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MODELLING AND OPTIMIZATION OF ELECTROSTATIC MEMBRANE-BASED ACTUATORS

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The paper presents methods of electrostatic membrane-based actuators modelling. Based on a combination of the classical and reduced model, a simplified approach fully describing the membrane behaviour under hydrostatic and electrostatic load for various conditions, is proposed. The obtained results have been verified experimentally for various shapes of membranes. Furthermore, a new complex tool for simulation and optimization has been developed.

1. INTRODUCTION

The use of MEMS devices has grown rapidly in last decades. They are used in various markets e.g. automotive, industrial, consumer, military. The most promising domain concerns the biomedical application, in which a variety of sensor types and actuators are commonly used. Due to these facts, a proper design method is very important for the project. It affects crucial factors such as project duration, cost and even its success. Thus, the main step performed in the project, i.e. simulations, should be fast and precise enough. In case of electrostatic actuator, the calculations are complicated due to nonlinear nature of electrostatic attraction. Therefore, the use of an analytical method of modelling instead of commonly used FEM simulations seems to be a good alternative. Consequently, the paper presents a new analytical approach to modelling and optimization of electrostatic membrane-based actuators.

2. MODELLING OF ELECTROSTATIC MEMBRANE-BASED ACTUATOR

Mechanical behaviour of a thin plate ($10 < a/h < 80$, named later 'a membrane') is given by Newton-Lagrange equation [1]:

$$\mathrm{D}\Delta\Delta\big(\mathrm{w}(\mathrm{x},\mathrm{y})-\mathrm{w}_0(\mathrm{x},\mathrm{y})\big)+\sigma_0\mathrm{h}\Delta\mathrm{w}(\mathrm{x},\mathrm{y})=\mathrm{P} \tag{1}$$

where a is a membrane length, h is a membrane thickness, w is a total membrane deflection, w_0 is an initial membrane deflection, P is an applied pressure, σ is a residual stress, α is a coefficient of anisotropy, v is a Poisson ratio, E is a Young's modulus, D_0 is a membrane rigidity, D is a flexural rigidity given by:

$$D = D_0 h^3, \qquad D_0 = \frac{E}{12\left(1-v^2\right)} \tag{2}$$

Δ is a Laplace operator and $\Delta\Delta$ is an operator as follows:

$$\Delta\Delta = \frac{\partial^4}{\partial x^4} + 2\alpha\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

Equation (1) is solved using Galerkin [2] method taking the following membrane form [3]:

$$w(u,v) = \left(1-u^2\right)^2\left(1-v^2\right)^2 \sum_{i,j=0}^{n} K_{ij} u^{2i} v^{2j} \tag{3}$$

where u and v are coordinates normalized to membrane length a and width b. The dependence between the applied pressure and a membrane deflection is linear. Thus, the model can be reduced into linear form:

$$k_{mem} w_{\max} = P + P_{w0_\max}, \qquad k_{mem} = C_1 \frac{D}{b^4} + C_2 \frac{\sigma_0 h}{b^2} \tag{4}$$

where C_1 and C_2 are constants dependent on ratio $R=a/b$ and C_2 on coefficient of anisotropy also. Since this model operates on maximal membrane deflection w_{max} only, it does not give any information about the membrane form. Moreover, the model is valid for a specific range of residual stress.

In case of electrostatic actuation of the membrane [4], equation (1) takes the following form:

$$D\Delta\Delta\big(w(x,y) - w_0(x,y)\big) + \sigma_0 h \Delta w(x,y) = P + \varepsilon \frac{V^2}{2\big(d - w(x,y)\big)^2} \tag{5}$$

where V is an applied voltage and d is a distance between the membrane and the bottom electrode. Equation (5) is solved in the same way as equation (1). However, it needs an iterative procedure (electrostatic force depends on membrane position), in which the following integral has to be calculated numerically [5]:

$$\iint_{\Omega} \frac{1}{(d-w)^2} \varphi_{i,j} du dv \tag{6}$$

Therefore, the use of the reduced model is desirable in order to shorten calculation time:

$$k_{mem} w_{\max} = P_h + \varepsilon \frac{V^2}{2(d - w_{\max})^2} \tag{7}$$

Simulations showed that this model is not correct in whole range of applied voltage (Fig. 1a). To correct this model, two additional coefficients A and B were introduced into equation (7), which depends on membrane ratio R and material properties:

$$k_{mem} w_{\max} = P_h + B\varepsilon \frac{V^2}{2(d - Aw_{\max})^2} \tag{8}$$

Moreover, the from of characteristic $w = f(V)$ does not fit the real one and has to be corrected by a function showed in (Fig. 1).

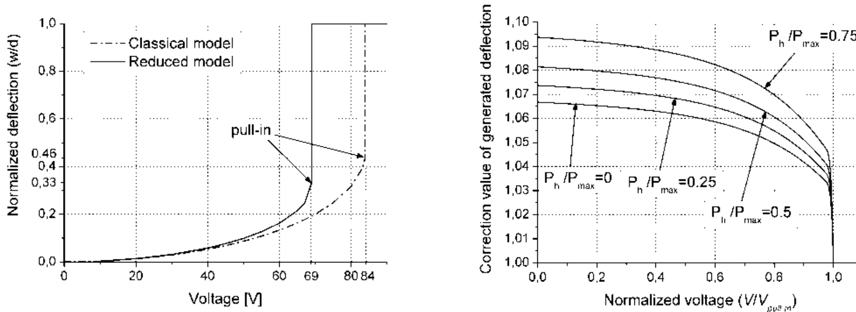

Fig. 1. a) comparison of reduced and classical model, b) function correcting the form of characteristic $w = f(V)$

FEM simulations showed that the corrected model gives accurate results and that calculation time is much shorter than for a FEM simulator (approximately 50 ms instead of 6 minutes needed to trace a full characteristic of the actuator).

3. OPTIMIZATION AND STATISTICAL APPROACH

An optimization phase is very important because it affects the project success (cost, duration, etc.). This step should contain precise simulations, which would not be time consuming. Therefore, the use of the corrected reduced model is a good alternative. The optimization loop contains then only fast calculations and only one FEM simulation is needed to verify the results (Fig. 2).

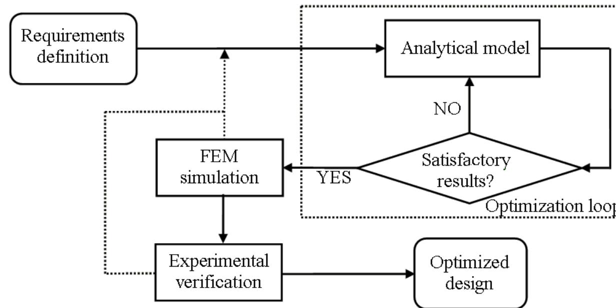

Fig. 2. Optimization phase

The reduced model may be used also in a statistical simulation in order to characterize a technological process. The optimization phase gives us the information about optimal design. However, it is not possible to fabricate such device as the technological process has some dispersion due to inaccuracy of machines and human errors. It may affect the device performance significantly and result in a decrease of yield production. Therefore, a fast statistical simulation giveing the information about the device performance before the fabrication, may be useful in finding project weaknesses (Fig. 3).

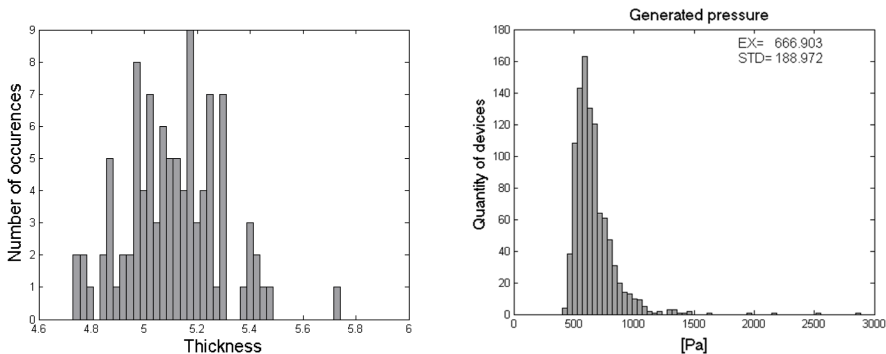

Fig. 3. a) Dispersion in membrane thickness (measurements of SOI thickness made by a vendor) and b) its influence on generated pressure

4. FABRICATION AND CHARACTERIZATION

The fabrication of test structures was performed using classical technological process [6]. The main steps were: oxidation of Si wafers and cavity etching, bonding with SOI wafers and creating contacts on topside. The use of Si wafers as a substrate eliminates the necessity of electrode deposition on cavity bottom. The SOI wafers simplify the membrane fabrication and define the proper thickness of the membrane (SOI layer forms the membrane).

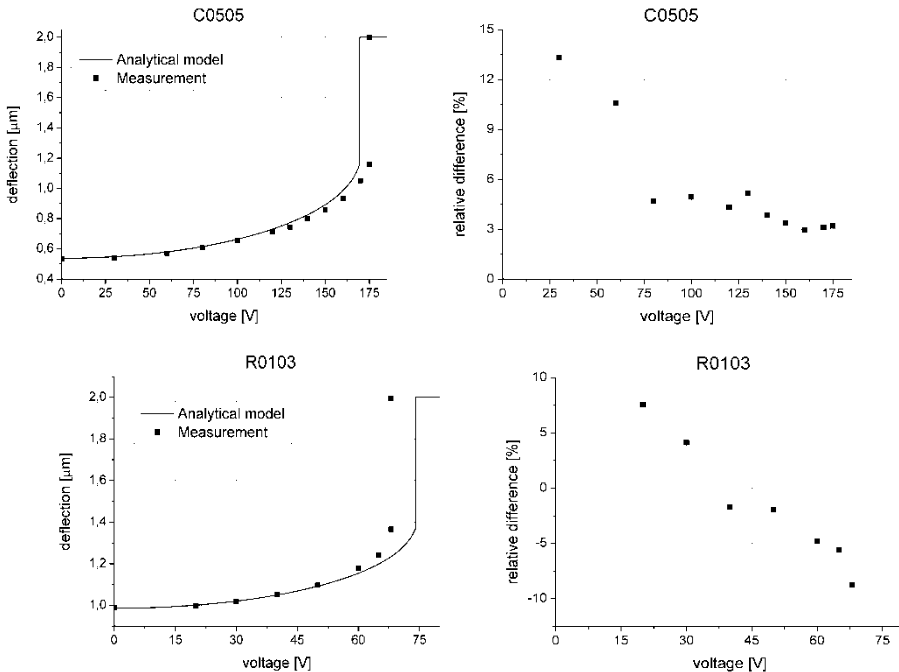

Fig. 4. Exemplary membrane deflection in function of applied voltage (left) and corresponding relative difference between analytical model and measurements (right) for square (top) and rectangular (bottom) membranes

The characterization required the following steps: measurement of membrane initial deflection in vacuum (vacuum inside the cavity), measurement of membrane response to the hydrostatic pressure in order to estimate the value of residual stress, measurement of membrane response to applied voltage. In our experiment square and rectangular membranes, fabricated in different conditions, were measured. A membrane initial deflection varied in range of -310 nm to 250 nm showing that fabrication process was not very reproductive (bonding especially) and that membrane parameters depended on its position on the wafer. Residual stress was quite reproductive and was about -25 MPa and

-50 Mpa for square and rectangular membrane, respectively. The measurements under applied voltage verified successfully the proposed analytical model. The difference did not exceed 20%, even though the analytical model has some limitations (the most important is ideal clamping).

5. CONCLUSIONS

In this paper:

1. Analytical models of electrostatic membrane-based actuators have been studied in terms of accuracy and calculation time.
2. The existing reduced model has been corrected to be fully applicable for electrostatic load.
3. The corrected model has been used in optimization of an actuator and in statistical simulation of technological process. Dedicated tools have been written in MATLAB® environment.
4. The developed model has been successfully verified by the conducted experiment.

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MODELOWANIE I OPTYMALIZACJA MIKROSYSTEMÓW STEROWANYCH SIŁĄ ELEKTROSTATYCZNĄ OPARTYCH NA MEMBRANIE

Streszczenie

W niniejszym artykule zostały przedstawione metody modelowania aktuatorów sterowanych elektrostatycznie, opartych na membranie. Zaproponowane

podejście, które w pełni opisuje zachowanie membrany pod wymuszeniem hydrostatycznym i elektrostatycznym dla wielu założeń, oparte jest na połączeniu modelu klasycznego i zredukowanego. Wyniki zostały potwierdzone eksperymentalnie dla różnych kształtów membran. Dodatkowo zostały stworzone narzędzia do kompletnej symulacji i optymalizacji.

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