The analysed object is an induction micromotor with a short-circuited auxiliary phase (shaded-pole micromotor - SPM). The motor rating data are: output power 3 W, two poles, one short-circuited coil per pole. Due to flat – parallel distribution of the field and radial direction of flux in SPM, the computational analysis was limited to the cross section plane. Field calculations were done with OPERA 2d and FEMM software, using finite element method. Several models of the SPM motor have been defined basing on numerical simulations. They were different in: the topology of the magnetic shunt, the number of short-circuited coils per pole, also the number and shape of rotor’s rods. Taking into account the manufacturing conditions of the SPM motor, the shunt set up and geometry were proposed. The optimum solution of motor’s magnetic circuit with the largest value of flux density in air gap and also the largest starting torque basing on steady general dimensions has been achieved.

1. INTRODUCTION

Induction motors with a shorted phase are called shaded-pole motors (SPM). The most common structure of SPM induction motor is the following: two or four salient poles with stator winding, and a single or double shorted coil in the recess on the pole. With present technology an SPM motor requires a
magnetic shunt. It is most often designed as an elongation of the salient poles. Proper cross-section of the shunt and the location of the shorted coil improves gap induction distribution. This directly influences the produced electromagnetic torque.

Relatively simple technology and motor structure contrasts with its complicated electromagnetic computing. The computing is complex, in particular due to multiple asymmetries of the magnetic circuit and winding.

The core of the problem is to develop an evolutionary optimization algorithm for a particular motor. This algorithm has to be efficient and compatible with chosen field computing program. The space for optimization is the air gap and a magnetic shunt with shorted coil. It is defined by boundary conditions functions. Optimization was conducted with the goal of maximizing starting torque of SPM. After finishing field modelling the optimization process begins at starting torque of $T_s = 0.412 \times 10^{-3}$ Nm.

2. CHARACTERISTICS OF COMPUTATION METHOD

One of the most useful environments for complex geometries is Final Element Method (FEM). The electromagnetic problem in FEM requires accurate defining due to the generation of multiple nodes for space discretisation of an object.

![Fig. 1. A part of the mesh of the SPM computational model](image)
Several dozen of SPM motor geometries were studied using 2D electromagnetic field analysis (Fig. 1). Phenomena occurring in a micromotor shaft were approximated using standard FEM procedures implemented in Opera 2D. In an induction motor with a shorted winding a flat – parallel field distribution and a radial direction of flux occur.

The assumptions concerning the computational analysis was limited to the cross section plane of the motor. Dimensions of the core, stator and rotor were also included.

Three layers of the air gap were designed. A uniform mesh was generated. The following field calculations are for 12 SPM motor models. different materials and the shapes of the magnetic circuit were analysed. Comparison of the results showed the following differences between particular solutions:

- the change in the shorted coil plane cross section in the recess on the pole.
- the introduction of the additional shorted coil at each pole and its changed span across the pole.
- change of the number and shape of rotor slots.
- change of the number of rotor slots in relation to basic model. \( Q_r = 13 \) to \( Q_r = 12 \) and \( Q_r = 24 \), respectively.
- change of the shape of magnetic shunt at the external recess of the pole.
- the introduction of the new shape of magnetic shunt with circular recess inside its structure. This is so-called technological shunt.

After introducing those changes in an SPM motor it can be noticed that the main element affecting its parameters is a magnetic shunt. The change of its geometry influences starting torque. It can be best seen in 1NMES, 2MES and 12MES (chart 1).

Introduction of the optimized magnetic shunt - model 12MES leads to the highest starting torque \( T_s = 1.408 \times 10^{-3} \) Nm. This example of computation is the verification of the optimization method.

### 3. FIELD CALCULATIONS BY FEMM SOFTWARE

The next environment for an SPM motor analysis is FEMM. The goal of employing two equivalent environments is confirming the convergence of the results. Also the goal of testing FEMM cooperation with Matlab.
In FEMM space the key element is the possibility of redefining the properties of particular domains after changing the geometry. It enables making changes to geometry without the necessity of defining new material and structure of domain. The decisive factor in computing is the time required for creating new model after every iteration. For models developed in FEMM the air gap is divided into two layers due to the possibility of computing the Maxwell stress tensor for median line between two layers of air. This line is determined by division triangle sides. (Fig. 2). In this case the Maxwell stress tensor represents the force generated by electromagnetic field described by equation [1]:

$$\vec{F} = \oint_S T \cdot dS = \frac{1}{2} \oint_S (H \cdot B) + B(H \cdot n) - (H \cdot B)n\,dS$$  \hspace{1cm} (1)$$

where:
T – Maxwell stress tensor,
n – vector normal to integration area S,
H – vector of magnetic field strength,
B – vector of magnetic field induction.
Calculations done by FEMM encompass different SPM motor solutions. These include shaping of its magnetic circuit and air gap. The relation points were Opera models. The chart 2 presents the best FEMM solutions.

In the scope of asymmetric air gap models it can be noticed that:
- Moving of asymmetric air gap to the proximity of magnetic shunt may increase the starting torque. This effect is further enhanced by a recess at the external side of the shunt. The model 2FEM confirms that effect.
- Simulation research was done for several dozens of SPM motors with asymmetric air gap. The shape of the air gap was changed. The results indicate slight changes in the starting torque. An additional asymmetric air gap decreases the starting torque by 1 to 7 per cent.

4. OPTIMIZATION ALGORITHM

The main element of optimization is the choice of proper algorithm. This in connection with executing blocks of operation creates a closed cycle until reaching an optimal solution. The solution presented in this paper includes an evolutionary algorithm in Matlab. It is implemented as a genetic algorithm function.

Fig. 3. SPM motor model for optimization process with solution space boundaries
This in turn generates sequential computational cycles according to the defined criteria. The genetic algorithm used in this paper uses Matlab functions included in Genetic Algorithm Toolbox (GA-Toolbox), [10]. An independent tool used here is Toolbox GAOT in Matlab employed with changes for optimization process. The Implemented algorithm takes into account binary coding in the first phase. The alleles of all chromosomes are 0 or 1. It was found that the most valuable chromosome features are for the real float number coding.

The task of solving an SPM air gap and magnetic shunt was reduced to optimization of the gain function of the starting torque \( m(x) \) for the motor with shorted winding (Fig. 3).

While solving an optimization task, it cannot be assumed with certainty that gain function will be a positive number for every \( X \). This caused a non-negative criterion of gain function value defined by an adaptation function.

For adaptation function characteristics presented in Fig 4 heuristic cross-over and edge mutation genetic operators were used. This combination of genetic operators leads to a quick finding of the final solution. The Optimal SPM solution is situated at the very edge of the solution space. This was proved by all combinations of genetic operators.

![Fig. 4. The curve \( F_p = f(L_p) \) of adaptation function for variant IIc calculations and optimal SPM solution](image-url)
5. OPTIMIZATION RESULTS

An analysis of the optimization calculations realized by evolutionary algorithms ought to be connected with the field calculations of SPM motor.

Fig. 5 shows the optimization results of the starting torque. FEMM and Matlab cooperation is a tool of analyzed object optimization. This object is an SPM induction motor with shorted winding. Chart 1 shows the comparison of optimization results, genetic operators and coding type influence on the final solution.

The optimization process described above is aimed at maximizing the starting torque. Two kinds of coding were included: binary and real number. In consequence of that approach it was possible to use more precise optimization operators for a solved task.

In the scope of binary coding the task was defined in variants Ia and Ib. It was possible to conclude that:

- Binary coding leads to frequent reaching of the Hamming’s edges. This is setting the algorithm in the proximity of a local solution which is not the optimization process proper answer.
- In order to achieve proper selection of operators and the size of basic population many parallel optimization calculations must be performed.
On the basis of the adaptation function course it can be noticed that the size of basic population heavily influences the final solution. The best result was reached by an algorithm with 40 individuals in each population. The number of populations was 300.

Table 1. The set of decision variables, adaptation function and starting torque.
Different variants of SPM optimization

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>variant Ia</th>
<th>variant Ib</th>
<th>variant IIa</th>
<th>variant IIb</th>
<th>variant IIc</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_1</td>
<td>0.725</td>
<td>0.756</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D_2</td>
<td>0.832</td>
<td>0.945</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D_3</td>
<td>0.654</td>
<td>0.365</td>
<td>0.942</td>
<td>0.986</td>
<td>0.985</td>
</tr>
<tr>
<td>D_4</td>
<td>0.730</td>
<td>0.685</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D_5</td>
<td>0.815</td>
<td>0.741</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D_6</td>
<td>0.724</td>
<td>0.452</td>
<td>0.956</td>
<td>0.923</td>
<td>0.978</td>
</tr>
<tr>
<td>K_1</td>
<td>0.072</td>
<td>0.036</td>
<td>0.009</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>K_2</td>
<td>0.081</td>
<td>0.025</td>
<td>0.012</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>F_p</td>
<td>0.810</td>
<td>0.760</td>
<td>1.215</td>
<td>1.217</td>
<td>1.220</td>
</tr>
<tr>
<td>T_s [Nm*10^{-3}]</td>
<td><strong>0.972</strong></td>
<td><strong>0.912</strong></td>
<td><strong>1.458</strong></td>
<td><strong>1.460</strong></td>
<td><strong>1.464</strong></td>
</tr>
</tbody>
</table>

In the scope of real number coding the task was defined in variants IIa and IIb. Thus it was possible to conclude that:

- Real number coding with adopted combination of cross-over and mutation operators enables to reach a global optimum.
- In order to achieve a proper operation of evolutionary algorithm and to compensate for the selection pressure the cross-over operator must affect population uniformly. The mutation operator has to be of a wide range.
- Adopting the combination of genetic operators in variant IIc for well-defined SPM motor optimization makes it possible to reach a global solution in solution space determined by boundary functions (right column Chart1)
- The value of starting torque showed in Chart1 for variant IIc $T_s = 1.464 \times 10^{-3}$ is close to starting torque values obtained from field calculations done by FEMM and Opera. It confirms the correctness of the optimization method.
6. CONCLUSIONS

Obtained results allow to conclude the following:

- The convergence of field calculations coming from two different FEMM and Opera systems confirms the correctness of adapting much simpler FEMM system. FEMM is a part of the optimization module in the evolutionary algorithm.
- Introducing an optimized magnetic shunt to the SPM motor structure – model 12MES permits to achieve the highest starting torque value $T_s = 1,408 \times 10^{-3}$ Nm. This field calculations result verifies the optimization process solution.
- Obtained from float number coding optimization calculations starting torque value $T_s = 1,464 \times 10^{-3}$ Nm is close to field calculations done by Opera.
- As a result of optimization process starting torque value of studied motor was increased from 0.4 x 10^-3 Nm to 1,464 x 10^-3 Nm.
- The set of genetic operators proposed here leads to a quick and correct final solution which is the global solution. An optimal SPM solution is located at the very edge of acceptable solution space. This was confirmed by each combination of genetic operators.

REFERENCES


Streszczenie

Celem pracy było przedstawienie procesu poszukiwania optymalnej konstrukcji silnika ze zwojem zwartym. W tym celu zastosowano obliczenia polowe przy pomocy pakietów Opera i FEMM. Niezależnym elementem wykorzystanym w pracy jest dodatkowy Toolbox GAOT, wprowadzony do Matlaba, a następnie z odpowiednimi zmianami wykorzystany w procesie optymalizacyjnym. Zdefiniowano funkcję optymalizacyjną, która w określonej przestrzeni poszukiwań wyszukiwała największą wartość momentu rozruchowego. W procesie optymalizacji został wprowadzony do MATLABA zadany algorytm ewolucyjny z odpowiednimi operatorami genetycznymi.

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