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EFFECT OF LIGHT BEAM DEVIATION FROM BRAGG ANGLE ON THE INTENSITY OF LIGHT DIFFRACTED ON ACOUSTIC WAVE

The paper describes the influence of the effect of light beam deviation from Bragg angle on the intensity of light diffracted on acoustic wave. The analysis was based on the results of numerical calculations. The calculations were made for several characteristic cases.

Keywords: Acousto-optics, Bragg regime.

1. INTRODUCTION

Acousto-optic interaction was predicted in 1922 by L.N. Brillouin less than hundred year ago. Since then, thanks to the work of many researchers the interaction between light and acoustic wave is better understood [1]. That fact enabled to construct a lot of devices which exploit acousto-optic effect [2]. Often, the acousto-optic cells are used for deflecting the beam of light. The advantage of such approach is the lack of mechanical parts, so the deflector can work more efficient and robust. However, in many cases, the deflected light beam should not change its intensity when it changes in direction. But when the intensity of the sound wave is constant the intensity of deflected light can change. It takes place because the direction of the beam is changed due to the change in frequency of sound. In turn, variation of sound wavelength causes change of the value of Bragg angle. As a result, the beam no longer meets the Bragg condition in the medium. In this paper we will focus on the influence of such deviation from exact Bragg incidence on the energy exchange between incident beam and diffracted beams.

2. THEORETICAL BACKGROUND OF THE ACUSTO-OPTIC EFFECT

It is commonly known that Maxwell’s equations Eqs. (1-4) can be employed to describe the light propagation in the medium. Below, the equations are used to analyze the interaction between light and ultrasound waves in acousto-optic cell. Presented theoretical approach follows the works [3,4,5].

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j} \tag{1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2}$$

$$\nabla \cdot \vec{D} = \rho \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \tag{4}$$

where $\vec{E}$, $\vec{D}$, $\vec{B}$, $\vec{H}$, $\vec{j}$, ρ denote electric field intensity, electric flux density, magnetic flux density, magnetic field intensity, electric current density and electric charge density, respectively. All these quantities can vary in time and space. Because we will consider only linear and nonmagnetic media the constitutive relations Eqs. (5-6) are true.

$$\vec{D} = \varepsilon_o \varepsilon_r \vec{E} \tag{5}$$

$$\vec{B} = \mu_o \vec{H} \tag{6}$$

where ε_0, μ_0 are universal constants, called the permittivity of free space and permeability of free space, respectively and ε_r is relative permittivity of the medium (material).

Substituting Eqs. (5-6) to Eqs. (1-4) and assuming that $\vec{j}$, ρ are equal to zero Maxwell’s equations can be written in the following form:

$$\nabla \times \vec{H} = \varepsilon_o \varepsilon_r \frac{\partial \vec{E}}{\partial t} \tag{7}$$

$$\nabla \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t} \tag{8}$$

$$\varepsilon_o \nabla \cdot \varepsilon_r \vec{E} = 0 \tag{9}$$

$$\nabla \cdot \vec{H} = 0 \tag{10}$$

Thanks to Eq. (9) which gives:

$$\varepsilon_o \nabla \cdot \varepsilon_r \vec{E} = \varepsilon_o \varepsilon_r \nabla \cdot \vec{E} + \varepsilon_o \vec{E} \cdot \nabla \varepsilon_r = 0 \tag{11}$$

Taking into account that plane wave of sound is propagating in z-directionand light represented by electric field $\vec{E}(x,z,t) = E(x,z,t) \cdot \vec{j}$is polarized along y-direction, one can spotthat $\vec{E} \cdot \nabla \varepsilon_r = 0$ because both vectors are mutually perpendicular. It is more clear when we look at the typical layout of acousto-optic cell which is sketched on Fig. 1.

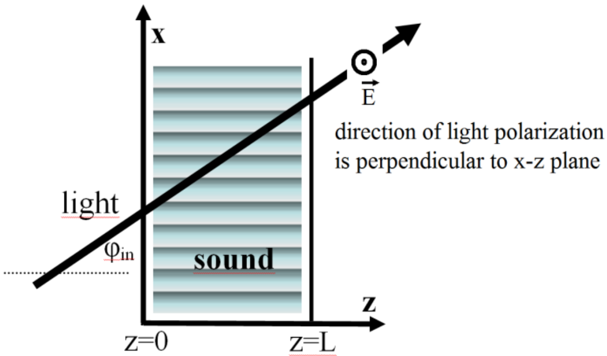

Fig. 1. The typical layout of the acousto-optic cell. The angle of incidence φ_{in}, for clarity, is drawn outside the acousto-optic cell but in fact it is measured within the medium inside the cell

Additionally, taking the curl of Eq. (8) and using both Eq. (7) and well known vector calculus identity:

$$\nabla \times \nabla \vec{E} = \nabla(\nabla \cdot \vec{E}) - \Delta \vec{E} \qquad (12)$$

We obtain the following equation:

$$\Delta \mathrm{E} = \mu_o \varepsilon_0 \frac{\partial^2}{\partial t^2}(\varepsilon_r \mathrm{E}) = \mu_o \varepsilon_0 \left(\mathrm{E} \frac{\partial^2 \varepsilon_r}{\partial t^2} + 2 \frac{\partial \varepsilon_r}{\partial t} \frac{\partial \mathrm{E}}{\partial t} + \varepsilon_r \frac{\partial^2 \mathrm{E}}{\partial t^2} \right) \qquad (13)$$

Because the sound frequency is several orders lower than light frequency, so both first and second terms in the parenthesis can be omitted.

$$\Delta \mathrm{E} = \mu_o \varepsilon_0 \varepsilon_r \frac{\partial^2 \mathrm{E}}{\partial t^2} \qquad (14)$$

It can be assumed that incident light waveform is harmonic against time and distance:

$$\mathrm{E_i}(x, z, t) = \left(\mathcal{E}_i(x, z)\exp(i\omega_0 t) + \mathcal{E}_i^*(x, z)\exp(-i\omega_0 t)\right)/2 \qquad (15)$$

ω_0 – angular frequency of the incident light, asterisk denotes complex conjugate.

The relative permittivity ε_r of the substance in the of acousto-optic cell can be divided on two parts. ε_c is constant and $\varepsilon(x,t)$ is varying in space and time.

$$\varepsilon_r = \varepsilon_c + \varepsilon(x, t) \qquad (16)$$

Variation of $\varepsilon(x,t)$ will be assumed in the similar form as above was assumed $E_i(x,z,t)$. Because relative permittivity ε_r of the substance in acousto-optic cell changes due to propagation of sound S(x)exp(-iΩt). Ω stands forangular frequency of the sound. In fact in such cells ultrasounds are used.

$$\frac{\varepsilon(x,t)}{\varepsilon_c} = \big(S(x)\exp\,(i\Omega t) + S^*(x)\exp\,(-i\Omega t)\big)/2 \tag{17}$$

S(x) can be expressed as follows:

$$S(x) = A \cdot \exp(-iKx) \tag{18}$$

where: K = 1/Λ – wavenumber of the sound,

Λ – wavelength of the sound,

A – a complex constant depends on initial phase and intensity of the sound and also depends on the properties of the acousto-optic cell.

It is worth to predict the total field in the form:

$$E(x,z,t) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left[\mathcal{E}_n(x,z)\exp(i(\omega_0 + n\Omega)t) + \mathcal{E}_n^*(x,z)\exp(-i(\omega_0 + n\Omega)t)\right] \tag{19}$$

Introducing Eq. (17) into Eq. (16) and then together with Eq. (19) into Eq. (13) we obtain infinite system of coupled differential equations:

$$\Delta\mathcal{E}_n(x,z) + k_0^2\mathcal{E}_n(x,z) + \frac{1}{2}k_0^2 S(x)\mathcal{E}_{n-1}(x,z) + \frac{1}{2}k_0^2 S^*(x)\mathcal{E}_{n+1}(x) = 0 \tag{20}$$

where: n is a member of integers,

$k_0 = 1/\lambda_0$ – wavenumber of the light in the acousto-optic cell,

λ_0 – wavelength of the light in the acousto-optic cell.

The incident light wave can be described in the form:

$$\mathcal{E}_{in}(x,z) = \psi_{in} \cdot \exp(-ik_0(z \cdot \cos\varphi_{in} + x \cdot \sin\varphi_{in})) \tag{21}$$

where ψ_{in} is the amplitude of the incident light and φ_{in} is angle of light incidence and it is measured within the medium inside the acusto-optis cell.

The solution to Eqs. (20) we anticipate (by analogy to Eqs. (21)) in the following form:

$$\mathcal{E}_n(x,z) = \psi_n \cdot \exp(-ik_0(z \cdot \cos\varphi_n + x \cdot \sin\varphi_n)) \tag{22}$$

where φ_n satisfies the grating equation:

$$\sin\varphi_n = \sin\varphi_{in} + \frac{nK}{k_0} \tag{23}$$

where Λ = 1/K corresponds to the distance between slits in the classical diffraction grating. After considerable calculation we can obtain:

$$\frac{d\psi_n}{dz} = -\frac{i\cdot k_0}{4\cos\varphi_n}\left[A\cdot\psi_{n-1}\cdot\exp\left(-ik_0 z(\cos\varphi_{n-1}-\sin\varphi_n)\right)\right.$$
$$\left.+A^*\psi_{n+1}\cdot\exp\left(-ik_0 z(\cos\varphi_{n+1}-\sin\varphi_n)\right)\right] \tag{24}$$

where: n – is integer,

ψ_n – is amplitude of diffracted light in n-th order.

The boundary condition for Eq. (24) can be written as follows:

$$\begin{aligned}&\psi_n = \psi_{in} \quad \text{at } z \le 0 \quad \text{for } n = 0\\&\psi_n = 0 \quad \text{at } z \le 0 \quad \text{for } n \ne 0\end{aligned} \tag{25}$$

The relative permittivity ε_r can be expressed by the refractive index n of the medium in acousto-optic cell.

$$\varepsilon_r(x,t) = (n_0 + \Delta n(x,t))^2 \cong n_0^2\left(1+\frac{2\cdot\Delta n(x,t)}{n_0}\right) \tag{26}$$

where: n_0 – the constant part of the refractive index,

Δn – the part of refractive index which is varying in time due to the sound wave.

Because the variations of the refractive index Δn are caused by the sound, as mentioned above, we can write.

$$S(x) = \frac{2\cdot\Delta n(x,t)}{n_0} \tag{27}$$

Denoting the maximum of that variation by Δn_{max}. We can determine the maximum of phase shift α due to the passage of the light through the cell.

$$\alpha = \frac{L\cdot k_0\cdot\Delta n_{max}}{n_0} = \frac{L\cdot|A|\cdot k_0}{2} \tag{28}$$

L is shown in Fig. 1.

Expanding the terms in Eq. (24) which contain trigonometric functions in Taylor series and employing Eq. (23) we obtain the final form of Eq. (24)[6].

$$\frac{d\psi_n}{dz} = -\frac{i\cdot\alpha\cdot L}{2}\cdot\left[\begin{array}{c}\psi_{n-1}\cdot\exp\left(-\frac{iQz}{2L}\left(\frac{\varphi_{in}}{\varphi_B}+(2n+1)\right)\right)+\\ +\psi_{n+1}\cdot\exp\left(\frac{iQz}{2L}\left(\frac{\varphi_{in}}{\varphi_B}+(2n+1)\right)\right)\end{array}\right] \tag{29}$$

where: φ_{in} – angle of incidence is shown in Fig. 1,

φ_B – Bragg angle which satisfies following equation: $\sin\varphi_B = \frac{K}{2k_0}$,

n – diffraction order.

Q is called Klein-Cook parameter and equals:

$$Q = \frac{K^2 \cdot L}{k_0} \tag{30}$$

We can introduce parameter δ which describes the deviation of the incident light beam away from the Bragg angle.

$$\varphi_{in} = -(1 + \delta)\varphi_B \tag{31}$$

Δ = 0 means that light reaches acousto-optic cell exactly at Bragg angle.

3. RESULTS OF NUMERICAL CALCULATIONS AND CONCLUSIONS

Numerical calculations were made based on Eqs. (29). Fourteen amplitudes ψ_n of diffracted light beam was taken into account (n = {-6,-5,…,0…,6,7}). It was assumed that the remaining orders of Bragg diffraction are negligible. To assess the efficiencyof energy exchangebetween diffracted beams the following expression was calculated $\left(\frac{|\psi_n|}{|\psi_{in}|}\right)^2$.

While performing all the calculations necessary to plot Fig. 2 it was assumed that parameter Q = 100. That means, that acousto-optic cell operates in Bragg regime.

Fig. 2a) presents what part of energy of incident beam is transferred to 0-th order of Bragg diffraction. So $\left(\frac{|\psi_0|}{|\psi_{in}|}\right)^2$ is plotted as a function of parameters αand δ. In turn Fig. 2b) presents what part of energy of incident light beam is transferred to 1-st order of Bragg diffraction. Additionally, since parameter α depends inter alia on sound intensity it is worth to note that for α = π/2, the light can be efficiently modulated in both 0-th and 1-st order of diffraction.

Comparing the two graphs shows that a strong increase in δ reduces the intensity of light diffracted to 1-th order. Fig. 2c) shows cross section| of Figs. (2a-2b) in δ direction at α = sπ. Fig. 2c) confirms the observation that the increase in δ causes the loss of energy in the light diffracted in 1-th order. Similar information are obtained from the analysis of Fig. 2d).

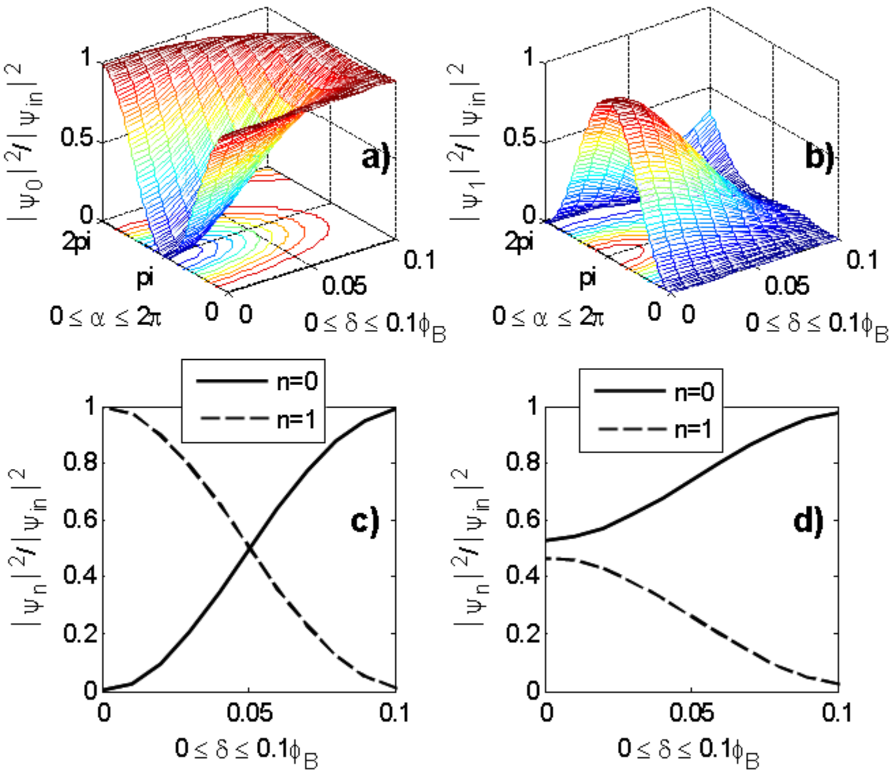

Fig. 2a) shows what part of incident light energy is received by light in 0-th order of diffraction versus two parameters δ and α, b) shows what part of incident light energy is received by light in 1-st order of diffraction versus two parameters δ and α. The Klein-Cook parameter Q = 100, c) shows a plot of the cross section of the pictures a) and b) in direction δ at α = π, d) shows a plot of the cross section of the pictures a) and b) in direction δ at α = π/2s

Fig. 3 and Fig. 4 show that for smaller Q much more energy is transferred into higher orders of diffraction (2-nd order is additionally shown). If Q parameter slightly exceeds 1 it can be said that acousto-optic cell operates in near Bragg regime.

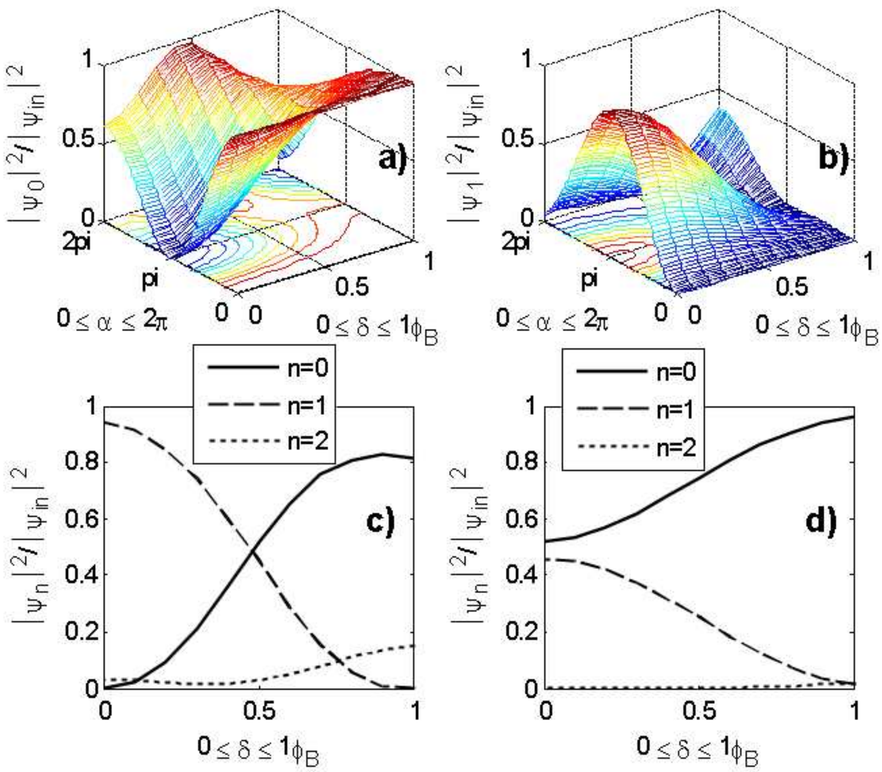

Fig. 3a) shows what part of incident light energy is received by light in 0-th order of diffraction versus two parameters δ and α, b) shows what part of incident light energy is received by light in 1-st order of diffraction versus two parameters δ and α. The Klein-Cook parameter Q = 10, c) shows a plot of the cross section of the pictures a) and b) in direction δ at α = π, d) shows a plot of the cross section of the pictures a) and b) in direction δ at α = π/2

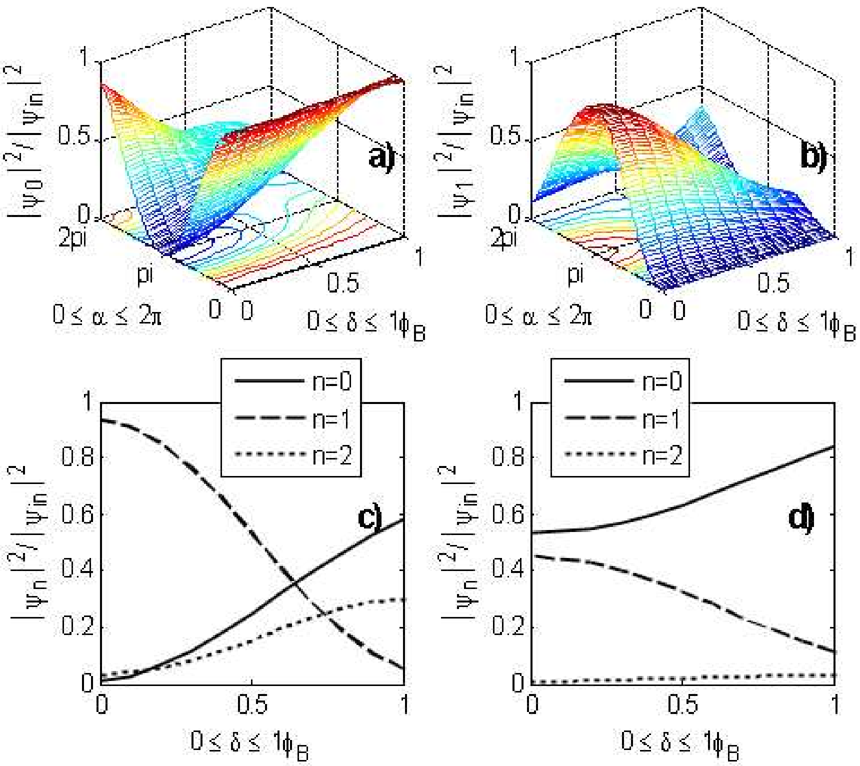

Fig. 4a) shows what part of incident light energy is received by light in 0-th order of diffraction versus two parameters δ and α, b) shows what part of incident light energy is received by light in 1-st order of diffraction versus two parameters δ and α. The Klein-Cook parameter Q = 7, c) shows a plot of the cross section of the pictures a) and b) in direction δ at α = π, d) shows a plot of the cross section of the pictures a) and b) in direction δ at α = π/2

Analysis of all the graphs discussed so far shown that for small values of Q cell becomes less sensitive to changes in δ. To describe above observation quantitatively parameter δ_{FWHM} is introduced. This parameter is defined by the deviation δ at which diffracted light in n-th order receives half of its peak available energy. The change the parameter δ_{FWHM} as a function of Q is shown in Fig. 5.

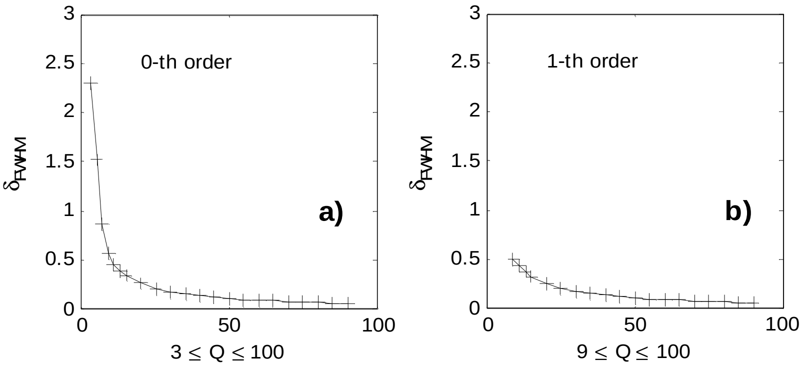

Fig. 5. The change of the parameter δ_{FWHM} as a function of Q. Right graph starts with a larger value of Q, since the beginning of the curve has a different shape. It is caused by the fact that higher diffracted orders receive energy from incident light beam. This is clearly visible in Fig 4

Since the change of the parameter δ_{FWHM} is nonlinear the Fig. 6 presents reciprocal of δ_{FWHM} as a function of Q.

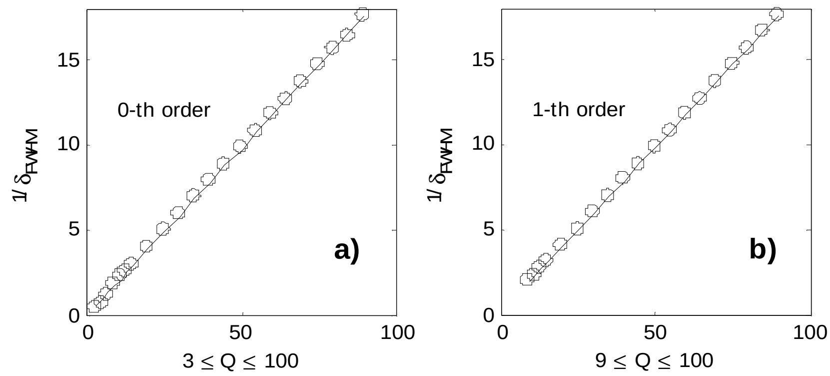

Fig. 6. The change of the $1/\delta_{FWHM}$ as a function of Q

The linearity of the graphs in Fig. 4 indicates that δ_{FWHM} is inversely proportional to Q. The above analysis may be useful in the design of light deflectors. It should be noted, however, that the deviation of the incident beam δ was measured, for convenience, in multiples the Bragg angle rather than sin radians.

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WPŁYW ODCHYLENIA WIĄZKI ŚWIATŁA OD KĄTA BRAGGA NA NATĘŻENIE UGIĘTEGO ŚWIATŁA NA FALI AKUSTYCZNEJ

Streszczenie

W pracy opisano wpływ odchylenia wiązki światła od kąta Bragga na natężenie ugiętego światła na fali akustycznej. Analizy dokonano w oparciu o wyniki obliczeń numerycznych. Obliczeń dokonano dla kilku charakterystycznych przypadków.