# BUCKLING AND VIBRATIONS OF COMPOSITE COLUMN-BEAMS

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This paper attends to analyse the modes for static buckling and vibrations of column-beams with rectangular and trapezoidal cross-sections with the help of the finite elements method. In the case of forced vibrations as well as in static buckling, the bending or eccentric compression loadings are investigated.

## 1. INTRODUCTION

The paper deals with static buckling, natural free and forced vibrations for beams with rectangular trapezoidal cross-sections, subjected to eccentric compression or pure bending, using ANSYS 12.1 FEM program [1]. In order to determine dynamic critical load, the Budiansky-Hutchinson [2] criterion was used. Many works describe behaviour of steel and composite columns and column-beams with closed and open cross-sections with respect to buckling modes and their interactions in aspect of dynamic buckling under axial compression, but the world subject literature does not presents generally the dynamic buckling of bent or eccentrically compressed thin-walled structures. In work [3] Kubiak investigated rectangular girder subjected to pure bending, using the finite elements method (FEM) and analytical-numerical method.

## 2. NUMERICAL MODELS

In both numerical models, the multi-layered shell finite element (SHELL99) was used (see Fig. 1) to include composite material properties only with one layer of lamina.

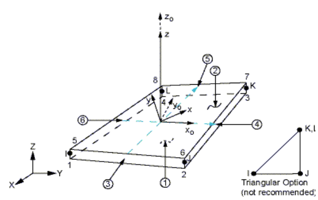


Fig. 1. SHELL99 element geometry [1]

This element has 4 nodes and 6 degrees of freedom at each node. The numerical models (Fig. 2.1, 2.2) of two beams were prepared as one half of the analyzed column taking in mind symmetry condition assumed at the half length of column. Thickness of all walls is identical and equals 1 mm (Table 1). Load distribution at the ends of beams causes pure bending to be developed (only bending moment is applied) or eccentric compression (combined load composed of bending moment and compressive force). In case of eccentric compression, there is uniformly distributed force applied to each node. The nodes that belong to the plate under tension are not loaded, but nodes at the edges that belong to the compressed plate (bottom plate) are loaded with 1 N. For rectangular beam, in case of eccentric compression, it gives moment value of 2.34 Nm and compression force of 60 N. For trapezoidal beam the bending moment equals 0.034 Nm and axial force is 1.36 N. For pure bending, bending moments are specified at 4.64 Nm for rectangular beam and 1.45 Nm for trapezoidal beam. These values are reference values used in static buckling and forced vibrations analysis. The columns is investigated as simply supported at the loaded ends in plane of neutral axis, but top and bottom edges can move in 'z' direction ('z' axis is parallel to neutral axis) and all loaded edges belonging to top and bottom plates, remain straight. The applied load is assumed as a dynamic pulse. The shape of this pulse is a rectangle and pulse duration corresponds to the period of natural fundamental flexural vibrations of analyzed beams. The structures are modeled using the assumption of constant total mass of rectangular beam. It means that trapezoidal beam is made in the way that its mass is equal to the mass of rectangular beam.

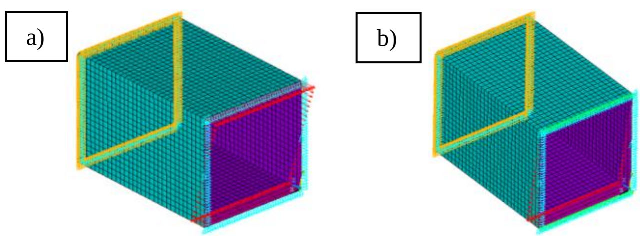


Fig. 2.1. Numerical model and boundary conditions (half model): a) pure bending, b) eccentric compression

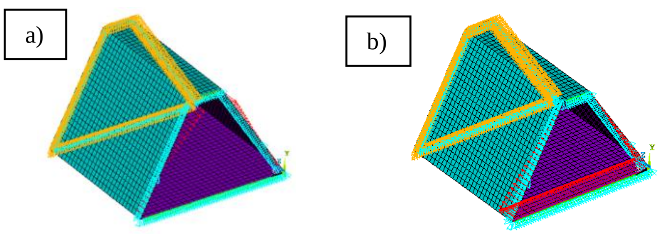


Fig. 2.2. Numerical model and boundary conditions (half model) for two cases of load: a) pure bending, b) eccentric compression

Table 1. Dimensions of beam-columns

| Type of section | **a** [mm] | $\mathbf{b_1}$ [mm] | $\mathbf{b_2}$ [mm] | $\mathbf{b_3}$ [mm] | $\alpha$ [deg] |
|---|---|---|---|---|---|
| trapezoidal | 300 | 150 | 112 | 27 | 15 |
| rectangular | 300 | 100 | 100 | 100 | 0 |

where: $a$ – total length, $b_1$ – width of bottom plate, $b_2$ – width of inclined or vertical plates, $b_3$ – width of top plate, α - angle of inclination (inclined or vertical plates).

Table 2. Mechanical properties of composite material (1 – compression direction)

| $E_1$ [GPa] | $E_2$ [GPa] | $G_{12}$ [GPa] | $\nu_{12}$ [-] | ρ [kg/m$^3$] |
|---|---|---|---|---|
| 76 | 5.5 | 2.1 | 0.34 | 7800 |

3. 3. RESULTS OF CALCULATIONS

Calculations, obtained with the finite element method, were conducted for static buckling (the critical static loads) and frequencies of natural vibrations, for two cases of the load.

*3.1. STATIC BUCKLING ANALYSIS*

The modes of buckling are shown for pure bending in figures 3 and 5 and for the eccentric compression in figures 4 and 6. The figures show first three modes of buckling. Static critical moments were calculated for two types of load (Tables 3a, 3b).

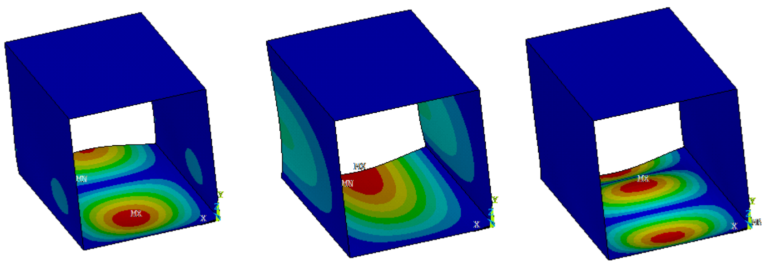

Fig. 3. Buckling modes (half model) - for pure bending case: modes 1-3 (numbering from left to right)

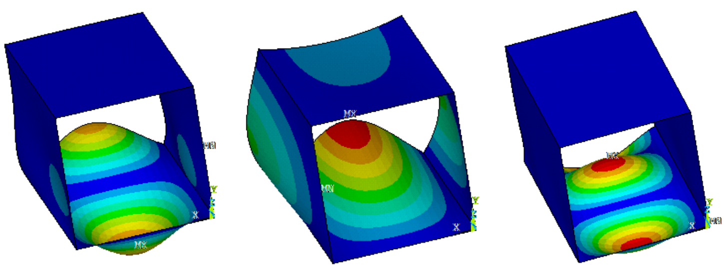

Fig.4. Buckling modes (half model) - for eccentric compression case: 1-3 modes (numbering from left to right)

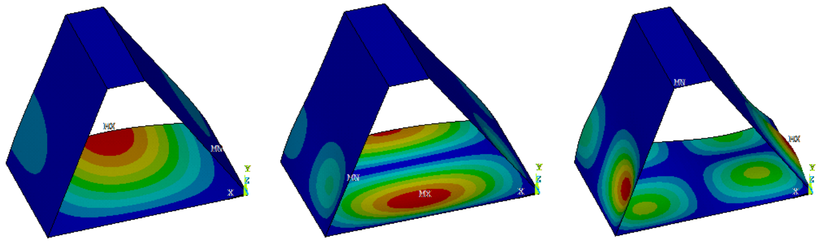

Fig.5. Buckling modes (half model) - for pure bending case: modes 1-3 (numbering from left to right)

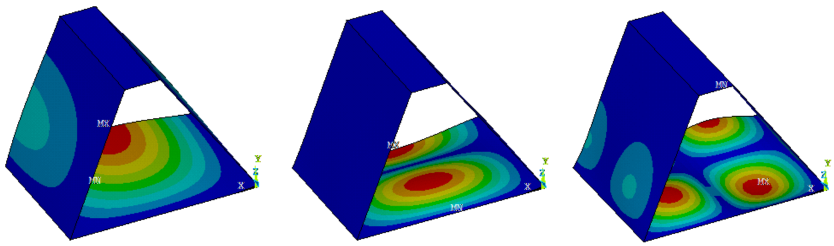

Fig.6. Buckling modes (half model) - for eccentric compression case: modes 1-3 (numbering from left to right)

Table 3a. Critical static moments for rectangular beam

| Number of half-waves | Critical buckling moment [Nm] | Case of load |
|---|---|---|
| 3 | 107.7 | pure bending |
| 3 | 54.2 | ecc. compr. |

Table 3b. Critical static moments for trapezoidal beam

| Number of half-waves | $M_{cr}$ [Nm] | Case of load |
|---|---|---|
| 1 | 221.8 | pure bending |
| 1 | 19.2 | eccentric compression |

### *3.2. VIBRATION ANALYSIS*

Analysis of vibrations was conducted for first three modes obtained for natural and forced vibrations. In Figs. 7 and 8, the modes of natural vibrations are shown for rectangular and trapezoidal beams. Frequencies for each mode are shown in Table 4.1 and 4.2 for rectangular and trapezoidal cross sections, respectively.

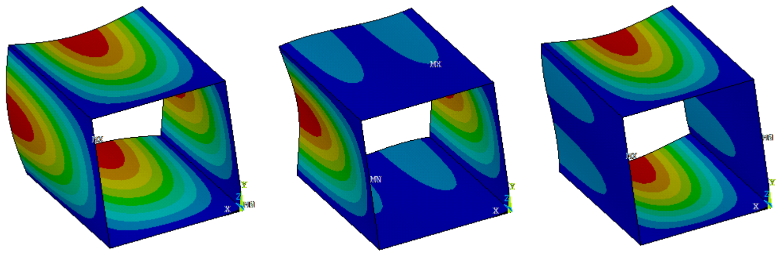

Fig.7. Modes of natural vibrations (half model) for rectangular beam

Table 4.1. Natural frequencies for rectangular beam

| Number of half-waves | *f* [Hz] |
|---|---|
| 1 | 45.4 |
| 1 | 64.7 |
| 1 | 64.8 |

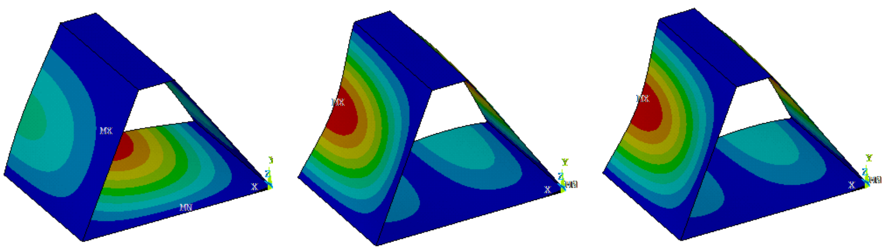

Fig.8. Modes of natural vibrations (half model) for trapezoidal beam

Table 4.2. Natural frequencies for trapezoidal beam

| Number of half-waves | *f* [Hz] |
|---|---|
| 1 | 33.8 |
| 1 | 52.5 |
| 1 | 57.2 |

Next, frequencies of forced vibrations were calculated for the beams (Fig. 9). Moreover, in the case of trapezoidal one, the second mode of buckling was included.

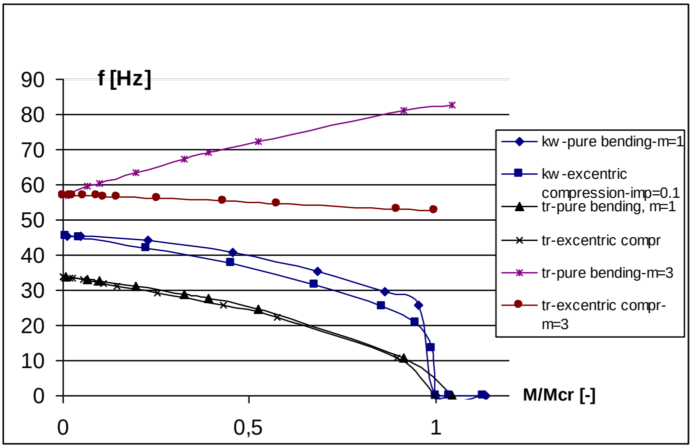


Fig.9. Frequencies of forced vibrations vs. moment ratio: $M/M_{cr}$

## 4. CONCLUSIONS

For the second mode of buckling, in case of eccentric compression, the frequency decreases, but it does not reach value equal to zero. In the case of pure bending, the frequency increases. Comparing the modes of vibrations and buckling, one can conclude that all the modes for appropriate numbers of modes are different except first modes for trapezoidal beam.

## REFERENCES

[1] ANSYS Theory Reference, 12th Edition, www.ansys.com.

[2] Kubiak T., Criteria of dynamic buckling estimation of thin-walled structures, Thin-Walled Structures, 45, 2007, 888-892.

[3] Kubiak T., Dynamic response of girders subjected to pure pulse bending loading, Acta Mech. et Autom., Vol. 8, No. 4 (2008), 57-61.