ŁUKASZ DOMAGALSKI, MICHAŁ GAJDZICKI
JAROSŁAW JĘDRYSIAK

Department of Structural Mechanics
Lodz University of Technology

ON A CERTAIN NONLINEAR MODEL OF THIN PERIODIC PLATES

Reviewer: **professor Ewaryst Wierzbicki**

The object under consideration are thin plates, which structure is periodic in planes parallel to the midplane. Plates of this kind consist of many small, repetitive elements, called periodicity cells, that can be treated as thin plates. The microstructure size is characterized by the diameter of the cell, which is called the microstructure parameter l. It is assumed that mechanical properties (bending and membrane stiffness tensors' components) of such plates are periodic, highly-oscillating, non-continuous functions. The main aim is to propose a mathematical model describing moderately large static deflections problem of considered plates, which is based on the tolerance modelling technique. A calculational example for a specific problem is included. The results are compared with results obtained within the linear model and with Finite Element Method.

1. Introduction

The objects of considerations are thin linear-elastic plates with a periodic structure in planes parallel to the plate midplane, cf. Fig. 1, subjected to large deflections. Governing equations of static problems of such plates have non-continuous, highly oscillating, functional coefficients. Exact solutions to these equations are very difficult to obtain. Therefore, various simplified approaches, introducing effective plate properties, are proposed. Amongst them there have to be mentioned models based on the asymptotic homogenization, cf. Kohn and Vogelius [7]. Unfortunately, governing equations of these models usually neglect the effect of the microstructure size on the plate behaviour.
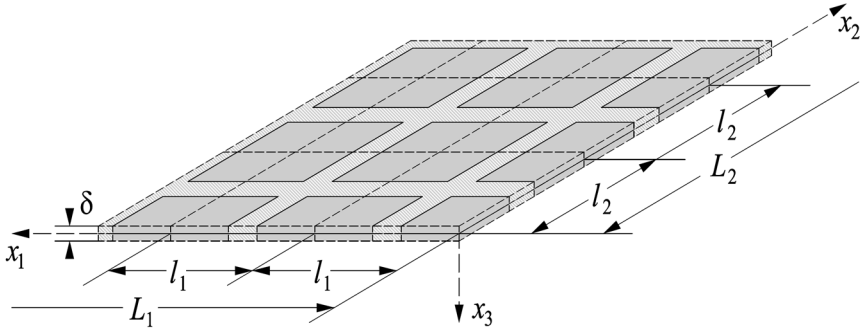
Fig. 1. Fragment of a thin periodic plate

In this paper, in order to obtain a model that takes this effect into account, the tolerance modelling approach is applied, cf. the books edited by Woźniak, Michalak and Jędrysiak (eds.) [14] and by Woźniak et al. (eds.) [13]. Applications of this method to other problems of periodic plates are shown in a series of papers, e.g. for vibrations of periodic wavy-type plates by Michalak [9], for periodically stiffened plates by Nagórko and Woźniak [10], for the buckling of periodic thin plates by Jędrysiak [4], for plates with the inhomogeneity period of an order of the plate thickness by Baron [1], for stability and vibrations of periodic plates by Jędrysiak [5, 6], for some problems of bending of thin periodic plates by Domagalski and Jędrysiak [2, 3].

The main aim of this note is to present the nonlinear tolerance model of elastostatic problems for thin periodic plates with large deflections. The paper contains also an illustrative example of a rectangular periodic plate. The results are calculated within proposed nonlinear model and the known linear tolerance model, and then compared with those obtained by a finite element program.

2. Fundamental equations

Let $Ox_1x_2x_3$ be an orthogonal Cartesian coordinate system; subscripts i, j, k, l run over 1, 2, 3 and α, β, γ, ω run over 1, 2. Denote $x = (x_1,x_2)$ and $z = x_3$. The undeformed plate occupies the region $\Omega \equiv \{(x,z): -\delta(x)/2 \leq z \leq \delta(x)/2, x \in \Pi\}$, with midplane Π and the plate thickness $\delta(\cdot)$.

It is assumed that periodic plates under consideration consist of many small repetitive elements called *periodicity cells*. The cell is defined as a plane region $\Omega \equiv [-l_1/2,l_1/2] \times [-l_2/2,l_2/2]$, where l_1, l_2 are the cell dimensions along the x_1-, x_2-axis. The size of the microstructure of the plate is described by the diameter of the periodicity cell, given by $l=[(l_1)^2+(l_2)^2]^{1/2}$ and satisfying the condition $\max(\delta) << l << \min(L_1,L_2)$, ($L_1$ and L_2 are characteristic dimensions of the plate

along the x_1- and x_2-axis). This diameter is called *the microstructure parameter*. Hence, the cell can be treated as a thin plate. Let us denote the partial derivatives with respect to a space coordinate by $\partial_\alpha = \partial/\partial x_\alpha$.

Our considerations are based on the well-known nonlinear theory of thin plates (cf. Timoshenko and Woinowsky-Krieger [11], and Woźniak (ed.) [12]). Let $w(x)$ be a plate midplane deflection, $u_{0\alpha}(x)$ be the in-plane displacements along the x_α-axes, $F(x)$ be the stress function, and $q(x)$ be the total loadings in the z-axis; $x \in \Pi$. Thickness $\delta(\cdot)$ can be a periodic function in x and elastic moduli $a_{ijkl} = a_{ijkl}(\cdot, z)$ can be also periodic functions in x and even functions in z. Let $a_{\alpha\beta\gamma\omega}$, $a_{\alpha\beta33}$, a_{3333} be the non-zero components of the elastic moduli tensor. Denote $c_{\alpha\beta\gamma\omega} \equiv a_{\alpha\beta\gamma\omega} - a_{\alpha\beta33}a_{33\gamma\omega}(a_{3333})^{-1}$.

Define the mean plate properties, being periodic functions in **x**, i.e. shell stiffnesses $b_{\alpha\beta\gamma\omega}$ and bending stiffnesses $d_{\alpha\beta\gamma\omega}$, in the form:

$$b_{\alpha\beta\gamma\omega}(\mathbf{x}) = \int_{-\delta(\mathbf{x})/2}^{\delta(\mathbf{x})/2} c_{\alpha\beta\gamma\omega}(\mathbf{x},z)dz, \quad d_{\alpha\beta\gamma\omega}(\mathbf{x}) = \int_{-\delta(\mathbf{x})/2}^{\delta(\mathbf{x})/2} c_{\alpha\beta\gamma\omega}(\mathbf{x},z)z^2 dz. \tag{1}$$

From the well-known assumptions of the nonlinear thin plate theory, e.g. the strains $E_{0\alpha\beta}$ of the plate midplane and curvatures $\kappa_{\alpha\beta}$ written as:

$$E_{0\alpha\beta} = \tfrac{1}{2}(\partial_\beta u_{0\alpha} + \partial_\alpha u_{0\beta} + \partial_\alpha w \partial_\beta w), \quad E_{0\alpha\beta} = \widetilde{b}_{\alpha\beta\gamma\omega}\Re_{\gamma\omega}F,$$
$$\kappa_{\alpha\beta} = -\partial_{\alpha\beta}w, \tag{2}$$

where $\widetilde{b}_{\alpha\beta\xi\eta}b_{\xi\eta\gamma\omega} = \delta_{\alpha\gamma}\delta_{\beta\omega}$, $\Re_{\alpha\beta}(\cdot) \equiv (\nabla^2\delta_{\alpha\beta} - \partial_{\alpha\beta})(\cdot)$, we obtain for periodic plates the following equations for the deflection w and the stress function F:

$$\Re_{\alpha\beta}(\widetilde{b}_{\alpha\beta\gamma\omega}\Re_{\gamma\omega}F) = \tfrac{1}{2}(\partial_{\alpha\beta}w\partial_{\alpha\beta}w - \partial_{\alpha\alpha}w\partial_{\beta\beta}w),$$
$$\partial_{\alpha\beta}(d_{\alpha\beta\gamma\omega}\partial_{\gamma\omega}w) - \Re_{\alpha\beta}F\partial_{\alpha\beta}w = q, \tag{3}$$

having highly oscillating, non-continuous, periodic in x functional coefficients.

3. The tolerance modelling approach

3.1. Introductory concepts

In the course of modelling, some introductory concepts of the tolerance modelling technique, such as the averaging operation $<\cdot>$, the slowly-varying (*SV*) function, the fluctuation shape (*FS*) function, are used. These concepts are described in books edited by Woźniak, Michalak and Jędrysiak (eds.) [14] and by Woźniak et al. (eds.) [13].

3.2. Fundamental assumptions

Following books [13, 14] and using the previously mentioned introductory concepts, the fundamental modelling assumptions can be formulated.

First of them is the *micro-macro decomposition* of the basic unknowns, where there is assumed:

1) for the out-of-plane deflection:

$$w(\mathbf{x}) = W(\mathbf{x}) + h^A(\mathbf{x})V^A(\mathbf{x}), \quad A = 1, \ldots, N, \tag{4}$$

2) for the in-plane displacements:

$$u_{0\alpha}(\mathbf{x}) = U_{0\alpha}(\mathbf{x}), \tag{5}$$

3) for the stress function:

$$F(\mathbf{x}) = \Phi(\mathbf{x}) + g^K(\mathbf{x})\Psi^K(\mathbf{x}), \quad K = 1, \ldots, M, \tag{6}$$

and $W(\cdot), V^A(\cdot), \Phi(\cdot), \Psi^K(\cdot) \in SV_d^2(\Pi, \Omega)$, $U_\alpha(\cdot) \in SV_d^1(\Pi, \Omega)$ are basic unknowns; $h^A(\cdot), g^K(\cdot) \in FS_d^2(\Pi, \Omega)$ are the known fluctuation shape functions. Functions $W(\cdot), \Phi(\cdot), V^A(\cdot)$ and $\Psi^K(\cdot)$ are called *the macrodeflection*, *the macrostress function*, *the fluctuation amplitudes* of the deflection and of the stress function, respectively; $U_{0\alpha}(\cdot)$ are the *in-plane macrodisplacements*.

The additional assumption is the decomposition of the load $q(\mathbf{x})$ in the form $q(\mathbf{x}) = q^0(\mathbf{x}) + \tilde{q}(\mathbf{x})$, where $q^0 = <q>$ is the slowly-varying averaged load, and $\tilde{q}$ is the oscillating part, $<\tilde{q}> \cong 0$.

4. Model equations

Applying the modelling procedure described in [13, 14], under denotations

$$
\begin{aligned}
&\tilde{B}_{\alpha\beta\gamma\omega} \equiv <\tilde{b}_{\alpha\beta\gamma\omega}>, &&\tilde{B}_{\gamma\omega}^K \equiv <\tilde{b}_{\alpha\beta\gamma\omega}\Re_{\alpha\beta}g^K>, &&\tilde{B}^{KL} \equiv <\tilde{b}_{\alpha\beta\gamma\omega}\Re_{\alpha\beta}g^K\Re_{\gamma\omega}g^L>, \\
&D_{\alpha\beta\gamma\omega} \equiv <d_{\alpha\beta\gamma\omega}>, &&D_{\alpha\beta}^A \equiv <d_{\alpha\beta\gamma\omega}\partial_{\gamma\omega}h^A>, &&D^{AB} \equiv <d_{\alpha\beta\gamma\omega}\partial_{\gamma\omega}h^A\partial_{\alpha\beta}h^B>, \quad (7)\\
&Q \equiv <q^0>, &&Q^A \equiv <\tilde{q}h^A>l^{-2}, &&G_{\alpha\beta}^{AB} \equiv <\partial_\alpha h^A\partial_\beta h^B>l^{-2},
\end{aligned}
$$

we arrive at the following system of equations for the macrostress function $\Phi(\cdot)$, the fluctuation amplitudes of the stress function $\Psi^K(\cdot)$, the macrodeflection $W(\cdot)$, the fluctuation amplitudes of the deflection $V^A(\cdot)$:

$$\Re_{\alpha\beta}(\widetilde{B}^{eff}_{\alpha\beta\gamma\omega}\Re_{\gamma\omega}\Phi) = \tfrac{1}{2}\Re_{\alpha\beta}(\partial_\alpha W\partial_\beta W) + \underline{\tfrac{1}{2}l^2 G^{AB}_{\alpha\beta}\Re_{\alpha\beta}(V^B V^A)},$$

$$\partial_{\alpha\beta}(D_{\alpha\beta\gamma\omega}\partial_{\gamma\omega}W + D^B_{\alpha\beta}V^B) - \Re_{\alpha\beta}\Phi\partial_{\alpha\beta}W = Q,$$

$$D^A_{\gamma\omega}\partial_{\gamma\omega}W + D^{AB}V^B + \underline{l^2 G^{AB}_{\alpha\beta}V^B\Re_{\alpha\beta}\Phi} = \underline{l^2 Q^A}, \tag{8}$$

$$\Psi^K = -(\widetilde{B}^{LK})^{-1}\widetilde{B}^L_{\alpha\beta}\Re_{\alpha\beta}\Phi; \quad \alpha,\beta,\gamma,\omega = 1,2; \quad A,B = 1,...,N; \quad K,L = 1,...,M,$$

where $\widetilde{B}^{eff}_{\alpha\beta\gamma\omega} \equiv \widetilde{B}_{\alpha\beta\gamma\omega} - \widetilde{B}^L_{\alpha\beta}(\widetilde{B}^{KL})^{-1}\widetilde{B}^K_{\gamma\omega}$.

Equations (8) together with micro-macro decompositions (4)-(6) constitute *the nonlinear tolerance model* of thin periodic plates. This model describes the effect of the microstructure size on the overall plate behaviour by the underlined terms. For considered plates there have to be formulated boundary conditions only for the macrodeflection W and the macrostress function Φ. Moreover, the basic unknowns of equations (8) have to satisfy the following conditions: $W(\cdot), V^A(\cdot), \Phi(\cdot), \Psi^K(\cdot) \in SV^2_d(\Pi,\Omega)$. To evaluate obtained results we recall the governing equations of *the linear tolerance model* of thin periodic plates:

$$D_{\alpha\beta\gamma\omega}\partial_{\alpha\beta\gamma\omega}W + D^A_{\alpha\beta}\partial_{\alpha\beta}V^A = Q,$$

$$D^A_{\alpha\beta}\partial_{\alpha\beta}W + D^{AB}V^B = \underline{l^2 Q^A}, \tag{9}$$

cf. Jędrysiak [4], Woźniak, Michalak and Jędrysiak (eds.) [14]. It can be observed that in this model the effect of the microstructure size is taken into account only by the term related to the oscillating part $l^2 Q^A$ of the load.

5. Example of application

5.1. Formulation of the problem

The object under consideration is a simply supported rectangular plate with constant thickness δ and length dimensions L_1 and $L_2 = \eta L_1$ along the x_1- and x_2-axis, respectively. It is also assumed that all edges of the plate are immovable. The plate is made of two isotropic materials (a matrix – M, a rib – R), having Young's modulus E_M and E_R and Poisson's ratio ν_M and ν_R, periodically distributed along the x_1- and x_2-axis.

Solutions $W(\cdot), \Phi(\cdot)$ to the model equations (8) have to satisfy boundary conditions for the simply supported plate, i.e. $W = \partial_{11}W = 0$ for $x_1 = 0, L_1$; $W = \partial_{22}W = 0$ for $x_2 = 0, L_2$; $\partial_{22}\Phi = N_{11}$ for $x_1 = 0, L_1$ and $\partial_{11}\Phi = N_{22}$ for $x_2 = 0, L_2$. Therefore, denoting $\xi_m = m\pi/L_1$, $\zeta_n = n\pi/L_2$, the above mentioned solutions can be assumed in the form of double sine or cosine trigonometric series ([8, 11]):

$$W(x_1, x_2) = \sum_n \sum_m W_{mn} \sin \xi_m x_1 \sin \zeta_n x_2,$$

$$V^A(x_1, x_2) = \sum_n \sum_m V^A_{mn} \sin \xi_m x_1 \sin \zeta_n x_2,$$

$$\Phi(x_1, x_2) = \sum_n \sum_m \Phi_{mn} \cos \xi_m x_1 \cos \zeta_n x_2 + \tfrac{1}{2} p_1 x_2^2 + \tfrac{1}{2} p_2 x_1^2, \tag{10}$$

$$\Psi^K(x_1, x_2) = \sum_n \sum_m \Psi^K_{mn} \cos \xi_m x_1 \cos \zeta_n x_2,$$

where constants p_1 and p_2 represent average edge membrane tensions in the x_1- and x_2-direction. The transversal average loads $Q(x_1, x_2)$ and $Q^A(x_1, x_2)$ can be expanded into double sine series:

$$Q(x_1, x_2) = \sum_n \sum_m Q_{mn} \sin \xi_m x_1 \sin \zeta_n x_2,$$

$$Q^A(x_1, x_2) = \sum_n \sum_m Q^A_{mn} \sin \xi_m x_1 \sin \zeta_n x_2. \tag{11}$$

Application of the Galerkin method leads to the following set of nonlinear, coupled algebraic equations for coefficients of series (10):

$$\widetilde{b}^{eff}_{kl} \overline{\Phi}_{kl} = \sum_{p,q,r,s} \Omega^{pqrs}_{kl} \overline{W}_{pq} \overline{W}_{rs} + \eta^2 \lambda^2 \sum_{A,B} \sum_{p,q,r,s} \Gamma^{AB,pqrs}_{kl} \overline{V}^A_{pq} \overline{V}^B_{rs},$$

$$d_{mn} \overline{W}_{mn} + \pi^2 (m^2 \overline{p}_1 + \eta^2 n^2 \overline{p}_2) \overline{W}_{mn} - \sum_{i,j,k,l} \mathrm{M}^{ijkl}_{mn} \overline{W}_{ij} \overline{\Phi}_{kl} + \eta \sum_B d^B_{mn} \overline{V}^B_{mn} - q_{mn} = 0,$$

$$\eta \sum_B d^{AB} \overline{V}^B_{mn} + d^A_{mn} \overline{W}_{mn} - \eta \lambda^2 \sum_B \sum_{i,j,k,l} \mathrm{N}^{AB,ijkl}_{mn} \overline{V}^B_{ij} \overline{\Phi}_{kl} + \tag{12}$$

$$+ \lambda^2 \eta \sum_B [G^{AB}_{11} \overline{p}_1 + G^{AB}_{22} \overline{p}_2] \overline{V}^B_{mn} - \lambda^2 q^A_{mn} = 0,$$

where overlined terms represent dimensionless forms of unknown coefficients:

$$\overline{W}_{mn} \equiv \frac{W_{mn}}{\delta}, \quad \overline{V}^A_{mn} \equiv \frac{L_1 L_2}{\delta} V^A_{mn}, \quad \overline{\Phi}_{kl} \equiv \frac{\Phi_{kl}}{E_M \delta^3}, \tag{13}$$

dimensionless load-dependent terms are as follows:

$$[q_{mn}, q^A_{mn}] \equiv \frac{L_1^4}{E_M \delta^4} [Q_{mn}, Q^A_{mn}], \quad \overline{p}_\alpha \equiv \frac{L_1^2}{E_M \delta^3} p_\alpha, \tag{14}$$

where λ is dimensionless microstructure parameter $\lambda = l/L_1$. The other coefficients occurring in (12) are obtained as a result of applying the Galerkin method and, in order to limit the length of this paper, will not be presented here.

Dropping the nonlinear terms appearing in equations (12) or applying the Galerkin method to equations (9), algebraic equations of the linear tolerance model are obtained:

$$d_{mn}\overline{W}_{mn} + \eta\sum_B d^B_{mn}\overline{V}^B_{mn} - q_{mn} = 0,$$

$$\eta\sum_B d^{AB}\overline{V}^B_{mn} + d^A_{mn}\overline{W}_{mn} - \lambda^2 q^A_{mn} = 0. \tag{15}$$

In the next section, some numerical results calculated in the framework of the nonlinear and linear tolerance models are presented. These results are also compared with results obtained using a finite element program.

5.2. Calculational results

Numerical results are calculated for a square plate ($\eta=1$), made of two different isotropic materials. The periodicity cell is a square and is defined as $\Omega \equiv [-l/2, l/2] \times [-l/2, l/2]$, cf. Fig. 2. It is assumed that the Young's modulus and Poisson's ratio are given by

$$E(\mathbf{x}), \nu(\mathbf{x}) = \begin{cases} E_M, \nu_M & \text{if} & \mathbf{x} \in [-l/2, -\alpha l/2) \times [-l/2, -\alpha l/2) \cup \\ & & \cup [-l/2, -\alpha l/2) \times (\alpha l/2, l/2] \cup \\ & & \cup (\alpha l/2, l/2] \times [-l/2, -\alpha l/2) \cup \\ & & \cup (\alpha l/2, l/2] \times (\alpha l/2, l/2], \\ E_R, \nu_R & \text{if} & \mathbf{x} \in [-l/2, -\alpha l/2) \times [-\alpha l/2, \alpha l/2] \cup \\ & & \cup [-\alpha l/2, \alpha l/2] \times [-l/2, l/2] \cup \\ & & \cup (\alpha l/2, l/2] \times [-\alpha l/2, \alpha l/2], \end{cases} \tag{16}$$

where α is a dimensionless parameter describing distribution of material properties in the periodicity cell, cf. Fig. 2.
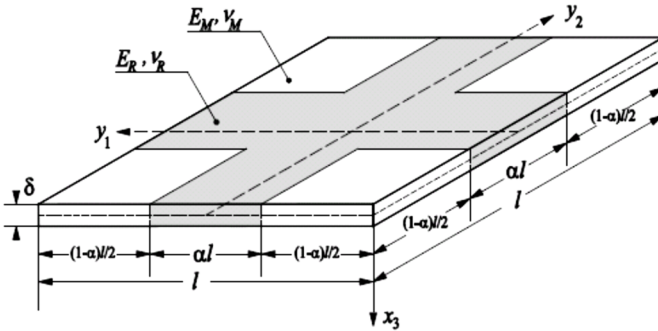

Fig. 2. A basic periodicity cell

The fluctuation shape functions h^A and g^K are assumed in the similar form:

$$h^A = g^A = \tfrac{1}{3}l^2[(1 - \cos\frac{2A\pi y_1}{l})(1 - \cos\frac{2A\pi y_2}{l}) - 1], \quad A = 1,...,N, \tag{17}$$

satisfying conditions $< h^A > = 0$.

In calculations we assume for the matrix: the Young's modulus E_M=69 GPa, the Poisson's ratio ν_M=0.316. Moreover, assume the length L_1=1.0 m, the thickness δ=1×10^{-3} m. For the rib it is assumed: the Young's modulus $E_R=\varepsilon E_M$, the Poisson's ratio $\nu_R=\nu_M$. Results presented here are obtained for following values of parameters: dimensionless width of the rib α=0.2; dimensionless microstructure parameter λ=0.05; Young's modulus' ratio ε=4.0, 3.0, 2.0. A case of uniformly distributed transversal load is considered, $q(x)=q$, with the maximum value applied 34.5 Pa.
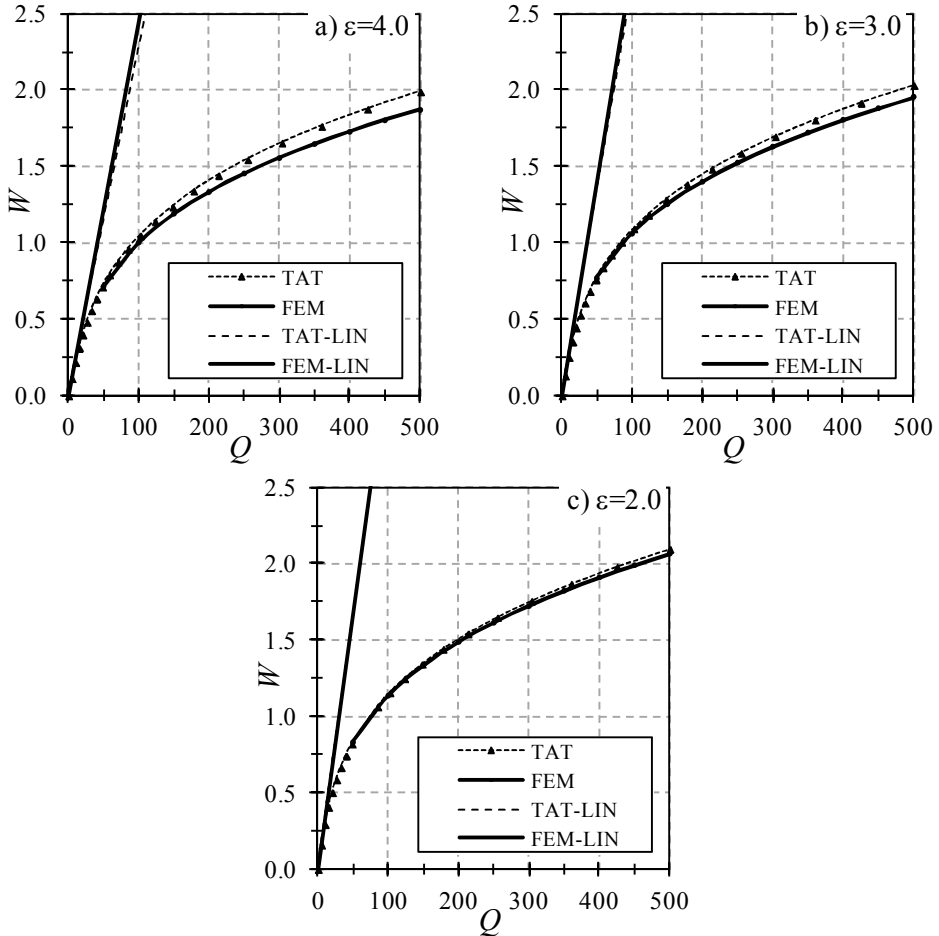

Fig. 3. Load-deflection curves

Figure 3 shows magnitudes of central deflection calculated in the framework of the tolerance model (TAT) and the finite element method (FEM) with various

values of Young's moduli ratio ε. The results are shown in their dimensionless form, cf. formulas (13), (14).

Table 1. Deflections at maximum load computed –
comparison of results

ε	type of analysis	FEM	TAT	Δ
4.00	nonlinear	1.869	1.989	6.41%
	linear	12.28	11.34	-7.64%
3.00	nonlinear	1.954	2.030	3.87%
	linear	14.11	13.35	-5.40%
2.00	nonlinear	2.064	2.097	1.58%
	linear	16.86	16.41	-2.65%

Comparison of center deflection calculated within FEM and TAT models under maximum load applied is shown in table 1. It has to be mentioned that, both for nonlinear and linear model, differences between center deflection under maximum load computed are less than 0.1%.

6. Remarks

Using the tolerance modelling to the known differential equations of thin periodic plates with large deflections the averaged equations of the nonlinear tolerance model are derived. This technique makes it possible to replace the governing equations with non-continuous, periodic, highly oscillating coefficients by the system of differential equations with constant coefficients.

Analysing results presented in the previous section it can be noted that:
- differences between deflections calculated within the nonlinear tolerance model and FEM are, for computed load and Young's moduli ratio, less than 10%;
- these differences are less for the nonlinear than for linear analysis and decrease with decreasing parameter ε;
- the linear tolerance model seems to underestimate, while the nonlinear model tends to overestimate calculated deflections.

Then, some general remarks can be formulated:
- the derived equations of the nonlinear tolerance model involve terms, which take into account the effect of the microstructure size on the overall behaviour of periodic plates;

- the governing equations of the linear tolerance model take into account the effect of the microstructure size only by the term dependent of the oscillating part of the load.

References

[1] **Baron E.:** On modelling of periodic plates having the inhomogeneity period of an order of the plate thickness. J. Theor. Appl. Mech., 44, 1, 2006, 3-18.

[2] **Domagalski Ł., Jędrysiak J.:** Modelling of thin periodic plates subjected to large deflections. Civ. Environm. Engnrg Rep., 5, 2010, 121-136.

[3] **Domagalski Ł., Jędrysiak J.:** On the elastostatics of thin periodic plates with large deflections. Meccanica (in the course of publication: MECC779R1).

[4] **Jędrysiak J.:** The length-scale effect in the buckling of thin periodic plates resting on a periodic Winkler foundation. Meccanica, 38, 2003, 435451.

[5] **Jędrysiak J.:** The tolerance averaging model of dynamic stability of thin plates with one-directional periodic structure. Thin Walled Struct., 45, 2007, 855-860.

[6] **Jędrysiak J.:** Higher order vibrations of thin periodic plates. Thin Walled Struct., 47, 2009, 890-901.

[7] **Kohn R.V., Vogelius M.:** A new model of thin plates with rapidly varying thickness. Int. J. Solids Struct., 20, 1984, 333-350.

[8] **Levy S.:** Bending of rectangular plates with large deflections. NACA Rep. No. 737, NACA Tech. Note No. 846.

[9] **Michalak B.:** The meso-shape functions for the meso-structural models of wavy-plates, ZAMM, 81, 2001, 639-641.

[10] **Nagórko W., Woźniak Cz.:** Nonasymptotic modelling of thin plates reinforced by a system of stiffeners. Electronic J. Polish Agric. Univ., Civ Engnrg, 5, 2, 2002: online: www.ejpau.media.pl.

[11] **Timoshenko S., Woinowsky-Krieger S.:** Theory of plates and shells. New York, McGraw-Hill 1959.

[12] **Woźniak Cz. (ed.):** Mechanics of Elastic Plates and Shells. Warszawa, PWN 2001 (in Polish).

[13] **Woźniak Cz. et al. (eds.):** Mathematical modelling and analysis in continuum mechanics of microstructured media. Gliwice (Poland), Wydawnictwo Politechniki Śląskiej 2010.

[14] **Woźniak Cz., Michalak B, Jędrysiak J. (eds.):** Thermomechanics of micro-heterogeneous solids and structures. Tolerance averaging approach. Łódź (Poland), Wydawnictwo Politechniki Łódzkiej 2008.

O PEWNYM NIELINIOWYM MODELU CIENKICH PŁYT PERIODYCZNYCH

Streszczenie

Rozważane są cienkie płyty o strukturze periodycznej w płaszczyznach równoległych do płaszczyzny środkowej. Płyty tego rodzaju składają się z wielu małych, powtarzalnych elementów, zwanych komórkami periodyczności, z których każda może być traktowana jak cienka płyta. Wielkość mikrostruktury jest charakteryzowana poprzez średnicę (największy liniowy wymiar) komórki. Wymiar ten jest nazywany parametrem mikrostruktury i oznaczany przez *l*. Przyjęto, że własności mechaniczne płyty, reprezentowane przez składowe tensorów sztywności płytowych i tarczowych, są periodycznymi, nieciągłymi, silnie oscylującymi funkcjami. Głównym celem opracowania jest zaproponowanie matematycznego modelu opisującego zagadnienie umiarkowanie dużych ugięć rozważanych płyt, opartego na tzw. technice modelowania tolerancyjnego. Praca zawiera przykład obliczeniowy dla pewnego przypadku szczególnego. Dokonano porównania wyników uzyskanych w ramach proponowanego modelu nieliniowego, modelu liniowego oraz Metody Elementów Skończonych.