STABILITY OF THIN UNIPERIODIC CYLINDRICAL SHELLS

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The aim of this contribution is to propose a new averaged model for stability analysis of thin linear-elastic cylindrical shells having the periodic structure along one direction tangent to the shell midsurface. In contrast with the known homogenized models the proposed one makes it possible to describe the effect of the periodicity cell size on the overall shell behavior (a length-scale effect). In order to derive the governing equations with constant or slowly varying coefficients the known tolerance averaging procedure is applied. The comparison between the proposed model and the model without the length-scale effect is presented.

1. INTRODUCTION

In this paper a new average model of cylindrical shells having a periodic structure (a periodically varying thickness and/or periodically varying elastic and inertial properties) along one direction tangent to the undeformed shell midsurface $\mathcal{M}$ is presented. Shells like that are termed *uniperiodic*.

The exact equations of the shell (plate) theory are too complicated to constitute the basis for investigations of most engineering problems because they involve highly oscillating and often discontinuous coefficients. Thus many different approximated modelling methods for periodic (locally periodic) shells and plates have been formulated. Structures of this kind are usually described using homogenized models derived by means of asymptotic methods. Unfortunately, these models neglect the effect of periodicity cell length dimensions on the global structure behavior (the length-scale effect). The alternative nonasymptotic modelling procedure based on the notion of tolerance and leading to so-called the length-scale (or tolerance) models of dynamic and stationary problems for micro-periodic structures was proposed by Woźniak in a series of papers, e.g. [2,3]. These tolerance models have constant coefficients and take into account the effect of a periodicity cell size on the global body behavior. This effect is described by means of certain extra unknowns called *internal or fluctuation variables* and by known functions, which represent oscillations inside the periodicity cell. The length-scale model for stability analysis of cylindrical shells with two-directional periodic structure has been proposed in [1]. However, this model is not sufficient to analyze stability problems of uniperiodic cylindrical shells, which are not special case of those with a periodic structure in both directions tangent to $\mathcal{M}$.

The aim of this contribution is to derive an averaged model of uniperiodic cylindrical shell, which has constant coefficients in direction of periodicity and describes the effect of a cell size on the global shell stability. This model will be derived by using the tolerance averaging procedure proposed by Woźniak and Wierzbicki in [3].The proposed tolerance model will be compared with a simplified (homogenized) one in which the length-scale effect is neglected.

2. PRELIMINARIES

In this paper we will investigate thin linear-elastic cylindrical shells with periodic structure along one direction tangent to $\mathcal{M}$ and slowly varying structure along the perpendicular direction tangent to $\mathcal{M}$.

Denote by $\Omega \subset R^2$ a regular region of points $\Theta \equiv (\Theta^1, \Theta^2)$ on the $O\Theta^1\Theta^2$-plane, Θ^1, Θ^2 being the Cartesian orthogonal coordinates on this plane and let E^3 be the physical space parametrized by the Cartesian orthogonal coordinate system $Ox^1x^2x^3$. Let us introduce the orthogonal parametric representation of the undeformed smooth cylindrical shell midsurface $\mathcal{M}$ by means of:
$\mathcal{M} := \{ \mathbf{x} \equiv (x^1, x^2, x^3) \in E^3 : \ \mathbf{x} = \mathbf{x}(\Theta^1, \Theta^2), \Theta \in \Omega \}$, where $\mathbf{x}(\Theta^1, \Theta^2)$ is a position vector of a point on $\mathcal{M}$ having coordinates Θ^1, Θ^2. Throughout the paper indices $\alpha, \beta, \ldots$ run over 1,2 and are related to the midsurface parameters Θ^1, Θ^2; indices A, B, ... run over $1,2,\ldots,N$, summation convention holds for all aforesaid indices. To every point $\mathbf{x}=\mathbf{x}(\Theta)$, $\Theta \in \Omega$ we assign a covariant base vectors $\boldsymbol{a}_\alpha = \mathbf{x}_{,\alpha}$ and covariant midsurface first and second metric tensors denoted by $a_{\alpha\beta}$, $b_{\alpha\beta}$, respectively, which are given as follows: $a_{\alpha\beta} = \boldsymbol{a}_\alpha \cdot \boldsymbol{a}_\beta$, $b_{\alpha\beta} = \boldsymbol{n} \cdot \boldsymbol{a}_{\alpha,\beta}$, where $\boldsymbol{n}$ is a unit normal to $\mathcal{M}$. Let $\delta(\Theta)$ stand for the shell thickness. We also define t as the time coordinate. Taking into account that coordinate lines Θ^2=const. are parallel on the $O\Theta^1\Theta^2$-plane and that Θ^2 is an arc coordinate on $\mathcal{M}$ we define l as the period of shell structure in Θ^2-direction. The period l is assumed to be sufficiently large compared with the maximum shell thickness and sufficiently small as compared to the midsurface curvature radius R as well as the characteristic length dimension L of the shell midsurface along the direction of shell periodicity. Under given above assumptions for period l the shell under consideration will be referred to as *a mezostructured shell*, cf.[2], and the period l will be called *the mezostructured length parameter*. We shall denote by $\Lambda \equiv \{0\} \times (-l/2, l/2)$ the straight line segment on the $O\Theta^1\Theta^2$-plane along the $O\Theta^2$-axis direction, which can be taken as a representative cell of the shell periodic structure (the periodicity cell). To every $\Theta \in \Omega$ an arbitrary cell on $O\Theta^1\Theta^2$-plane will be defined by means of: $\Lambda(\Theta) \equiv \Theta + \Lambda$, $\Theta \in \Omega_\Lambda$, $\Omega_\Lambda := \{\Theta \in \Omega : \Lambda(\Theta) \subset \Omega\}$, where the point $\Theta \in \Omega_\Lambda$ is a center of a cell $\Lambda(\Theta)$ and Ω_Λ is a set of all the cell centers which are inside Ω.

A function $f(\Theta)$ defined on Ω_Λ will be called Λ-periodic if for arbitrary but fixed Θ^1 and arbitrary $\Theta^2, \Theta^2 \pm l$ it satisfies condition: $f(\Theta^1, \Theta^2) = f(\Theta^1, \Theta^2 \pm l)$ in the whole domain of its definition and it is not constant.

It is assumed that the cylindrical shell thickness as well as its material and inertial properties are Λ-periodic functions of Θ^2 and slowly varying functions of Θ^1. Shells like that are called *uniperiodic*, moreover under given above assumptions for period l they are referred to *mezostructured shells*.

For an arbitrary integrable function $\varphi(\cdot)$ defined on Ω, following [3], we define *the averaging operation*, given by

$$< \varphi >(\Theta) \equiv \frac{1}{l} \int_{\Lambda(\Theta)} \varphi(\Theta^1, \Psi^2)\, d\Psi^2, \qquad \Theta = (\Theta^1, \Theta^2) \in \Omega_\Lambda .$$

For a function φ, which is Λ-periodic in Θ^2 *this* formula leads to $< \varphi >(\Theta^1)$. If the functions φ is Λ-periodic in Θ^2 and is independent of Θ^1, its averaged value obtained from the above formula is constant.

Our considerations will be based on the simplified linear Kirchhoff-Love theory of thin elastic shells.

Let $u_\alpha(\Theta,t)$, $w(\Theta,t)$ stand for the midsurface shell displacements in directions tangent and normal to $\mathcal{M}$, respectively. We denote by $\varepsilon_{\alpha\beta}(\Theta,t)$, $\kappa_{\alpha\beta}(\Theta,t)$ the membrane and curvature strain tensors and by $n^{\alpha\beta}(\Theta,t)$, $m^{\alpha\beta}(\Theta,t)$ the stress resultants and stress couples, respectively. The properties of shell are described by 2D-shell stiffness tensors $D^{\alpha\beta\gamma\delta}(\Theta)$, $B^{\alpha\beta\gamma\delta}(\Theta)$ and let $\mu(\Theta)$ stand for a shell mass density per midsurface unit area. Let $f_\alpha(\Theta,t)$, $f(\Theta,t)$ be external force components per midsurface unit area, respectively tangent and normal to $\mathcal{M}$.

Functions $\mu(\Theta)$, $D^{\alpha\beta\gamma\delta}(\Theta)$, $B^{\alpha\beta\gamma\delta}(\Theta)$ and $\delta(\Theta)$ are Λ-periodic functions of Θ^2 and are assumed to be slowly varying functions of Θ^1.

We denote by $\overline{N}^{\alpha\beta}$ the compressive membrane forces in the shell midsurface, which satisfy the following equations of equilibrium $\overline{N}^{\alpha\beta}_{,\alpha} + f^\beta = 0$, $\quad b_{\alpha\beta}\overline{N}^{\alpha\beta} + f = 0$.

The equations of a shell theory under consideration consist of :

1) the strain-displacement equations

$$\varepsilon_{\gamma\delta} = u_{(\gamma,\delta)} - b_{\gamma\delta}\, w \;, \qquad \kappa_{\gamma\delta} = -\, w_{,\gamma\delta} \;, \tag{2.1}$$

2) the stress-strain relations

$$n^{\alpha\beta} = D^{\alpha\beta\gamma\delta}\,\varepsilon_{\gamma\delta} \;, \qquad m^{\alpha\beta} = B^{\alpha\beta\gamma\delta}\,\kappa_{\gamma\delta} \;, \tag{2.2}$$

3) the equations of equilibrium

$$n^{\alpha\beta}{}_{,\alpha} = 0 \;, \qquad m^{\alpha\beta}{}_{,\alpha\beta} + b_{\alpha\beta}\, n^{\alpha\beta} - \overline{N}^{\alpha\beta} w_{,\alpha\beta} = 0 \;, \tag{2.3}$$

In the above equations the displacements $u_\alpha = u_\alpha(\Theta, t)$ and $w = w(\Theta, t)$, $\Theta\in\Omega$, are the basic unknowns.

For mezostructured shells, $\mu(\Theta)$, $D^{\alpha\beta\gamma\delta}(\Theta)$ and $B^{\alpha\beta\gamma\delta}(\Theta)$, $\Theta\in\Omega$, are highly oscillating Λ-periodic functions; that is why equations (2.1)-(2.3) cannot be directly applied to the numerical analysis of special problems. From (2.1)-(2.3) an averaged model of uniperiodic cylindrical shells having coefficients, which are independent of Θ^2-midsurface parameter and are slowly varying functions of Θ^1 as well as describing the cell size effect on critical forces will be derived. In order to derive it the tolerance averaging procedure given by Woźniak and Wierzbicki in [3], will be applied. To make the analysis more clear, in the next section we shall outline the basic concepts and the main kinematic assumption of this approach, following the monograph [3].

3. BASIC CONCEPTS

The fundamental concepts of the tolerance averaging approach are that of a certain tolerance system, slowly varying functions, periodic-like functions and periodic-like oscillating functions. These functions will be defined with respect to the Λ-periodic shell structure defined in the foregoing section.

By *a tolerance system* we shall mean a pair $T=(\mathcal{F}, \varepsilon(\cdot))$, where $\mathcal{F}$ is a set of real valued bounded functions $F(\cdot)$ defined on $\overline{\Omega}$ and their derivatives, which represent the unknowns in the problem under consideration (such as unknown shell displacements tangent and normal to $\mathcal{M}$) and for which the tolerance parameters ε_F being positive real numbers and determining the admissible accuracy related to computations of values of $F(\cdot)$ are given ; by ε is denoted the mapping $\mathcal{F} \ni F \rightarrow \varepsilon_F$.

A continuous bounded differentiable function $F(\Theta,t)$ defined on $\overline{\Omega}$ is called Λ-*slowly_varying* with respect to the cell Λ and the tolerance system T, $F \in SV_\Lambda(T)$, if roughly speaking, can be treated (together with its derivatives) as constant on an arbitrary periodicity cell Λ. The continuous function $\varphi(\cdot)$ defined on $\overline{\Omega}$ will be termed *a Λ-periodic-like function*, $\varphi(\cdot)\in PL_\Lambda(T)$, with respect to the cell Λ and the tolerance system T, if for every $\Theta=(\Theta^1,\Theta^2)\in\Omega_\Lambda$ there exists a continuous Λ-periodic function $\varphi_\Theta(\cdot)$ such that $(\forall\Psi=(\Theta^1,\Psi^2))\ [\|\Theta - \Psi\| \leq l \Rightarrow \varphi(\Psi) \cong \varphi_\Theta(\Psi)]$, $\Psi\in\Lambda(\Theta)$, and the similar conditions are also fulfilled by all its derivatives. It means that the values of a periodic-like function $\varphi(\cdot)$ in an arbitrary cell $\Lambda(\Theta)$, $\Theta\in \Omega_\Lambda$, can be approximated, with sufficient accuracy, by the corresponding values of a certain Λ-periodic function $\varphi_\Theta(\cdot)$. The function $\varphi_\Theta(\cdot)$ will be referred to as a Λ-periodic approximation of $\varphi(\cdot)$ on $\Lambda(\Theta)$. Let $\mu(\cdot)$ be a positive value Λ-periodic function. The periodic-like function φ is called Λ-oscillating (with the weight μ), $\varphi(\cdot)\in PL^{\mu}_{\Lambda}(T)$, provided that the condition $<\mu\varphi>(\Theta) \cong 0$ holds for every $\Theta\in\Omega_\Lambda$.

If $F\in SV_\Lambda(T)$, $\varphi(\cdot)\in PL_\Lambda(T)$ and $\varphi_\Theta(\cdot)$ is a Λ-periodic approximation of $\varphi(\cdot)$ on $\Lambda(\Theta)$ then for every Λ-periodic bounded function $f(\cdot)$ and every continuous Λ-periodic differentiable function $h(\cdot)$ such that $\sup\{|h(\Psi^1,\Psi^2)|, (\Psi^1,\Psi^2)\in\Lambda\}\leq l$, the following *tolerance averaging relations* determined by the pertinent tolerance parameters hold for every $\Theta\in\Omega_\Lambda$:

(T1) $\langle fF\rangle(\Theta) \cong \langle f\rangle(\Theta)F(\Theta)$, **(T2)** $\langle f(hF)_{,2}\rangle(\Theta) \cong \langle fFh_{,2}\rangle(\Theta)$,

(T3) $\langle f\varphi\rangle(\Theta) \cong \langle f\varphi_\Theta\rangle(\Theta)$, **(T4)** $\langle h(f\varphi)_{,2}\rangle(\Theta) \cong -\langle f\varphi h_{,2}\rangle(\Theta)$.

In the tolerance averaging procedure, the left-hand sides of formulae **(T1)-(T4)** will be approximated by their right-hand sides, respectively - this operation will be called the *Tolerance Averaging Assumption.*

The main kinematic assumption of the tolerance averaging method is called *Conformability Assumption* and states that in every periodic solid the displacement fields have to conform to the periodic structure of this solid. It means that the displacement fields are periodic-like functions and hence can be represented by a sum of averaged displacements, which are slowly varying, and by highly oscillating periodic-like disturbances, caused by the periodic structure of the solid.

The aforementioned *Conformability Assumption* together with the *Tolerance Averaging Assumption* constitute the foundations of the tolerance averaging technique. Using this technique the tolerance model of stability problems for uniperiodic cylindrical shells will be derived in the subsequent section.

4. GOVERNING EQUATIONS

Let us assumed that there is a certain tolerance system $T=(\mathcal{F}, \varepsilon(\cdot))$, where the set $\mathcal{F}$ consists of the unknown shell displacements tangent and normal to $\mathcal{M}$ and their derivatives. From the *Conformability Assumption* , it follows that the unknown shell displacements $u_\alpha(\cdot,t)$, $w(\cdot,t)$ in Eqs.(2.1)-(2.3) have to satisfy the conditions: $u_\alpha(\cdot\,,t)\in PL_\Lambda(T)$, $w(\cdot\,,t)\in PL_\Lambda(T)$. Hence, we obtain what is called the *modelling decomposition*

$$u_\alpha(\cdot\,,t)= U_\alpha(\cdot\,,t)+d_\alpha(\cdot\,,t)\,, \quad w(\cdot\,,t)= W(\cdot\,,t)+p(\cdot\,,t)\,,$$
$$U_\alpha(\cdot\,,t),\ W(\cdot\,,t)\in SV_\Lambda(T), \quad d_\alpha(\cdot\,,t), p(\cdot\,,t)\in PL^{\mu}_{\Lambda}(T)\,, \tag{4.1}$$

which becomes under the normalizing conditions $<\mu\, d_\alpha(\cdot\,,t)> = <\mu p(\cdot\,,t)>=0$.

It can be shown, cf. [3], that the unknown Λ-slowly varying averaged displacements $U_\alpha(\cdot\,,t)$, $W(\cdot\,,t)$ in (4.1) are given by : $U_\alpha(\cdot\,,t)\equiv<\mu>^{-1}(\Theta^1)<\mu\, u_\alpha>(\cdot\,,t)$, $W(\cdot\,,t)\equiv<\mu>^{-1}(\Theta^1)<\mu\, w>(\cdot\,,t)$. The unknown displacement disturbances $d_\alpha(\cdot\,,t)$, $p(\cdot\,,t)$ in (4.1) being oscillating periodic-like functions are caused by the highly oscillating character of the shell mezostructure.

Substituting the right-hand side of (4.1) into (2.3) and after the tolerance averaging of the resulting equations, we arrive at the equations

$$[< D^{\alpha\beta\gamma\delta} > (\Theta^1)(U_{\gamma,\delta} - b_{\gamma\delta}W) + < D^{\alpha\beta\gamma\delta} d_{\gamma,\delta} > (\Theta,t) + b_{\gamma\delta} < D^{\alpha\beta\gamma\delta} p > (\Theta,t)]_{,\alpha} = 0,$$
$$[< B^{\alpha\beta\gamma\delta} > (\Theta^1) W_{,\gamma\delta} + < B^{\alpha\beta\gamma\delta} p_{,\gamma\delta} > (\Theta,t)]_{,\alpha\beta} - b_{\alpha\beta}[< D^{\alpha\beta\gamma\delta} > (\Theta^1)(U_{\gamma,\delta} - b_{\gamma\delta}W) + \tag{4.2}$$
$$+ < D^{\alpha\beta\gamma\delta} d_{\gamma,\delta} > (\Theta,t) - b_{\gamma\delta} < D^{\alpha\beta\gamma\delta} p >] - \overline{N}^{\alpha\beta} W_{,\alpha\beta} = 0$$

Multiplying Eqs.$(2.3)_1$ and $(2.3)_2$ by arbitrary Λ-periodic test functions d^*, p^*, respectively, such that $<\mu d^*>= <\mu p^*>=0$, integrating these equations over $\Lambda(\Theta)$, $\Theta\in\Omega_\Lambda$, and using the *Tolerance Averaging Assumption* as well as denoting by $\tilde{d}_\alpha$, $\tilde{p}$ the Λ- periodic approximations of d_α , p , respectively, on $\Lambda(\Theta)$, we obtain the periodic problem on $\Lambda(\Theta)$ for functions $\tilde{d}_\alpha(\Theta^1,\Psi^2,t)$, $\tilde{p}(\Theta^1,\Psi^2,t)$, $(\Theta^1,\Psi^2)\in\Lambda(\Theta) = \Lambda(\Theta^1,\Theta^2)$, given by the following variational conditions

$$- < d^*_{,2} D^{2\beta\gamma\delta} \tilde{d}_{\gamma,\delta} > + < d^* (D^{1\beta\gamma\delta} \tilde{d}_{\gamma,\delta})_{,1} > - b_{\gamma\delta}[- < d^*_{,2} D^{2\beta\gamma\delta} \tilde{p} > + < d^* (D^{1\beta\gamma\delta} \tilde{p})_{,1} >] =$$
$$= < d^*_{,\alpha} D^{\alpha\beta\gamma\delta} > (U_{\gamma,\delta} - b_{\gamma\delta}W) - [< d^* D^{1\beta\gamma\delta} > (U_{\gamma,\delta} - b_{\gamma\delta}W)]_{,1}\ ,$$
$$< p^*_{,22} B^{22\gamma\delta} \tilde{p}_{,\gamma\delta} > -2 < p_{,2} (B^{21\gamma\delta} \tilde{p}_{,\gamma\delta})_{,1} > + < p^* (B^{11\gamma\delta} \tilde{p}_{,\gamma\delta})_{,11} > + \tag{4.3}$$
$$- b_{\alpha\beta}[< p^* D^{\alpha\beta\gamma\delta} \tilde{d}_{\gamma,\delta} > - b_{\gamma\delta} < p^* D^{\alpha\beta\gamma\delta} \tilde{p} >] = b_{\alpha\beta} < p^* D^{\alpha\beta\gamma\delta} > (U_{\gamma,\delta} - b_{\gamma\delta}W) +$$
$$- < p^*_{,22} B^{22\lambda\delta} > W_{,\gamma\delta} + 2[(< p^*_{,2} B^{21\gamma\delta} >_{,1} - < p^*_{,21} B^{21\gamma\delta} >) W_{,\gamma\delta} + < p^*_{,2} B^{21\gamma\delta} > W_{,\gamma\delta 1}] +$$
$$- \{(< p^* B^{11\gamma\delta} >_{,1} - 2 < p^*_{,1} B^{11\lambda\delta} >)_{,1} + < p^*_{,11} B^{11\gamma\delta} >] W_{,\gamma\delta} + 2(< p^* B^{11\gamma\delta} >_{,1} +$$
$$- < p^*_{,1} B^{11\gamma\delta} >) W_{,\gamma\delta 1} + < p^* B^{11\gamma\delta} > W_{,\gamma\delta 11}\} + \overline{N}^{11} < p^* > W_{,11}$$

An approximate solution to this problem, which may be obtained by the orthogonalization method, will be assumed in the form

$$
\begin{aligned}
&\tilde{d}_{\alpha}(\Theta^1,\Psi^2,t)=h^A(\Theta^1,\Psi^2)Q^A_{\alpha}(\Theta^1,\Theta^2,t),\\
&\tilde{p}(\Theta^1,\Psi^2,t)=g^A(\Theta^1,\Psi^2)V^A(\Theta^1,\Theta^2,t),\quad A=1,2,\dots,N,
\end{aligned}
\tag{4.4}
$$

with $h^A(\Theta^1,\cdot)$, $g^A(\Theta^1,\cdot)$ as postulated Λ-periodic *shape functions* such that $<\mu h^A>(\Theta^1)=<\mu g^A>(\Theta^1)=0$, $\max|\, h^A(\Theta^1,\Psi^2)\,|\leq l$, $\max|\, g^A(\Theta^1,\Psi^2)\,|\leq l^2$, $h^A,\, lh^A_{,2},\, l^{-1}g^A,\, g^A_{,2},\, l\,g^A_{,22}\in O(l)$ and with $Q^A_{\alpha}(\Theta^1,\Theta^2,t), V^A(\Theta^1,\Theta^2,t)$ as new unknowns called *fluctuation variables*, being Λ-slowly varying functions in Θ^2, i.e. $Q^A_{\alpha}, V^A\in SV_{\Lambda}(T)$.

Substituting the right-hand sides of (4.4) into (4.2) and (4.3) and setting $d^*=h^A(\Theta^1,\Psi^2)$, $p^*=g^A(\Theta^1,\Psi^2)$, $A=1,2,\dots,N$, in (4.3), on the basis of the *Tolerance Averaging Assumption* we arrive at the *tolerance fluctuation variable model of stability problems for unperiodic cylindrical shells.* Under extra denotations

$$
\begin{aligned}
&\tilde{D}^{\alpha\beta\gamma\delta}\equiv<D^{\alpha\beta\gamma\delta}>, D^{A\alpha\beta\gamma}\equiv<D^{\alpha\beta\gamma\delta}h^A_{,\delta}>, \overline{D}^{A\alpha\beta\gamma}\equiv l^{-1}<D^{\alpha\beta\gamma 1}h^A>,\\
&L^{A\alpha\beta}\equiv l^{-2}b_{\gamma\delta}<D^{\alpha\beta\gamma\delta}g^A>, \tilde{B}^{\alpha\beta\gamma\delta}\equiv<B^{\alpha\beta\gamma\delta}>, K^{A\alpha\beta}\equiv<B^{\alpha\beta\gamma\delta}g^A_{,\gamma\delta}>,\\
&\overline{K}^{A\alpha\beta}\equiv l^{-1}<B^{\alpha\beta 1\delta}g^A_{,\delta}>,\quad \breve{K}^{A\alpha\beta}\equiv l^{-2}<B^{\alpha\beta 11}g^A>,\ C^{AB\beta\gamma}\equiv<D^{\alpha\beta\gamma\delta}h^A_{,\alpha}h^B_{,\delta}>,\\
&\overline{C}^{AB\beta\gamma}\equiv l^{-1}<D^{\alpha\beta\gamma 1}h^A_{,\alpha}h^B>,\ F^{AB\beta}\equiv l^{-2}b_{\gamma\delta}<D^{\alpha\beta\gamma\delta}h^A_{,\alpha}g^B>,\\
&\tilde{C}^{AB\beta\gamma}\equiv l^{-2}<D^{1\beta\gamma 1}h^A h^B>,\ \overline{F}^{AB\beta}\equiv l^{-3}b_{\gamma\delta}<D^{1\beta\gamma\delta}h^A g^B>, R^{AB}\equiv<B^{\alpha\beta\gamma\delta}g^A_{,\alpha\beta}g^B_{,\gamma\delta}>,\\
&\overline{L}^{AB}\equiv l^{-4}b_{\alpha\beta}b_{\gamma\delta}<D^{\alpha\beta\gamma\delta}g^A g^B>,\quad \breve{R}^{AB}\equiv l^{-1}<B^{1\beta\gamma\delta}g^A_{,\beta}g^B_{,\gamma\delta}>,\\
&\tilde{R}^{AB}\equiv l^{-2}<B^{11\gamma\delta}g^A_{,\gamma\delta}g^B>, \overline{R}^{AB}\equiv l^{-3}<B^{1\beta 11}g^A_{,\beta}g^B>,\\
&\hat{R}^{AB}\equiv l^{-4}<B^{1111}g^A g^B>,\qquad \tilde{S}^{AB}\equiv l^{-2}<B^{1\gamma 1\delta}g^A_{,\gamma}g^B_{,\delta}>,
\end{aligned}
\tag{4.5}
$$

this model is represented by :

1) *the constitutive equations*

$$
\begin{aligned}
N^{\alpha\beta}&=\tilde{D}^{\alpha\beta\gamma\delta}(U_{\gamma,\delta}-b_{\gamma\delta}W)+D^{B\alpha\beta\gamma}Q^B_{\gamma}+\underline{l\overline{D}^{B\alpha\beta\gamma}}Q^B_{\gamma,1}-\underline{\underline{l^2L^{B\alpha\beta}}}V^B,\\
M^{\alpha\beta}&=\tilde{B}^{\alpha\beta\gamma\delta}W_{,\gamma\delta}+K^{B\alpha\beta}V^B+\underline{2l\overline{K}^{B\alpha\beta}}V^B_{,1}+\underline{l^2\breve{K}^{B\alpha\beta}}V^B_{,11},\\
H^{A\beta}&=D^{A\beta\gamma\delta}(U_{\gamma,\delta}-b_{\gamma\delta}W)+C^{AB\beta\gamma}Q^B_{\gamma}+\underline{l\overline{C}^{AB\beta\gamma}}Q^B_{\gamma,1}-\underline{\underline{l^2F^{AB\beta}}}V^B,\\
\overline{H}^{A\beta}&\equiv\underline{l\overline{D}^{A\beta\gamma\delta}}(U_{\gamma,\delta}-b_{\gamma\delta}W)+\underline{l\overline{C}^{AB\beta\gamma}}Q^B_{\gamma}+\underline{l^2\tilde{C}^{AB\beta\gamma}}Q^B_{\gamma,1}-\underline{l^3\overline{F}^{AB\beta}}V^B,\\
G^A&\equiv-\underline{l^2L^{A\gamma\delta}}(U_{\gamma,\delta}-b_{\gamma\delta}W)+K^{A\alpha\beta}W_{,\alpha\beta}-\underline{\underline{l^2F^{AB\gamma}}}Q^B_{\gamma}-\underline{l^3\overline{F}^{AB\gamma}}Q^B_{\gamma,1}+\\
&+(R^{AB}+\underline{\underline{l^4\overline{L}^{AB}}}V^B+2\underline{l\breve{R}^{AB}}V^B_{,1}+\underline{l^2\tilde{R}^{AB}}V^B_{,11},\\
\tilde{G}^A&=\underline{l^2K^{A\alpha\beta}}W_{,\alpha\beta}+\underline{l^2\tilde{R}^{AB}}V^B+2\underline{l^3\overline{R}^{AB}}V^B_{,1}+\underline{l^4\hat{R}^{AB}}V^B_{,11},\\
\overline{G}^A&=\underline{l\overline{K}^{A\alpha\beta}}W_{,\alpha\beta}+\underline{l\breve{R}^{AB}}V^B+2\underline{l^2\tilde{S}^{AB}}V^B_{,1}+\underline{l^3\overline{R}^{AB}}V^B_{,11}\qquad ,
\end{aligned}
\tag{4.6}
$$

2) *the system of three averaged partial differential equations of equilibrium for averaged displacements* $U_{\alpha}(\Theta,t), W(\Theta,t)$

$$N^{\alpha\beta}_{,\alpha} = 0, \qquad M^{\alpha\beta}_{,\alpha\beta} - b_{\alpha\beta}N^{\alpha\beta} - \overline{N}^{\alpha\beta}W_{,\alpha\beta} = 0 \tag{4.7}$$

3) *the system of 3N partial differential equations for the fluctuation variables* $Q^{B}_{\alpha}(\Theta,t)$, $V^{B}(\Theta,t)$, $B=1,2,...,N$,

$$H^{A\beta} - \overline{H}^{A\beta}_{,1} = 0, \quad G^{A} + \tilde{G}^{A}_{,11} - 2\overline{G}^{A}_{,1} + N^{11}\underline{<g^{A}>}W_{,11} = 0, \quad A,B = 1,2,...,N. \tag{4.8}$$

The above model has a physical sense provided that the basic unknowns $U_{\alpha}(\Theta,t), W(\Theta,t), Q^{A}_{\gamma}(\Theta,t), V^{A}(\Theta,t) \in SV_{\Lambda}(T)$, $A=1,2,...,N$, i.e. they are Λ-slowly varying functions of Θ^{2} -midsurface parameter.

Taking into account (4.1) and (4.4) the shell displacement fields can be approximated by means of formulae

$$u_{\alpha}(\cdot,t) \approx U_{\alpha}(\cdot,t) + h^{A}(\cdot)Q^{A}_{\alpha}(\cdot,t), \quad w(\cdot,t) \approx W(\cdot,t) + g^{A}(\cdot)V^{A}(\cdot,t), A = 1,2,...,N, \tag{4.9}$$

where the approximation $\approx$ depends on the number of terms $h^{A}(\cdot)Q^{A}_{\alpha}(\cdot,t)$, $g^{A}(\cdot)V^{A}(\cdot,t)$.

The characteristic features of the derived model are:

1) The model takes into account the effect of the cell size on the overall shell behavior; this effect is describes by underlined coefficients dependent on the mezostructure length parameter l.

2) The model equations involve averaged coefficients which are independent of Θ^{2} -midsurface parameter (i.e. they are constant in direction of periodicity) and are slowly varying functions of Θ^{1}.

Assuming that the cylindrical shell under consideration has material and geometrical properties independent of Θ^{1} we obtain governing equations (4.7)-(4.8) with constant averaged coefficients. Moreover, in this case the mode-shape functions h^{A}, g^{A}, $A=1,2,...,N$, are also independent of Θ^{1} - midsurface parameter.

In the next section the homogenized model of uniperiodic cylindrical shells will be derived as a special case of Eqs.(4.6)-(4.8).

5. HOMOGENIZED MODEL

The simplified model of uniperiodic cylindrical shells can be derived directly from the tolerance model (4.6)-(4.8) by a limit passage $l \to 0$, i.e. by neglecting the underlined terms which depend on the mezostructure length parameter l. Hence, we arrive at the homogenized shell model governed by

1) equilibrium equations

$$D^{\alpha\beta\gamma\delta}_{eff}(U_{\gamma,\delta\alpha} - b_{\gamma\delta}W_{,\alpha}) = 0, \quad B^{\alpha\beta\gamma\delta}_{eff}W_{,\alpha\beta\gamma\delta} - b_{\alpha\beta}D^{\alpha\beta\gamma\delta}_{eff}(U_{\gamma,\delta} - b_{\gamma\delta}W) + \overline{N}^{\alpha\beta}W_{,\alpha\beta} = 0, \tag{5.1}$$

2) *constitutive equations*

$$N^{\alpha\beta} = D_{eff}^{\alpha\beta\gamma\delta}(U_{\gamma,\delta} - b_{\gamma\delta}W), \qquad M^{\alpha\beta} = -B_{eff}^{\alpha\beta\gamma\delta}W_{,\gamma\delta} \tag{5.2}$$

where $D_{eff}^{\alpha\beta\gamma\delta} \equiv \widetilde{D}^{\alpha\beta\gamma\delta} - D^{A\alpha\beta\eta}G_{\eta\xi}^{AB}D^{B\xi\gamma\delta}$, $B_{eff}^{\alpha\beta\gamma\delta} \equiv \widetilde{B}^{\alpha\beta\gamma\delta} - K^{A\alpha\beta}E^{AB}K^{B\gamma\delta}$, with $G_{\alpha\beta}^{AB}$ *and* E^{AB} defined by $G_{\alpha\beta}^{AB}C^{BC\beta\gamma} = \delta_{\alpha}^{\gamma}\delta^{AC}, E^{AB}R^{BC} = \delta^{AC}$.

The obtained above homogenized model governed by Eqs.(5.1),(5.2) is not able to describe the length-scale effect on the overall shell behavior being independent of the mezostructure length parameter l.

6. FINAL REMARKS

The subject-matter of this contribution is a thin linear-elastic cylindrical shell having a periodic structure in one direction tangent to the undeformed shell midsurface $\mathcal{M}$. Shells of this kind are termed uniperiodic. For these shells the equations governed of the Kirchhoff-Love shell theory involve highly oscillating periodic coefficients. In order to simplify the Kirchhoff-Love shell theory to the form which can be applied in engineering problems and also takes into account the effect of a periodicity cell on the overall shell behavior a new model of thin uniperiodic cylindrical shells has been proposed. In order to derive it the *tolerance averaging procedure* given by Woźniak and Wierzbicki in [3] was applied. This model called the tolerance model is represented by a system of partial differential equations (4.7)-(4.8) with coefficients, which are constant in direction of periodicity. The basic unknowns are : the *averaged displacements* U_α, W and the *fluctuation variables* $Q_\alpha^A, V^A, A = 1,2,...,N,$ which have to be *slowly varying functions* with respect to the cell and certain tolerance system. In order to obtain the governing equations the periodic shape *functions* $h^A, g^A, A = 1,2,...,N,$ should be postulated. *In contrast with the homogenized models the proposed one makes it possible to describe the effect of the periodicity cell on the critical forces (the length-scale effect).* Problems related to applications of the proposed Eqs.(4.6)-(4.8) to investigate the critical forces of uniperiodic cylindrical shells are reserved for a separate paper

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