JOURNAL OF APPLIED
COMPUTER SCIENCE
Vol. 17 No. 2 (2009), pp. 7–15

Particle Swarm Optimization: the Gradient Correction

Bożena Borowska[1], **Stefan Nadolski**[2]

[1]*Institute of Information Technology, Technical University of Łódź*
Wólczańska 215, 90-924 Łódź, Poland
bozena@ics.p.lodz.pl

[2]*Division of Computer Networks, Technical University of Łódź*
Stefanowskiego 18/22, 90-924 Łódź, Poland
snadolski@zsk.p.lodz.pl

Abstract. *In the following paper, the solution of the optimization problem that relied on the determination of the optimal geometry of two cylindrical coil arrangement evoking magnetic field of specific parameters was presented. The objective of the task was to generate, in a defined active area, the magnetic field of the largest possible gradient and simultaneously keep this gradient relatively constant. The computations were performed using the classical particle swarm optimization as well as the modified method with the correction sensitive to the fitness function gradient introduced to the formula describing the movement of the specific swarm particles. As a result, a considerable enhancement of the optimization process was achieved.*
Keywords: *Electromagnetism, optimization, evolutionary algorithms, Particle Swarm Optimization.*

1. Introduction

Evolutionary algorithms such as genetic algorithms GA or particle swarm optimization method PSO have found an application for solving several optimization problems in electromagnetism [1, 2]. The optimization relies on searching the minimum of a fitness function, which is a measure of the solution quality in a search space [1, 2, 3]. Obviously, a method of the fitness function designing depends on the problem that is expected to be solved.

In an optimization of source parameters (coils or their arrangements) of the static magnetic fields, there is a necessity to determine three-dimensional distributions of these fields. This leads to emerge very complex mathematical relationships, which results in extending the numerical computation time [4]. This problem can be simplified when the cylindrical symetry coil is taken into consideration as a magnetic field source.

In this paper a relatively simple coil arrangement were chosen in order to test the application of the modified PSO algorithm for solving a real physical optimization problem.

We consider a set of two identical coils forming the cylindrical symetry arrangement. The z-axis is a symetry axis of the arrangement, whereas the xy-plane represents its symetry plane. The coil cross section sides are 2a and 2b. The distance between the coil symetry planes and the xy-plane is Z_0, whereas the average radius of the coils is R_0 (Fig. 1).

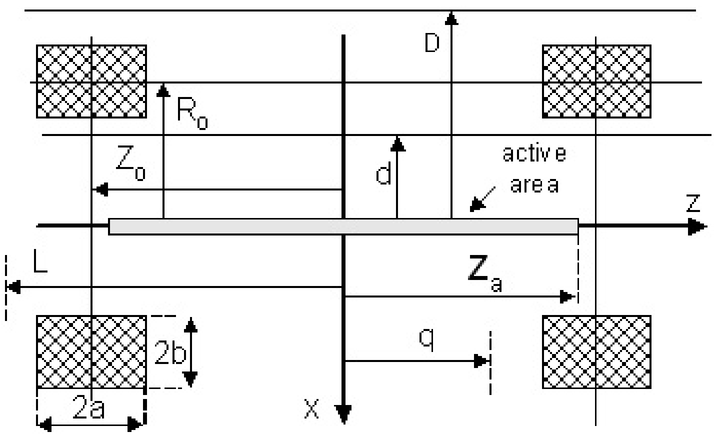


Figure 1. The cross section of the coil arrangement generating magnetic field with the controlled gradient

If the currents of the opposite directions flow through the coils, the magnetic field in the center of the symetry is H(0,0,0) = 0.

According to [4], the magnetic field along the z-axis is as follows:

$$H(0,0,z) = \frac{J_0}{2}\left(\begin{array}{c}\left(\begin{array}{c}(a{-}z{-}Z_0)ln\left[\frac{(R_0+b)+((R_0+b)^2+(a-z-Z_0)^2)^{1/2}}{(R_0-b)+((R_0-b)^2+(a-z-Z_0)^2)^{1/2}}\right]+\\ (a{+}z{+}Z_0)ln\left[\frac{(R_0+b)+((R_0+b)^2+(a+z+Z_0)^2)^{1/2}}{(R_0-b)+((R_0-b)^2+(a+z+Z_0)^2)^{1/2}}\right]\end{array}\right)-\\ \left(\begin{array}{c}(a{-}z{+}Z_0)ln\left[\frac{(R_0+b)+((R_0+b)^2+(a-z+Z_0)^2)^{1/2}}{(R_0-b)+((R_0-b)^2+(a-z+Z_0)^2)^{1/2}}\right]+\\ (a{+}z{-}Z_0)ln\left[\frac{(R_0+b)+((R_0+b)^2+(a+z-Z_0)^2)^{1/2}}{(R_0-b)+((R_0-b)^2+(a+z-Z_0)^2)^{1/2}}\right]\end{array}\right)\end{array}\right) \quad (1)$$

where J_0 represents density of the current flowing within the coil.

The optimization problem relies on finding the parameters a, b, R_0, Z_0 in such a manner to achieve the largest possible gradient of the magnetic field in an active area $2Z_a$ and to maintain simultaneously the maximal possible stability of this gradient. We consider the following additional parameters:
J_0=250 A/m^2, d=0.25m, D=0.6m, Z_a=0.7m, q=0.4m, L=1m and take into account geometrical constraints as follows: $R_0 + b \leq D$, $R_0 - b \geq d$, $Z_0 - a \geq q$, $Z_0 \leq$ L, and $abR_0 \leq 0.006$m^3. The last condition limits the maximal amount of material used for the coil construction.

The fitness function is defined according to the following formula:

$$F = \frac{1000G}{|H(0,0,z=Z_a/2)|^k} = \frac{1000\left[\left(\frac{4}{3}|H(0,0,z=0.75Z_a)|-|H(0,0,z=Z_a)|\right)^2+2(2|H(0,0,z=Z_a/4)|-|H(0,0,z=Z_a/2)|)^2\right]^{1/2}}{|H(0,0,z=Z_a/2)|^k|H(0,0,z=Z_a)|} \quad (2)$$

The factor G represents the field gradient stability, which is equalled to zero when the field gradient in the active area is constant. The factor $k = 0.15$ determines the priority of the field gradient quantity with reference to its stability. On the other hand, the fitness function F denominator is proportional to the field gradient in the active area. A minimization of the fitness function enables to solve the optimization problem expressed above.

2. Principle of the PSO algorithm operation

The Particle Swarm Optimization is a stochastic, evolutionary computational technique applied for solving complex optimization problems. It was developed in 1995 by Kennedy and Eberhard [5]. The PSO was inspired by a metaphor of the social behavior of animals that organize themselves in groups such as flock of birds, swarm of bees or shoal of fish. The exploration of the search space oriented to the area of the best fitted particles as well as utilizing the information of current achievements in a searching process is a basic control mechanism of the PSO technique. The main advantages of the PSO method include: a relatively low number of parameters used for the process optimization control, a convenient representation of the solutions and an unsophisticated method of transformation of one solution into another.

The PSO optimization process itself starts with generating the initial population of particles in the N-dimensional search space. The position of each particle is randomly generated within the range $< x_{min}, x_{max} >$ of the search space for every dimension. The initial value of the velocity vector is also randomly chosen. A measure of the goodness of a given solution represented by the particle is a fitness function. On its basis, each particle remembers its own best location pbest at which this particle has achieved the highest fitness value. For the entire swarm, among all the best locations pbest there is one particle of the highest fitness named gbest. At every successive iteration, the particles update the pbest values of their best current positions, and the gbest value for the entire swarm is also updated. The particle movement proceeds along the velocity vector, which determines the movement direction and the path length of the particles as follows:

$$v_{n+1} = wv_n + c_1 r_1 (pbest_n - x_n) + c_2 r_2 (gbest_n - x_n) \tag{3}$$

The new particle location is a function of a newly determined velocity and its previous position according to the following formula:

$$x_{n+1} = x_n + v_{n+1} \tag{4}$$

where w is inertial weight that determines the deviation of the particle original direction, c_1 and c_2 are acceleration factors that determine respectively how much the particle is influenced by the memory of its best location and by the rest of the swarm, whereas r_1 and r_2 represent randomly generated numbers in the range (0,1).

3. The new algorithm

In case of large, multidimensional optimization problems, an application of the standard PSO algorithm does not guarantee that the optimal solution will quickly be found. Moreover, it is expected that the algorithm will be converging slower and the computational expenditures will be much higher. The computational time will additionally increase when the optimizing search space contains many local solutions.

Chowdhury et. al. [6] proposed a hybrid method that combines gradient method with dynamic tunning method resulting in an efficient approach. Noel and Jannett [7] used a hybrid method, which incorporates gradient information into the velocity updating formula of each particle.

In order to improve the effectiveness of the standard PSO algorithm, and thereby accelerate the optimization process, a development of the original algorithm was introduced, and that was inspired by papers [6, 7] mentioned above. These changes refer to the equations of the particle velocity vector updating as well as to the way of the search space exploration. In the new algorithm, a Fletcher-Reeves gradient method was adopted. In this approach, the searching process is performed in the conjugate direction system [8, 9], which represents a combination of the gradient vector and the previous descent step vector. In the initial phase, the new algorithm (FR-PSO) makes use the combination of the original PSO with the equation (1) in order to find the first approximate local minimum. The best position of the swarm particle (gbest) obtained in this way becomes a starting point for the Fletcher-Reeves method.

The solutions achieved in this process are incorporated, as an additional component, into the equation of the velocity vector updating, which determines the movement direction and the path length of particles:

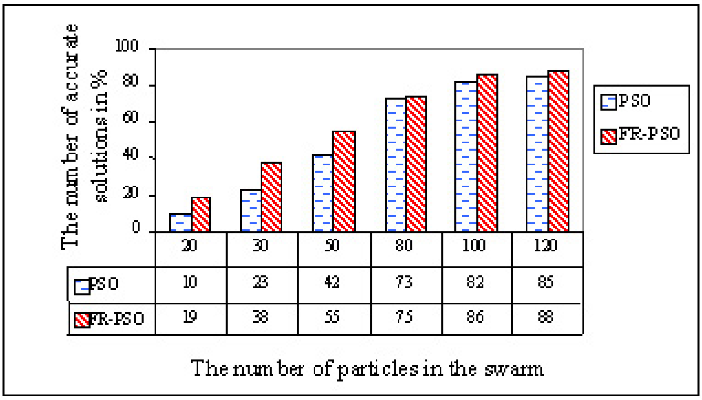
$$v_{n+1} = wv_n + c_1r_1(pbest_n - x_n) + c_2r_2(gbest_n - x_n) + c_3r_3(mfr_n - x_n) \quad (5)$$

which improves the algorithm performances. The necessity of additional computation of the gradient is compensated by much faster convergence of the algorithm to the minimum with lower number of iterations. The mfr_n factor introduced to the formula (5) represents the distance between the particle position and the position of the solution obtained by the Fletcher-Reeves method.

4. Results

The study on an effectiveness of the proposed method used to determine an optimal geometry of the coil arrangement was undertaken by means of a program written in C++. The tests were performed in order to evaluate the algorithm performance in terms both of the convergence velocity and the capability of finding global optimum. The results were then compared with the achievements of the standard PSO algorithm. All the computations were made for 1000 computations.

The exemplary results of the tests performed for 20, 30, 50, 80, 100, and 120 particles in the initial population are depicted in Fig. 2. All the values were averaged over 100 trials for each combination of the parameters.



	20	30	50	80	100	120
PSO	10	23	42	73	82	85
FR-PSO	19	38	55	75	86	88

Figure 2. The number of accurate solutions versus the swarm cardinality for PSO and FR-PSO

It was reported, that the best results were achieved for large swarms. In case of 100 and more particles, each investigated algorithm was able to find more than 80 % optimal solutions. The new algorithm turned out to be more effective than standard PSO since it could find a few percent more accurate solutions within a considerably lower iteration number (Table 1).

The results were poor for small swarms comprising of only a few particles. No optimal solution was managed to obtain for the swarm containing 10 or fewer particles. For the population comprising 20 particles, there were only 19 successful trials out of 100 that the FR-PSO algorithm found optimal solutions, but the convergence time was very long. For the same initial parameters, the PSO algorithm

Table 1. The relationship between the population cardinality and the number of iterations to achieve the accurate solutions for the standard PSO and FR-PSO

Algorithm	**The number of particles in the swarm**						
	10	**20**	**30**	**50**	**80**	**100**	**120**
PSO	0.0	601.3	646.71	561.8	527.02	453.57	376.31
FR-PSO	0.0	303.67	296.84	245.02	191.52	189.89	183.87

found only 10 % accurate solutions, and the number of iterations was twice larger.

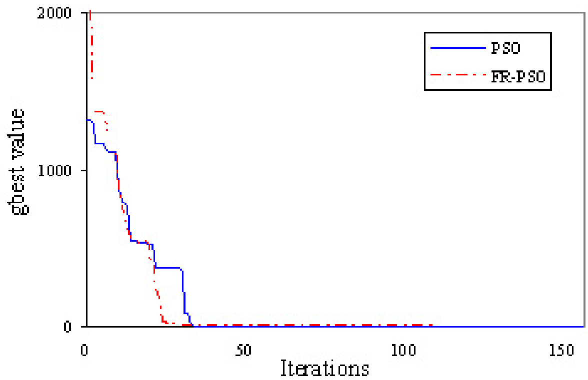


Figure 3. The average values of best solutions (gbest) in the following iterations for the population of 100 individuals

It was also discovered that the new algorithm gave more accurate solutions, and the exploration of the search space was faster and more precise. The average gbest values for 100 trials in the 150 initial iterations are presented in Fig. 3. The final part of the following study was to find optimal parameters of fitness, which are a=0.5254, b=0.0182, R_0=0.5818 and Z_0=0.9999. A distribution of magnetic field along the z-axis is depicted in Fig. 4, and this was plotted using the relationship (1) and the optimal parameters mentioned above.

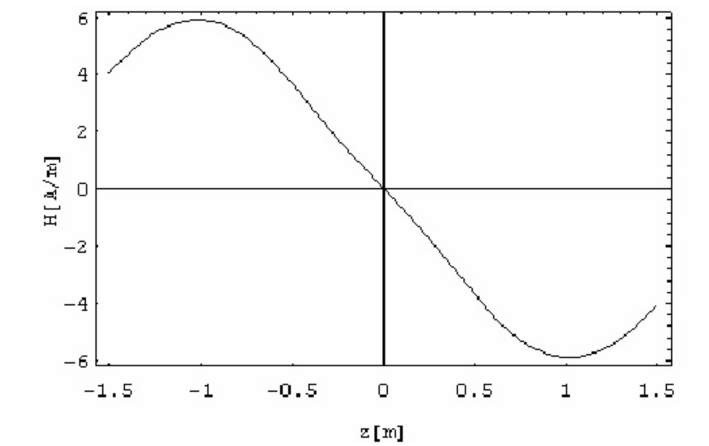


Figure 4. The distribution of magnetic field along the z-axis achieved for the optimal fitness parameters

5. Summary

In this work, a new optimization algorithm that combines the PSO algorithm with the Fletcher-Reeves method was proposed. The alterations referred to both equations for the particle velocity vector updating and methods of the search space exploration. The aim of the algorithm was to determine the optimal geometry of the coil arrangement that evoked the specific magnetic field. The experiments showed that the proposed FR-PSO algorithm is more efficient that PSO both in terms of the convergence velocity and the capability of finding global optimum. Furthermore, the results with the combined FR-PSO method are more accurate than the ones obtained using classical PSO.

The following work presents a continuation of the previous investigations performed by the authors [10, 11].

References

[1] Vasconcelos, J. A., Ramirez, J. A., Takahashi R. H. C. and Saldanha R. R., *Improvements in Genetic Algorithms*, IEEE Trans. Magn., Vol. 37, pp. 3414-3417, 2001.

[2] Robinson, J., and Rahmat-Samii, Y., *Particle Swarm Optimization in Electromagnetics*, IEEE Trans. Antennas and Propagation, Vol. 52, pp. 397-407, 2004.

[3] Sato, S., Sakaguchi, S., Futamata, K. and Katou, K., *Coil Optimization for Homogeneous Magnetic Field with Small Leakage Field,*, IEEE Trans. Magn., Vol. 36, pp. 649-653, 2000.

[4] Forbes, L. K., Crozier, S., Doddrell, M., *Rapid Computation of static Fields Produced by Thick circular solenoids*, IEEE Transaction on Magnetics, Vol. 33, pp. 4405-4410, 1997.

[5] Kennedy, J., Eberhart, R., *Particle swarm optimization*, IEEE International Conference on Neural Networks, Perth, Australia, pp. 1942-1948, 1995.

[6] Chowdhury, P. R., Singh Y. P., Chansarkar R. A, *Hybridization of Gradient Descent Algorithms with Dynamic Tunneling Methods for Global Optimization*, IEEE Transactions on Systems, Man and Cybernetics, Vol. 30, No. 3, pp. 384-390, 2000.

[7] Noel, M., Jannett, T. C., *Simulation of a New Hybrid Particle Swarm Optimization Algorithm*, Proceedings of the Thirty-Sixth Southeastern Symposium on System Theory, pp. 150-153, 2004.

[8] Findeisen, W., Szymanowski, J., Wierzbicki, A., *Teoria i metody obliczeniowe optymalizacji*, Wydawnictwo Naukowe PWN, Warszawa, 1980.

[9] Ostanin, A., *Metody i algorytmy optymalizacji*, Wydawnictwo Politechniki Białostockiej, Białystok, 2003.

[10] Borowska, B., Nadolski, S., *Application of the PSO Algorithm with Subdomain Approach for the Optimization of a Radio Telescope Array*, Journal of Applied Computer Science, Vol. 16, No. 1, pp. 7-14, Łódź, 2008.

[11] Borowska, B., Nadolski, S., *Genetic Algorithms in Electromagnetism: The Helmholtz Coils*, Journal of Applied Computer Science, Vol. 14, No. 2, pp. 109-118, Łódź, 2006.