TOLERANCE MODELLING OF DYNAMIC AND STABILITY PROBLEMS FOR THIN BIPERIODIC CYLINDRICAL SHELLS

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1. INTRODUCTION

Thin linearly elastic Kirchhoff-Love-type circular cylindrical shells with a periodically micro-inhomogeneous structure (a periodically varying thickness and/or periodically varying elastic and inertial properties) in circumferential and axial directions are analysed, cf. Fig. 1. Shells of this kind are termed *biperiodic*.

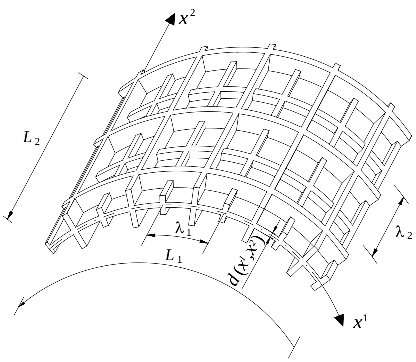

Fig. 1. Example of biperiodic cylindrical shell

The periods of heterogeneity are assumed to be very large compared with the maximum shell thickness and very small as compared to the midsurface curvature radius as well as the smallest characteristic length dimensions of the shell midsurface.

Dynamic and stability problems of such shells are described by partial differential equations with highly oscillating, non-continuous and periodic coefficients. Thus, these equations are too complicated to be applied to the investigations of engineering problems. To obtain averaged equations with constant coefficients, various approximate modelling procedures for shells of this kind have been proposed. Periodic cylindrical shells (plates) are usually described using *homogenized models* derived by means of *asymptotic methods*. Unfortunately, in the models of this kind *the effect of a cell size* on the overall shell behaviour is neglected. In order to analyse *the length-scale effect* in selected dynamic or/and stability problems for thin periodic cylindrical shells, the new averaged non-asymptotic models of such shells have been proposed and discussed by Tomczyk in a series of publications and summarized in [2]. These, so-called, tolerance models have been obtained by applying *the tolerance modelling technique* presented by Woźniak in many monographs and papers, e.g., in [3]. This technique is based on the concept of *tolerance relations between points and real numbers* related to the accuracy of the performed measurements and calculations. The second basic concept of this method is *a function slowly-varying within a cell*. A certain extended version of the tolerance modelling technique has been proposed by Tomczyk and Woźniak in [1]. This version is based on *a new notion of weakly slowly-varying functions* which is a certain extension of the well-known concept of *slowly-varying functions*.

The main aim of this contribution is to derive and discuss a new mathematical averaged *general tolerance model for the analysis of dynamics as well as stationary and dynamical stability of the biperiodic shells under consideration.* This model will be formulated on the basis of a notion of *weakly slowly-varying functions*. The second aim is to shown the differences and similarities between this *new general tolerance model* and the corresponding *known standard tolerance model* presented in [2], which was derived by applying the more restrictive notion of *slowly-varying functions*.

2. REMARKS AND CONCLUSIONS

The tolerance modelling technique based on the notion of *weakly slowly-varying function* is proposed as a tool to derive *a new mathematical non-asymptotic averaged model of dynamic and stability problems for thin biperiodic cylindrical shells* (cf. Fig. 1) subjected to time-dependent forces tangent to the shell midsurface.

Contrary to the starting well-known governing equations of Kirchhoff-Love second-order theory with highly oscillating, non-continuous and periodic coefficients, the general tolerance model equations proposed here have *constant coefficients depending also on a cell size*. Hence, the tolerance model makes it possible *to describe the effect of a length scale on the global shell dynamics and stability*.

The basic unknowns of the general tolerance model equations must be *weakly slowly-varying functions* in periodicity directions. This requirement can be verified only *a posteriori* and the pertinent values of tolerance parameters characterize both computational accuracy and physical reliability of the obtained solutions.

The general model equations contain a bigger number of terms depending on a cell size than *the known standard tolerance model equations* presented in [2], which were derived by applying 'classical' more restrictive concept of *the slowly-varying functions*. Thus, from the theoretical results it follows that the general model makes it possible to investigate the length-scale effect in more detail.

REFERENCES

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