POSTBUCKLING PROBLEMS OF THIN PERIODIC PLATES

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The objects under considerations are thin linear-elastic plates with periodic structure subjected to large deflections. The paper concerns the problem of periodic plates' postbuckling behaviour. The applied mathematical model describing geometrically nonlinear problems of such plates, proposed by Domagalski and Jędrysiak (2012), is based on the tolerance averaging technique, cf. Woźniak et al. (eds.) (2010).

1. INTRODUCTION

Plates considered in this paper are made of isotropic materials but as a result of changing thickness or using two or more materials with different elastic properties their behaviour is similar to behaviour of anisotropic or orthotropic ones with discontinuities of geometric or/and material properties, cf. Fig. 1. It leads to governing equations of these plates, which have non-continuous, highly oscillating, functional coefficients. Exact solutions to these equations are very difficult to obtain. Therefore, various simplified approaches, introducing effective plate properties, are proposed. Amongst them there have to be mentioned models based on the asymptotic homogenization, e.g. homogenized model of periodic plates proposed by Kohn and Vogelius [7].

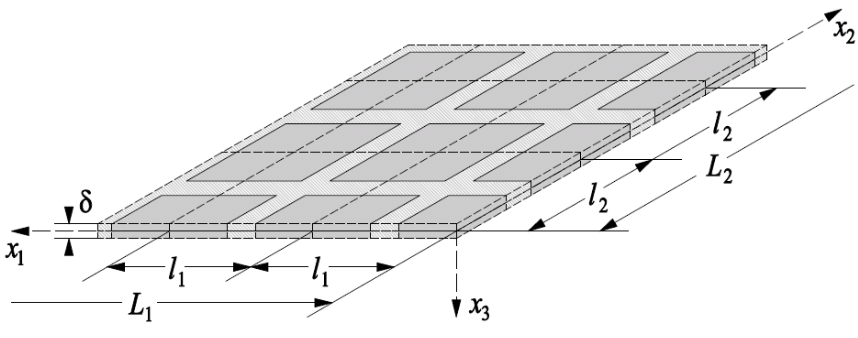

Fig. 1. Fragment of a thin periodic plate

In this paper, in order to take into account this effect in model equations, the tolerance modelling approach is applied, cf. the books edited by Woźniak, Michalak and Jędrysiak [14] and by Woźniak et al. [13]. Applications of this method to other problems of periodic plates are shown in a series of papers, e.g. for vibrations of periodic wavy-type plates by Michalak [9], for periodically stiffened plates by Nagórko and Woźniak [10], for the buckling of periodic thin plates by Jędrysiak [4], for plates with the inhomogeneity period of an order of the plate thickness by Baron [1], for stability and vibrations of periodic plates by Jędrysiak [5, 6], for some problems of bending of thin periodic plates by Domagalski and Jędrysiak [2, 3].

The aims of this contribution are: to present governing equations of the tolerance model of thin periodic plates subjected to large deflections, which take into account the effect of the microstructure size, and to apply this model to investigate some problems of plates subjected to inplane loads beyond their critical values.

2. FORMULATION OF THE PROBLEM

Let $Ox_1x_2x_3$ be an orthogonal Cartesian coordinate system; subscripts i, j, k, l run over 1, 2, 3 and α, β, γ, ω run over 1, 2. Denote $\mathbf{x}=(x_1,x_2)$ and $z=x_3$. The undeformed plate occupies the region $\Omega\equiv\{(\mathbf{x},z):-\delta(\mathbf{x})/2\le z\le\delta(\mathbf{x})/2, \mathbf{x}\in\Pi\}$, with midplane Π and the plate thickness δ(·). Let us also denote the partial derivatives with respect to a space coordinate by $\partial_\alpha=\partial/\partial x_\alpha$.

It is assumed that periodic plates under consideration consist of many small repetitive elements called *periodicity cells*. The cell is defined as a plane region $\square\equiv[-l_1/2,l_1/2]\times[-l_2/2,l_2/2]$, where l_1, l_2 are the cell dimensions along the x_1-, x_2-axis. The size of the microstructure of the plate is described by the diameter of the periodicity cell, given by $l=[(l_1)^2+(l_2)^2]^{1/2}$ and satisfying the condition $\max(\delta)<<l<<\min(L_1,L_2)$, ($L_1$ and L_2 are characteristic dimensions of the plate along the x_1- and x_2-axis). This diameter is called *the microstructure parameter*. Hence, the cell can be treated as a thin plate.

Our considerations are based on the well-known nonlinear theory of thin plates (cf. Timoshenko and Woinowsky-Krieger [11], and Woźniak (ed.) [12]). Let $w(\mathbf{x})$ be a plate midplane deflection, $u_{0\alpha}(\mathbf{x})$ be the in-plane displacements along the x_α-axes, $F(\mathbf{x})$ be the stress function, and $q(\mathbf{x})$ be the total loadings in the z-axis; $\mathbf{x}\in\Pi$. Thickness δ(·) can be a periodic function in **x** and elastic moduli $a_{ijkl}=a_{ijkl}(\cdot,z)$ can be also periodic functions in **x** and even functions in z. Let $a_{\alpha\beta\gamma\omega}$, $a_{\alpha\beta33}$, a_{3333} be the non-zero components of the elastic moduli tensor. Denote $c_{\alpha\beta\gamma\omega}\equiv a_{\alpha\beta\gamma\omega}-a_{\alpha\beta33}a_{33\gamma\omega}(a_{3333})^{-1}$.

Define the mean plate properties, being periodic functions in **x**, i.e. shell stiffnesses $b_{\alpha\beta\gamma\omega}$ and bending stiffnesses $d_{\alpha\beta\gamma\omega}$, in the form:

$$b_{\alpha\beta\gamma\omega}(\mathbf{x})=\int_{-\delta(\mathbf{x})/2}^{\delta(\mathbf{x})/2}c_{\alpha\beta\gamma\omega}(\mathbf{x},z)dz,\quad d_{\alpha\beta\gamma\omega}(\mathbf{x})=\int_{-\delta(\mathbf{x})/2}^{\delta(\mathbf{x})/2}c_{\alpha\beta\gamma\omega}(\mathbf{x},z)z^2dz. \tag{1}$$

From the well-known assumptions of the nonlinear thin plate theory, e.g. relations between the total strains $E_{\alpha\beta}$, membrane strains $E_{0\alpha\beta}$ and curvatures $\kappa_{\alpha\beta}$ written as:

$$\begin{gathered}E_{\alpha\beta}=E_{0\alpha\beta}+z\kappa_{\alpha\beta}\\ E_{0\alpha\beta}=\tfrac{1}{2}(\partial_\beta u_{0\alpha}+\partial_\alpha u_{0\beta}+\partial_\alpha w\partial_\beta w),\quad \kappa_{\alpha\beta}=-\partial_{\alpha\beta}w,\end{gathered} \tag{2}$$

the constitutive equations for membrane strains formulated as following:

$$E_{0\alpha\beta} = \tilde{b}_{\alpha\beta\gamma\omega} \Re_{\gamma\omega} F, \tag{3}$$

where $\tilde{b}_{\alpha\beta\xi\eta} b_{\xi\eta\gamma\omega} = \delta_{\alpha\gamma}\delta_{\beta\omega}$, $\Re_{\alpha\beta} F \equiv N_{\alpha\beta}$, $\Re_{\alpha\beta}(\cdot) \equiv (\nabla^2 \delta_{\alpha\beta} - \partial_{\alpha\beta})(\cdot)$, we obtain for periodic plates the following equations for the deflection *w* and the stress function *F*:

$$\begin{aligned} &\Re_{\alpha\beta}(\tilde{b}_{\alpha\beta\gamma\omega}\Re_{\gamma\omega}F) = \tfrac{1}{2}(\partial_{\alpha\beta}w\partial_{\alpha\beta}w - \partial_{\alpha\alpha}w\partial_{\beta\beta}w), \\ &\partial_{\alpha\beta}(d_{\alpha\beta\gamma\omega}\partial_{\gamma\omega}w) - \Re_{\alpha\beta}F\partial_{\alpha\beta}w = q. \end{aligned} \tag{4}$$

These equations have functional, highly oscillating, non-continuous, periodic in **x** coefficients.

3. THE TOLERANCE MODELLING APPROACH

3.1. INTRODUCTORY CONCEPTS

In the course of modelling, some introductory concepts of the tolerance modelling technique, such as the averaging operation <·>, the slowly-varying (*SV*) function, the fluctuation shape (*FS*) function, are used. These concepts are described in books edited by Woźniak, Michalak and Jędrysiak [14] and by Woźniak et al. [13].

3.2. FUNDAMENTAL ASSUMPTIONS

Following books [13, 14] and using the previously mentioned introductory concepts, the fundamental modelling assumptions can be formulated.

First of them is the *micro-macro decomposition* of the basic unknowns, where there is assumed:

– for the out-of-plane deflection:

$$w(\mathbf{x}) = W(\mathbf{x}) + h^A(\mathbf{x})V^A(\mathbf{x}), \quad A = 1,\ldots,N, \tag{5}$$

– for the in-plane displacements:

$$u_{0\alpha}(\mathbf{x}) = U_{0\alpha}(\mathbf{x}), \tag{6}$$

– for the stress function:

$$F(\mathbf{x}) = \Phi(\mathbf{x}) + g^K(\mathbf{x})\Psi^K(\mathbf{x}), \quad K = 1,\ldots,M, \tag{7}$$

and $W(\cdot), V^A(\cdot), \Phi(\cdot), \Psi^K(\cdot) \in SV_d^2(\Pi,\Omega)$, $U_\alpha(\cdot) \in SV_d^1(\Pi,\Omega)$ are basic unknowns; $h^A(\cdot), g^K(\cdot) \in FS_d^2(\Pi,\Omega)$ are the known fluctuation shape functions. Functions $W(\cdot)$, $\Phi(\cdot)$, $V^A(\cdot)$ and $\Psi^K(\cdot)$ are called *the macrodeflection*, *the macrostress function*, *the fluctuation amplitudes* of the deflection and of the stress function, respectively; $U_{0\alpha}(\cdot)$ are *the in-plane macrodisplacements*.

The additional assumption is the decomposition of the load $q(\mathbf{x})$ in the form $q(\mathbf{x}) = q^0(\mathbf{x}) + \tilde{q}(\mathbf{x})$, where $q^0 = < q >$ is the slowly-varying averaged load, and $\tilde{q}$ is the oscillating part, $< \tilde{q} > \cong 0$.

4. MODEL EQUATIONS

Applying the modelling procedure described in [13, 14], under denotations

$$\begin{aligned}
&\tilde{B}_{\alpha\beta\gamma\omega} \equiv < \tilde{b}_{\alpha\beta\gamma\omega} >, \quad \tilde{B}^K_{\gamma\omega} \equiv < \tilde{b}_{\alpha\beta\gamma\omega}\Re_{\alpha\beta} g^K >, \quad \tilde{B}^{KL} \equiv < \tilde{b}_{\alpha\beta\gamma\omega}\Re_{\alpha\beta} g^K \Re_{\gamma\omega} g^L >,\\
&D_{\alpha\beta\gamma\omega} \equiv < d_{\alpha\beta\gamma\omega} >, \quad D^A_{\alpha\beta} \equiv < d_{\alpha\beta\gamma\omega}\partial_{\gamma\omega} h^A >, \quad D^{AB} \equiv < d_{\alpha\beta\gamma\omega}\partial_{\gamma\omega} h^A \partial_{\alpha\beta} h^B >,\\
&Q \equiv < q^0 >, \quad Q^A \equiv < \tilde{q} h^A > l^{-2}, \quad G^{AB}_{\alpha\beta} \equiv < \partial_\alpha h^A \partial_\beta h^B > l^{-2},
\end{aligned} \tag{8}$$

we arrive at the following system of equations for the macrostress function $\Phi(\cdot)$, the fluctuation amplitudes of the stress function $\Psi^K(\cdot)$, the macrodeflection $W(\cdot)$, the fluctuation amplitudes of the deflection $V^A(\cdot)$:

$$\begin{aligned}
&\Re_{\alpha\beta}(\tilde{B}^{eff}_{\alpha\beta\gamma\omega}\Re_{\gamma\omega}\Phi) = \tfrac{1}{2}\Re_{\alpha\beta}(\partial_\alpha W \partial_\beta W) + \underline{\tfrac{1}{2} l^2 G^{AB}_{\alpha\beta}\Re_{\alpha\beta}(V^B V^A)},\\
&\partial_{\alpha\beta}(D_{\alpha\beta\gamma\omega}\partial_{\gamma\omega} W + D^B_{\alpha\beta} V^B) - \Re_{\alpha\beta}\Phi\partial_{\alpha\beta} W = Q,\\
&D^A_{\gamma\omega}\partial_{\gamma\omega} W + D^{AB} V^B + \underline{l^2 G^{AB}_{\alpha\beta} V^B \Re_{\alpha\beta}\Phi} = \underline{l^2 Q^A},\\
&\Psi^K = -(\tilde{B}^{LK})^{-1}\tilde{B}^L_{\alpha\beta}\Re_{\alpha\beta}\Phi; \quad \alpha,\beta,\gamma,\omega = 1,2; \quad A,B = 1,\ldots,N; \quad K,L = 1,\ldots,M,
\end{aligned} \tag{9}$$

where $\tilde{B}^{eff}_{\alpha\beta\gamma\omega} \equiv \tilde{B}_{\alpha\beta\gamma\omega} - \tilde{B}^L_{\alpha\beta}(\tilde{B}^{KL})^{-1}\tilde{B}^K_{\gamma\omega}$.

Equations (9) together with micro-macro decompositions (5)-(7) constitute *the nonlinear tolerance model* of thin periodic plates. This model describes the effect of the microstructure size on the overall plate behaviour by the underlined terms. For considered plates there have to be formulated boundary conditions only for the macrodeflection W and the macrostress function Φ. Moreover, the basic unknowns of equations (9) have to satisfy the following conditions: $W(\cdot), V^A(\cdot), \Phi(\cdot), \Psi^K(\cdot) \in SV_d^2(\Pi,\Omega)$. For comparison we recall the governing equations of *the linear tolerance model* of thin periodic plates:

$$\begin{aligned}
&D_{\alpha\beta\gamma\omega}\partial_{\alpha\beta\gamma\omega} W + D^A_{\alpha\beta}\partial_{\alpha\beta} V^A = Q,\\
&D^A_{\alpha\beta}\partial_{\alpha\beta} W + D^{AB} V^B = \underline{l^2 Q^A},
\end{aligned} \tag{10}$$

cf. Jędrysiak [4], Woźniak, Michalak and Jędrysiak [14]. It can be observed that in this model the effect of the microstructure size is taken into account only by the term related to the oscillating part l^2Q^A of the load.

5. EXAMPLE OF APPLICATION

5.1. PROBLEM FORMULATION

The object under consideration is a simply supported rectangular plate with constant thickness δ and length dimensions L_1 and $L_2=\eta L_1$ along the x_1- and x_2-axis, respectively. It is also assumed that all edges of the plate are immovable. The plate is made of two isotropic materials (a matrix – *M*, a rib – *R*), having Young's modulus E_M and E_R and Poisson's ratio ν_M and ν_R, periodically distributed along the x_1- and x_2-axis. A fragment of considered plate is illustrated in Fig. 1.

Solutions $W(\cdot)$, $\Phi(\cdot)$ to the model equations (8) have to satisfy boundary conditions for the simply supported plate, i.e. $W=\partial_{11}W=0$ for $x_1=0$, L_1; $W=\partial_{22}W=0$ for $x_2=0$, L_2; $\partial_{22}\Phi=N_{11}$ for $x_1=0$, L_1 and $\partial_{11}\Phi=N_{22}$ for $x_2=0$, L_2. Therefore, denoting $\xi_m=m\pi/L_1$, $\zeta_n=n\pi/L_2$, the above mentioned solutions can be assumed in the form of double sine or cosine trigonometric series ([8, 11]):

$$\begin{aligned}
W(x_1,x_2) &= \sum_n\sum_m W_{mn}\sin\xi_m x_1 \sin\zeta_n x_2,\\
V^A(x_1,x_2) &= \sum_n\sum_m V^A_{mn}\sin\xi_m x_1 \sin\zeta_n x_2,\\
\Phi(x_1,x_2) &= \sum_n\sum_m \Phi_{mn}\cos\xi_m x_1 \cos\zeta_n x_2 + \tfrac{1}{2}p_1 x_2^{\,2} + \tfrac{1}{2}p_2 x_1^{\,2},\\
\Psi^K(x_1,x_2) &= \sum_n\sum_m \Psi^K_{mn}\cos\xi_m x_1 \cos\zeta_n x_2,
\end{aligned} \tag{11}$$

where constants p_1 and p_2 represent average membrane forces per unit length in the x_1- and x_2-direction. The transversal average loads $Q(x_1,x_2)$ and $Q^A(x_1,x_2)$ can be expanded into double sine series:

$$Q(x_1,x_2) = \sum_n\sum_m Q_{mn}\sin\xi_m x_1 \sin\zeta_n x_2, \quad Q^A(x_1,x_2) = \sum_n\sum_m Q^A_{mn}\sin\xi_m x_1 \sin\zeta_n x_2. \tag{12}$$

Application of the Galerkin method leads to the following set of nonlinear, coupled algebraic equations for coefficients of series (11):

$$
\begin{aligned}
&\tilde{b}^{eff}_{kl}\overline{\Phi}_{kl} = \sum_{p,q,r,s}\Omega^{pqrs}_{kl}\overline{W}_{pq}\overline{W}_{rs} + \eta^2\lambda^2\sum_{A,B}\sum_{p,q,r,s}\Gamma^{AB,pqrs}_{kl}\overline{V}^{A}_{pq}\overline{V}^{B}_{rs},\\
&d_{mn}\overline{W}_{mn} + \pi^2(m^2\overline{p}_1 + \eta^2 n^2\overline{p}_2)\overline{W}_{mn} - \sum_{i,j,k,l}\mathrm{M}^{ijkl}_{mn}\overline{W}_{ij}\overline{\Phi}_{kl} + \eta\sum_{B} d^{B}_{mn}\overline{V}^{B}_{mn} - q_{mn} = 0,\\
&\eta\sum_{B} d^{AB}\overline{V}^{B}_{mn} + d^{A}_{mn}\overline{W}_{mn} - \eta\lambda^2\sum_{B}\sum_{i,j,k,l}\mathrm{N}^{AB,ijkl}_{mn}\overline{V}^{B}_{ij}\overline{\Phi}_{kl} +\\
&\qquad\qquad + \lambda^2\eta\sum_{B}[G^{AB}_{11}\overline{p}_1 + G^{AB}_{22}\overline{p}_2]\overline{V}^{B}_{mn} - \lambda^2 q^{A}_{mn} = 0,
\end{aligned}
\qquad (12)
$$

where overlined terms represent dimensionless forms of unknown coefficients:

$$
\overline{W}_{mn} \equiv \frac{W_{mn}}{\delta}, \quad \overline{V}^{A}_{mn} \equiv \frac{L_1 L_2}{\delta}V^{A}_{mn}, \quad \overline{\Phi}_{kl} \equiv \frac{\Phi_{kl}}{E_M\delta^3}, \qquad (13)
$$

dimensionless load-dependent terms are as follows:

$$
[q_{mn}, q^{A}_{mn}] \equiv \frac{L_1^{\,4}}{E_M\delta^4}[Q_{mn}, Q^{A}_{mn}], \quad \overline{p}_{\alpha} \equiv \frac{L_1^{\,2}}{E_M\delta^3}\,p_{\alpha}, \qquad (14)
$$

where λ is dimensionless microstructure parameter $\lambda = l/L_1$. The other coefficients of equations (12) are obtained as a result of applying the Galerkin method and, in order to limit the length of this paper, will not be presented here.

Some numerical results calculated in the framework of the nonlinear tolerance models are shown in the next section.

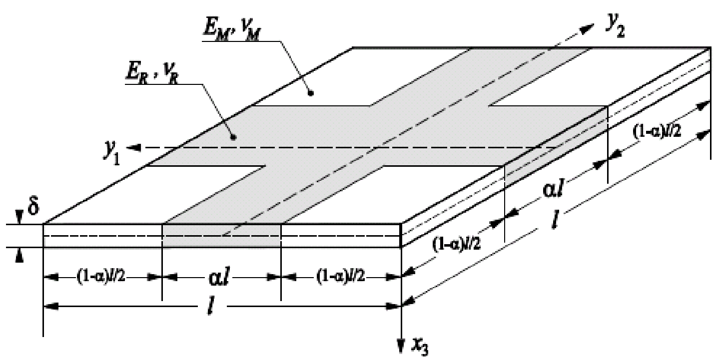

Fig. 2. A basic periodicity cell

5.2. CALCULATIONAL RESULTS

Numerical results are obtained for a square plate (η=1), made of two different isotropic materials. The periodicity cell is a square and is defined as $\square \equiv [-l/2, l/2]\times[-l/2, l/2]$, cf. Fig. 2. It is assumed that the Young’s modulus and Poisson’s ratio are given by:

$$E(\mathbf{x}),\nu(\mathbf{x}) = \begin{cases} E_M,\nu_M & \text{if} \quad \mathbf{x}\in[-l/2,-\alpha l/2)\times[-l/2,-\alpha l/2)\cup \\ & \cup[-l/2,-\alpha l/2)\times(\alpha l/2,l/2]\cup \\ & \cup(\alpha l/2,l/2]\times[-l/2,-\alpha l/2)\cup \\ & \cup(\alpha l/2,l/2]\times(\alpha l/2,l/2], \\ E_R,\nu_R & \text{if} \quad \mathbf{x}\in[-l/2,-\alpha l/2)\times[-\alpha l/2,\alpha l/2]\cup \\ & \cup[-\alpha l/2,\alpha l/2]\times[-l/2,l/2]\cup \\ & \cup(\alpha l/2,l/2]\times[-\alpha l/2,\alpha l/2], \end{cases} \tag{15}$$

where α is a dimensionless parameter describing distribution of material properties in the periodicity cell, cf. Fig. 2.

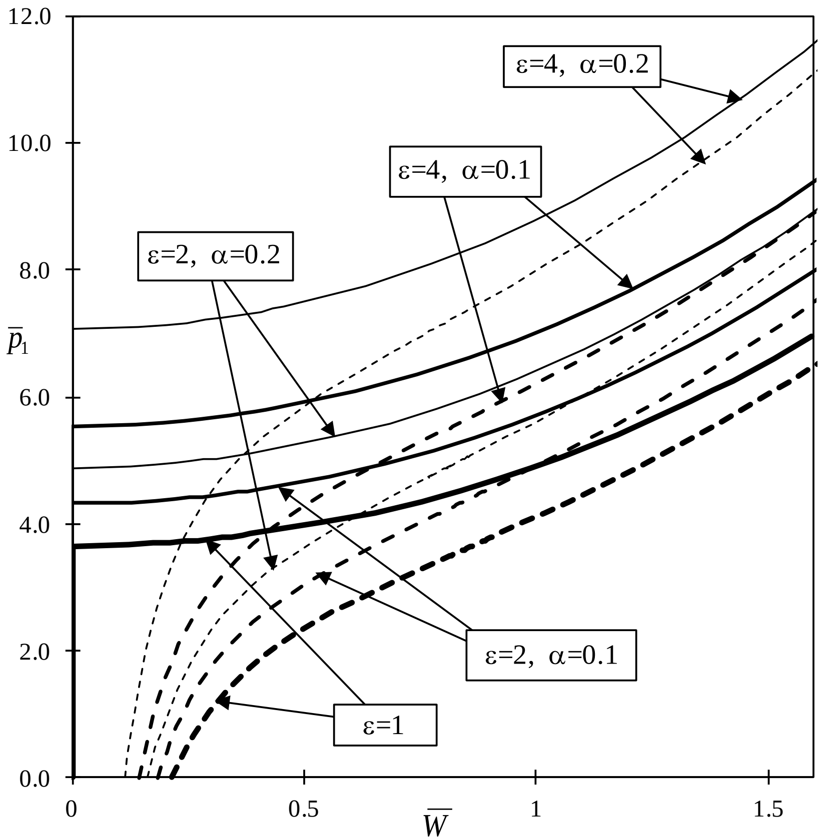

Fig. 3. Relations between uniaxial compression and maximum macrodeflection of initially flat (solid lines) and deflected due to transversal load (dashed lines) square plates

The fluctuation shape functions h^A and g^K are assumed in the similar form:

$$h^A = g^A = \tfrac{1}{3} l^2 [(1 - \cos\frac{2A\pi y_1}{l})(1 - \cos\frac{2A\pi y_2}{l}) - 1], \quad A = 1,\ldots,N, \tag{16}$$

satisfying conditions $<h^A>=0$.

In calculations we assume for the matrix: the Young's modulus E_M, the Poisson's ratio $\nu_M=0.316$. For the rib it is assumed: the Young's modulus $E_R=\varepsilon E_M$, the Poisson's ratio $\nu_R=\nu_M$. Results presented here are obtained for following values of parameters: dimensionless width of the rib $\alpha=0.1$, 0.2; dimensionless microstructure parameter $\lambda=0.05$; Young's modulus' ratio $\varepsilon=2.0$, 3.0, 4.0.

The deflections of a plate subjected to various combinations of inplane and lateral loads were calculated for the values of geometrical (α) and material (ε) parameters of the plate's microstructure given above.

The minimal values of critical inplane load $(\bar{p}^1_{11})_{crit}$ for a plate with periodically distributed material inhomogeneities are presented in the dimensioneless form, in Table 1. This loads correspond to a linear stability problem's eigenvalues, as the macrodeflection is assumed as $W(x_1,x_2)=W_{11}\sin\xi_1 x_1 \zeta_1 x_2$. The critical load values were calculated using the tolerance modelling technique, as described by Jędrysiak [4]. The value $\varepsilon=1.0$ refers to a homogeneous plate.

The graphs in Fig. 3 illustrate the inplane load - center macrodeflection relationships initially flat (solid lines) and transversally loaded plates (dashed lines). The dimensionless intensity of the transversal load was assumed as $q=5.0$.

Table 1. Minimal values of critical load for a square plate

$\varepsilon=E_R/E_M$	1.0	2.0		4.0	
α	-	0.1	0.2	0.1	0.2
$(\bar{p}^1_{11})_{crit}$	3.655	4.322	4.886	5.547	7.069

Following Levy [8], let us define the ratio of elastic effective width to initial width as the ratio of the actual load carried by the plate to the load the plate would have carried if the stress had been uniform and equal to the Young's modulus of the matrix material E_M multiplied by the average edge strain ε_1. Here, this ratio is calculated from the following formula:

$$\frac{L_2^0}{L_2} \equiv \tilde{B}^{eff}_{2222} \frac{p_1}{\varepsilon_1}. \tag{17}$$

The graphs in Fig. 4 illustrate the dependence between ratios L_2^0/L_2 and $\varepsilon_1/\varepsilon_{1.crit}$, where the value of critical strain corresponds to the case of no transverse load. The results were calculated for $\varepsilon=E_R/E_M=1.0$, 4.0, $\alpha=0.1$, 0.2. The results calculated for a homogeneous plate ($\varepsilon=1.0$) are identical with those obtained by Levy.

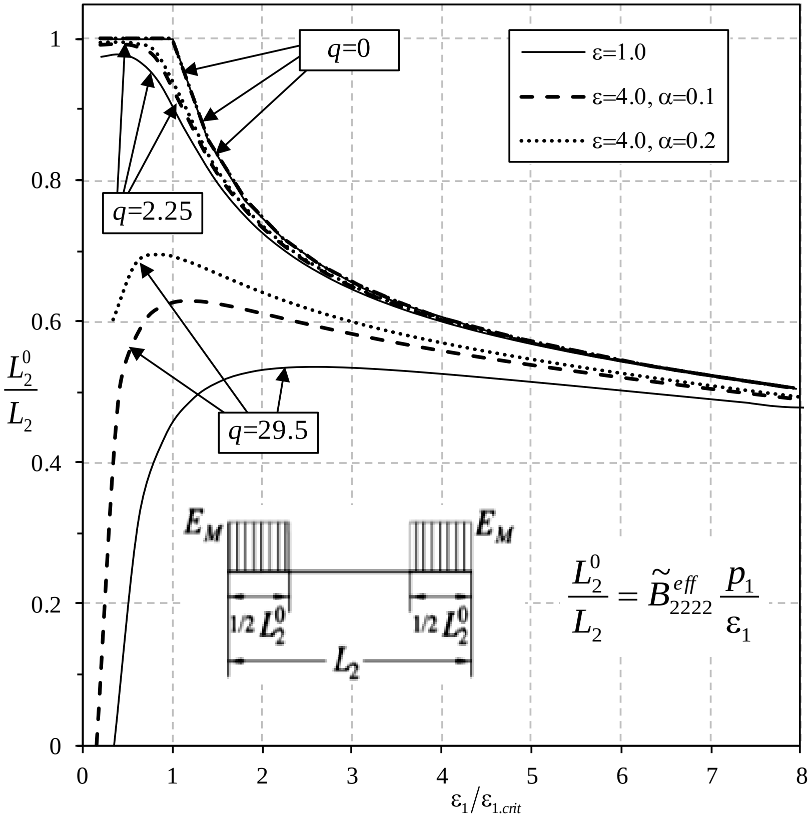

Fig. 4. Effect of transversal load on effective width of a square plate loaded by edge compression

6. REMARKS

Analyzing graphs in Fig. 3 and Fig. 4 it can be observed that:

- for the transversally unloaded plates, the macrodeflection is equal to zero until the inplane load does not exceed the critical value;
- the bifurcation points correspond to the critical values of the load calculated within the linear tolerance model;
- occurence of the transversal load results in a significant quantitative and qualitative differences in the calculated deflections and its maximal influence occurs for the critical values of inplane load;
- increasing values of the parameters α and ε causes: increasing of the critical loads and the effective width of the plate for $q>0$ and decreasing of the calculated deflections.

Furthermore, some general remarks can be formulated:

– applying the tolerance modelling to the known differential equations of thin periodic plates with large deflections the averaged equations of the nonlinear tolerance model are derived.
– this technique makes it possible to replace the governing equations with non-continuous, periodic, highly oscillating coefficients by the system of differential equations with constant coefficients;
– the derived equations of the nonlinear tolerance model involve terms, which take into account the effect of the microstructure size on the overall behaviour of periodic plates;
– the governing equations of the linear tolerance model take into account the effect of the microstructure size only by the term dependent of the oscillating part of the load.

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