^0 = \langle p \rangle,
 \end{aligned}$$

$$\begin{aligned}
 & \langle D^{11\gamma\mu} h^A \rangle_{|11} > w^0_{|\gamma\mu} + \langle D^{1122} h^A \rangle_{|11} h^B > V_{B|22} + \langle D^{1111} h^A \rangle_{|11} h^B \rangle_{|11} > V_B + \\
 & + (\langle D^{22\gamma\mu} h^A > w^0_{|\gamma\mu} \rangle_{|22} + (\langle D^{1122} h^A h^B \rangle_{|11} > V_B \rangle_{|22} + (\langle D^{2222} h^A h^B > V_{B|22} \rangle_{|22} + \\
 & - 2(\langle D^{12\gamma\mu} h h^A \rangle_{|1} > w^0_{|\gamma\mu} \rangle_{|2} - 4(\langle D^{1212} h^A \rangle_{|1} h^B \rangle_{|1} > V_{B|2} \rangle_{|2} + \langle k_z h^A h^B > V_B + \\
 & + \langle k_z h^A > w^0 - \frac{h^2}{4} (\langle k_t h^A > \delta^{2\beta} w^0_{|\beta} \rangle_{|2} + \frac{h^2}{4} \langle k_t h^A \rangle_{|1} > \delta^{11} w^0_{|\beta} \rangle_{|2} \\
 & - \frac{h^2}{4} (\langle k_t h^A h^B > \delta^{22} V_{B|2} \rangle_{|2} + \frac{h^2}{4} \langle k_t h^A \rangle_{|1} h^B \rangle_{|1} > \delta^{11} V_B \\
 & - (N^{22} \langle h^A h^B > V_{B|2} \rangle_{|2} + N^{11} \langle h^A \rangle_{|1} h^B \rangle_{|1} > V_B + \langle \mu h^A h^B > \ddot{V}_B = \langle p h^A >
 \end{aligned} \tag{12}$$

We have assume that forces $n^{\alpha\beta}$ can be represented by a decomposition

$$n^{\alpha\beta}(\xi^\gamma) = N^{\alpha\beta}(\xi^\gamma) + \tilde{n}^{\alpha\beta}(\xi^\gamma), \tag{13}$$

where $N^{\alpha\beta} = \langle n^{\alpha\beta} \rangle$ is a slowly varying function and $\tilde{n}^{\alpha\beta}(\cdot)$ is a fluctuating part of the forces $n^{\alpha\beta}(\cdot)$, such that $\langle \tilde{n}^{\alpha\beta} \rangle = 0$. In Eq.(10) we have assumed that the fluctuating part $\tilde{n}^{\alpha\beta}(\cdot)$ of the forces $n^{\alpha\beta}(\cdot)$ is very small compared to their averaged part $N^{\alpha\beta}(\cdot)$, and hence $\langle n^{22} h^A h^B \rangle \cong N^{22} \langle h^A h^B \rangle$.

The above equations have the smooth and functional coefficients in contrast to equations in direct description with the discontinuous and highly oscillating coefficients. Equations (12) together with micro-macro decomposition of displacement field (8) constitute the tolerance model of the plate under consideration.

3. ASYMPTOTIC MODEL

For asymptotic modelling procedure we retain only the concept of highly oscillating function. We shall not deal with the concept of the tolerance periodic function as well as slowly-varying function. For every parameter $\varepsilon = 1/n$, $n = 1, 2, \dots$ we define a scaled cell $\Delta_\varepsilon \equiv (-\varepsilon l/2, \varepsilon l/2)$ and by $\Delta_\varepsilon(x) = x + \Delta_\varepsilon$ the scaled cell with a centre at $\xi^\alpha \in \bar{\Pi}$.

The mass density $\mu(\cdot)$, moduli of the foundation $k_z(\cdot)$, $k_t(\cdot)$ and tensor of elasticity $D^{\alpha\beta\gamma\delta}(\cdot)$ are assumed to be highly oscillating discontinuous functions $\mu(\cdot)$, $k_z(\cdot)$, $k_t(\cdot)$, $D^{\alpha\beta\gamma\delta}(\cdot) \in HO^0_\delta(\Xi, \Delta)$ for almost every $\xi^\alpha \in \bar{\Pi}$. If $\mu(\cdot)$, $k_z(\cdot)$, $k_t(\cdot)$, $D^{\alpha\beta\gamma\delta}(\cdot) \in HO^0_\delta(\Xi, \Delta)$ then for every $\xi^\alpha \in \bar{\Pi}$ there exist functions $\mu(y, \xi^2)$, $k_z(y, \xi^2)$, $k_t(y, \xi^2)$, $D^{\alpha\beta\gamma\delta}(y, x^2)$ which are periodic approximation of functions $\mu(\cdot)$, $k_z(\cdot)$, $k_t(\cdot)$, $D^{\alpha\beta\gamma\delta}(\cdot)$, respectively.

The fundamental assumption of the asymptotic modelling is that we introduce decomposition of displacement as family of fields

$$w_\varepsilon(y, \xi^2, t) = w^0(y, \xi^2, t) + \varepsilon^2 \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) V_A(y, \xi^2, t), \quad y \in \Delta_\varepsilon(\xi^\alpha) \quad (14)$$

where $\tilde{h}^A(y, \xi^2)$ are periodic approximation of highly oscillating functions $h^A(\cdot)$. From formula (14) we obtain

$$\begin{aligned} \partial_\alpha w_\varepsilon(y, \xi^2, t) &= \partial_\alpha w^0(y, \xi^2, t) + \varepsilon \partial_1 \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) V_A(y, \xi^2, t) + \varepsilon^2 \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) \partial_2 V_A(y, \xi^2, t) \\ \partial_{\alpha\beta} w_\varepsilon(y, \xi^2, t) &= \partial_{\alpha\beta} w^0(y, \xi^2, t) + \partial_{11} \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) V_A(y, \xi^2, t) + \\ &\quad + 2\varepsilon \partial_1 \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) \partial_2 V_A(y, \xi^2, t) + \varepsilon^2 \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) \partial_{22} V_A(y, \xi^2, t) \\ \dot{w}_\varepsilon(y, \xi^2, t) &= \dot{w}^0(y, \xi^2, t) + \varepsilon^2 \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) \dot{V}_A(y, \xi^2, t) \end{aligned} \quad (15)$$

Bearing in mind that by means of property of the mean value, Jikov et al. (1994), function $\tilde{h}^A(y/\varepsilon, \xi^2)$, $y \in \Delta_\varepsilon(\xi^\alpha)$, is weakly bounded and has under $\varepsilon \rightarrow 0$ weak limit. Under limit passage $\varepsilon \rightarrow 0$ for $y \in \Delta_\varepsilon(\xi^\alpha)$ we obtain

$$\begin{aligned} w^0(y, \xi^2, t) &= w^0(\xi^\alpha, t) + O(\varepsilon), & \partial_\alpha w^0(y, \xi^2, t) &= \partial_\alpha w^0(\xi^\alpha, t) + O(\varepsilon) \\ \partial_{\alpha\beta} w^0(y, \xi^2, t) &= \partial_{\alpha\beta} w^0(\xi^\alpha, t) + O(\varepsilon) \\ V_A(y, \xi^2, t) &= V_A(\xi^\alpha, t) + O(\varepsilon), & \partial_2 V_A(y, \xi^2, t) &= \partial_2 V_A(\xi^\alpha, t) + O(\varepsilon) \\ \partial_{22} V_A(y, \xi^2, t) &= \partial_{22} V_A(\xi^\alpha, t) + O(\varepsilon) \\ \dot{w}^0(y, \xi^2, t) &= \dot{w}^0(\xi^\alpha, t) + O(\varepsilon), & \dot{V}_A(y, \xi^2, t) &= \dot{V}_A(\xi^\alpha, t) + O(\varepsilon) \end{aligned} \quad (16)$$

By means of (16) we rewrite formulae (14) and (15) in the form

$$\begin{aligned} w_\varepsilon(y, \xi^2, t) &= w^0(y, \xi^2, t) + O(\varepsilon) \\ \partial_\alpha w_\varepsilon(y, \xi^2, t) &= \partial_\alpha w^0(y, \xi^2, t) + O(\varepsilon) \\ \partial_{\alpha\beta} w_\varepsilon(y, \xi^2, t) &= \partial_{\alpha\beta} w^0(y, \xi^2, t) + \partial_{11} \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) V_A(y, \xi^2, t) + O(\varepsilon) \\ \dot{w}_\varepsilon(y, \xi^2, t) &= \dot{w}^0(y, \xi^2, t) + O(\varepsilon) \end{aligned} \quad (17)$$

For a periodic approximation of Lagrangian Λ we have

$$\begin{aligned} & \tilde{\Lambda}_\varepsilon\left(\frac{y}{\varepsilon}, \xi^2, w^0\left(\frac{y}{\varepsilon}, \xi^2, t\right) + O(\varepsilon), \partial_\alpha w^0\left(\frac{y}{\varepsilon}, \xi^2, t\right) + O(\varepsilon), \dot{w}^0\left(\frac{y}{\varepsilon}, \xi^2, t\right) + O(\varepsilon), \right. \\ & \left. \partial_{\alpha\beta} w^0\left(\frac{y}{\varepsilon}, \xi^2, t\right) + \partial_{11} \tilde{h}^A\left(\frac{y}{\varepsilon}, \xi^2\right) V_A\left(\frac{y}{\varepsilon}, \xi^2, t\right) + O(\varepsilon)\right) \end{aligned} \quad (18)$$

If $\varepsilon \rightarrow 0$ then $\tilde{\Lambda}_\varepsilon$ by means of property of the mean value, Jikov et al. (1994), tends weakly in to

$$\begin{aligned} & \Lambda_0\left(\xi^\alpha, w^0\left(\xi^\alpha, t\right), \partial_\alpha w^0\left(\xi^\alpha, t\right), \dot{w}^0\left(\xi^\alpha, t\right), \partial_{\alpha\beta} w^0\left(\xi^\alpha, t\right), V_A\left(\xi^\alpha, t\right)\right) = \\ & = \frac{1}{|\Delta|} \int_{\Delta(x)} \tilde{\Lambda}\left(y, \xi^\alpha, w^0\left(\xi^\alpha, t\right), \partial_\alpha w^0\left(\xi^\alpha, t\right), \dot{w}^0\left(\xi^\alpha, t\right), \partial_{\alpha\beta} w^0\left(\xi^\alpha, t\right) + \partial_{11} \tilde{h}^A\left(y, \xi^2\right) V_A\left(\xi^\alpha, t\right)\right) dy. \end{aligned} \quad (19)$$

Asymptotic action functional has the form

$$A_\varepsilon^0\left(w^0, V_A\right) = \int_{t_0}^{t_1} \int_{\Pi} \left(\Lambda_{0\eta}\left(\xi^\alpha, w^0(\cdot), \partial_\alpha w^0(\cdot), w_{|\alpha\beta}^0(\cdot), V_A(\cdot), \dot{w}^0(\cdot)\right)\right) d\xi^\alpha dt \quad (20)$$

where Lagrangian is given by

$$\begin{aligned} & \Lambda_{0\eta}\left(\xi^\alpha, w^0, \partial_\alpha w^0, w_{|\alpha\beta}^0, V_{A|22}, V_{A|2}, V_A, \dot{w}^0\right) = \\ & = \frac{1}{2} \langle D^{\alpha\beta\eta\mu} \rangle w_{|\alpha\beta}^0 w_{|\eta\mu}^0 + \langle D^{11\eta\mu} \rangle h^A_{|11} > V_A w_{|\eta\mu}^0 + \frac{1}{2} \langle D^{1111} \rangle h^A_{|11} h^B_{|11} > V_A V_B + \\ & \frac{1}{2} \langle k_z \rangle w^0 w^0 + \frac{h^2}{8} \langle k_t \rangle \delta^{\alpha\beta} \partial_\alpha w^0 \partial_\beta w^0 + \frac{1}{2} \langle n^{\alpha\beta} \rangle \partial_\alpha w^0 \partial_\beta w^0 - \frac{1}{2} \langle \mu \rangle w^0 \dot{w}^0 - \langle p \rangle w^0. \end{aligned} \quad (21)$$

From principle stationary action we can derive the Euler-Lagrange equations

$$\begin{aligned} & \frac{\partial}{\partial t} \frac{\partial \Lambda_0}{\partial \dot{w}^0} - \left(\frac{\partial \Lambda_0}{\partial w_{|\alpha\beta}^0} \right)_{|\alpha\beta} + \partial_\alpha \left(\frac{\partial \Lambda_0}{\partial w^0_{,\alpha}} \right) - \frac{\partial \Lambda_0}{\partial w^0} = 0 \\ & \frac{\partial \Lambda_0}{\partial V_A} = 0, \quad A = 1, \dots, N \end{aligned} \quad (22)$$

Substituting formulae (21) into equations (22), governing equations of the plate under consideration take the form

$$\begin{aligned}
 & (\langle D^{\alpha\beta\gamma\mu} \rangle w^0_{|\gamma\mu} \rangle)_{|\alpha\beta} + (\langle D^{11\alpha\beta} h^A_{|11} \rangle V_A)_{|\alpha\beta} + \langle k_z \rangle w^0 - \frac{h^2}{4} \partial_\alpha (\langle k_t \rangle \delta^{\alpha\beta} \partial_\beta w^0) + \\
 & - (N^{\alpha\beta} w^0_{|\beta})_{|\alpha} + \langle \mu \rangle \ddot{w}^0 = \langle p \rangle \\
 & \langle D^{11\alpha\beta} h^A_{|11} \rangle w^0_{|\alpha\beta} + \langle D^{1111} h^A_{|11} h^B_{|11} \rangle V_B = 0
 \end{aligned} \tag{23}$$

Eliminating V_A from second equation (23)

$$V_A = - \frac{\langle D^{11\gamma\mu} h^B_{|11} \rangle}{\langle D^{1111} h^A_{|11} h^B_{|11} \rangle} w^0_{|\gamma\mu}, \tag{24}$$

and denoting effective elastic moduli

$$D_{eff}^{\alpha\beta\gamma\mu} = \langle D^{\alpha\beta\gamma\mu} \rangle - \frac{\langle D^{11\gamma\mu} h^B_{|11} \rangle}{\langle D^{1111} h^A_{|11} h^B_{|11} \rangle} \langle D^{11\alpha\beta} h^A_{|11} \rangle, \tag{25}$$

we arrive the following equation of motion for the averaged displacement of the plate midplane $w^0(\xi^\alpha, t)$

$$\begin{aligned}
 & (\langle D_{eff}^{\alpha\beta\gamma\mu} \rangle w^0_{|\gamma\mu} \rangle)_{|\alpha\beta} + \langle k_z \rangle w^0 - \frac{h^2}{4} \partial_\alpha (\langle k_t \rangle \delta^{\alpha\beta} \partial_\beta w^0) - (N^{\alpha\beta} w^0_{|\beta})_{|\alpha} \\
 & + \langle \mu \rangle \ddot{w}^0 = \langle p \rangle
 \end{aligned} \tag{26}$$

Equations (24)-(24) represent the asymptotic model of the stability behaviour of the plate interacting with microheterogeneous subsoil.

The general results of contribution will be illustrated by analysis of the stability of an annular plate resting on elastic heterogeneous foundation. Coefficients of model equations (12), (26) are a smooth functions of radial coordinate $\rho \in (R_0, R_1)$, and in most cases numerical methods have to be used in order to obtain solutions. In this contribution in order to obtain the approximate solution of equations (26) will be used the Galerkin method. Example of the obtained results will be given during presentation.

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