

## Dynamic analysis of functionally graded sandwich shells resting on elastic foundations

SHMATKO TETYANA<sup>1\*</sup>, KURPA LIDIYA<sup>2</sup>, AWREJCEWICZ JAN<sup>3</sup>

1. Department of Higher Mathematics, National Technical University "KhPI", Kharkov, Ukraine [ORCID 0000-0003-3386-8343].
2. Department of Applied Mathematics, National Technical University "KhPI", Kharkov, Ukraine [ORCID 0000-0001-8380-1521].
3. Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, Lodz, Poland [ORCID 0000-0003-0387-921X]

\*Presenting author (ktv\_ua@yahoo.com)

**Abstract:** Free vibrations of sandwich shallow shells resting on elastic foundations are investigated. It is assumed that shell consists of three layers of the different thickness. Different schemes of arrangement of layers are considered. Namely, the core is made of ceramics or metals, and the upper and lower layers are made of FGM. The volume fractions of metal and ceramic are described by the power law. Shear deformation shell theory of the first (FSDT) that includes interaction with elastic foundations is applied. To study shells with an arbitrary plan form the R-functions theory combined with the variational Ritz method are used. Validation of the proposed method and developed software has been examined on test problems for FG shell with rectangular plan form. New results for shallow shells with complex plan form were obtained. Effects of the power law index, boundary conditions, thickness of core and face sheet layers, elastic foundations on fundamental frequencies are studied in this work.

**Keywords:** FGM shell, R-functions, Ritz's method, elastic foundation.

### 1. Introduction

Functionally graded materials (FGM) are one of advanced inhomogeneous composite materials. Usually they are fabricated from a mixture of metal and ceramics. This combination of material constituents makes the design lighter, provide heat resistant and strength of a construction very well [1, 2]. Among the different types of functionally graded (FG) structures FG sandwich structures resting on elastic foundation have shown its advantages in mechanical strength, thermal high-gradient insulation, and lightweight demands. By literature review we conclude that numerical of studies devoted to FG sandwich shallow shells resting on elastic foundation is very limited. As for shells with a complex plan shape, the authors are not aware of such works. In this paper functionally graded sandwich shallow shells supported by elastic foundations are considered by application RFM [3, 4] approach.

### 2. Formulation of the problem

We will consider the sandwich shallow shells with FGM face –sheets and ceramic core (model A) or metal core (model B). Denote the total thickness of the shells by  $h$ , thickness of face–sheet by  $h_f$  and thickness of core (central layers) by  $h_c$  (Fig. 1, Fig. 2).

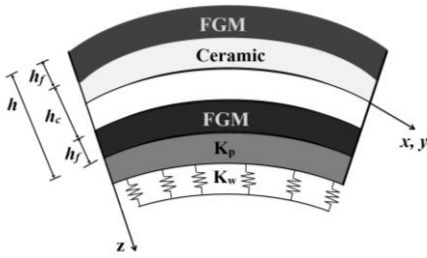


Fig. 1. (Model A)

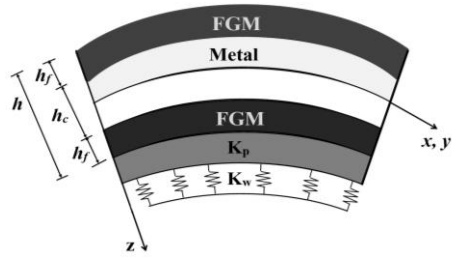


Fig. 2. (Model B)

The effective material properties: elastic modulus  $E$ , Poisson's ratio  $\nu$ , and the density  $\rho$  of the FGM are defined by the following relations

$$E = (E_c - E_m)V_c + E_m, \quad \nu = (\nu_c - \nu_m)V_c + \nu E_m, \quad \rho = (\rho_c - \rho_m)V_c + \rho_m. \quad (1)$$

Indexes  $c$  and  $m$  correspond to characteristics of ceramics and metal relatively. Fraction of ceramic  $V_c$  and metal phases  $V_m$  are related by formula  $V_c + V_m = 1$ . For considered problems the corresponding expressions  $V_c$  are:

$$\begin{array}{l} \text{Model A} \\ \left\{ \begin{array}{l} V_c = \left( \frac{h + 2z}{h - h_c} \right)^p, \quad z \in \left[ -\frac{h}{2}, -\frac{h_c}{2} \right], \\ V_c = 1, \quad z \in \left[ -\frac{h_c}{2}, \frac{h_c}{2} \right], \\ V_c = \left( \frac{h - 2z}{h - h_c} \right)^p, \quad z \in \left[ \frac{h_c}{2}, \frac{h}{2} \right] \end{array} \right. \\ \text{Model B} \\ \left\{ \begin{array}{l} V_c = \left( \frac{h_c + 2z}{h_c - h} \right)^p, \quad z \in \left[ -\frac{h}{2}, -\frac{h_c}{2} \right], \\ V_c = 0, \quad z \in \left[ -\frac{h_c}{2}, \frac{h_c}{2} \right], \\ V_c = \left( \frac{h_c + 2z}{h + h_c} \right)^p, \quad z \in \left[ \frac{h_c}{2}, \frac{h}{2} \right] \end{array} \right. \end{array} \quad (2)$$

In formula (2) index  $p$  denotes the volume fraction exponent (gradient index). Influence of the foundation is taken into account through relation  $p_0 = K_w w - K_p \nabla^2 w$ . Where  $K_w, K_p$  are the Winkler and Pasternak parameters relatively for elastic foundation.

Mathematical statement of the problem is carried out within the first and higher order shear deformation theory of shallow shells: FSDT and HSDT. To solve this problem we apply variational Ritz's method combined with the R-functions theory (RFM method) [3]. In order to illustrate the possibilities of the proposed approach we have considered the shallow shells with cutout of the complex form. Influence of the different parameters on fundamental frequencies are studied in this work.

## References

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