STABILITY OF PERIODIC BEAMS

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1. INTRODUCTION

Composite structures are often applied in modern engineering. Using a present technology to combine some various materials into one heterogeneous structure, characterized by physical and mechanical properties, which are unreachable for homogeneous materials, optimizing of these properties for special engineering purposes can be possible. A kind of the simplest cases of periodic structures are beams consisting of many identical repeated elements. Because they have specific properties that they can find many applications in civil and mechanical engineering, e.g. in noise or vibration protection. Unfortunately, a mathematical description of such beams is rather difficult.

In this note stability of slender periodic beams, with the length of the cell l, interacting with an elastic foundation is considered. Beams of this kind are described by equations with highly-oscillating, periodic, non-continuous functional coefficients, e.g.:

$$\partial\partial(EJ\partial\partial w) - \partial(n\partial w) + kw = q, \tag{1}$$

where:

w - a deflection of the beam,
n - an axial force,
q - a load normal to the beam axis,
EJ - a periodic beam stiffness,
k - a Winkler's coefficient.

In order to solve special problems of these beams, there are applied different methods. Among them it is necessary to distinguish those based on the asymptotic homogenization. In this approach the periodic structure is modelled as a homogeneous structure with some effective properties, e.g. for beams by Kolpakov [1]. Other models, based on complicated displacement's field theories, e.g. broken line hypothesis, were used to describe buckling and dynamic stability of sandwich beams with microheterogeneous core e.g. by Grygorowicz et al. [2-3]. However, most of these averaging approaches lead to equations, which neglect the effect of the microstructure size. In order to take into account this effect *the tolerance modelling* can be applied, which allows to replace governing equations with periodic, non-continuous coefficients by equations with constant coefficients. This approach is developed for various problems of microstructured media, e.g. for stability of periodic plates by Jędrysiak [4], for vibrations of periodic plate strips by Marczak and Jędrysiak [5], for non-linear vibrations of periodic beams by Domagalski and Jędrysiak [6].

2. TOLERANCE MODEL

In the tolerance modelling procedure some concepts and assumptions are used, e.g.: a slowly-varying function, a tolerance-periodic function or an averaging operation, defined on an interval (-l/2,l/2), cf. [4-6]. The main modelling assumption, called *the micro-macro decomposition*, is that the beam deflection $w(x)$, is decomposed in the form: $w(x)=W(x)+h^A(x)V^A(x)$, where: $h^A(\cdot)$ are the known fluctuation shape functions (being periodic in x); W and V^A are macrodeflection and fluctuation variables of the beam axis, respectively (being slowly-varying in x).

Using these concepts and modelling assumptions, applying the modelling procedure, cf. [4-6], and introducing coefficients:

$$
\begin{aligned}
&D \equiv < EJ >, \quad D^A \equiv < EJ\partial\partial h^A >, \quad D^{AB} \equiv < EJ\partial\partial h^B \partial\partial h^A >, \quad l^2 H^{AB} \equiv < \partial\partial h^A h^B > \\
&K \equiv < k >, \quad l^2 K^A \equiv < kh^A >, \quad l^4 K^{AB} \equiv < kh^A h^B >, \quad Q \equiv < q >, \quad l^2 Q^A \equiv < qh^A >,
\end{aligned}
$$

the tolerance model equations take the form:

$$
\begin{aligned}
&D\partial\partial\partial\partial W + D^A\partial\partial V^A + KW + l^2 K^A V^A - \partial(N\partial W) - Q = 0, \\
&(D^{AB} + l^4 K^{AB})V^B + D^A\partial\partial W - l^2 H^{AB} N V^B + l^2 K^A W - l^2 Q^A = 0,
\end{aligned}
\tag{2}
$$

where: N - averaged axial force.

The above system of N+1 differential equations has averaged constant coefficients, in contrary to equation (1). Some of these coefficients involve the length of the periodicity cell. Hence, equations (2) describe the stability of the periodic beams, with taking into account the effect of the microstructure size. Some numerical results will be shown in forthcoming notes.

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REFERENCES

[1] Kolpakov A.G., Application of homogenization method to justification of 1-D model for beam of periodic structure having initial stresses, Int. J. Solids Struct., 35, 22, 1998, pp. 2847-2859.

[2] Grygorowicz M., Magnucki K., Malinowski M., Elastic buckling of a sandwich beam with variable mechanical properties of the core, Thin-Walled Struct., 87, 2015, pp. 127-132.

[3] Grygorowicz M., Magnucka-Blandzi E., Mathematical modelling for dynamic stability of sandwich beam with variable mechanical properties of core, Appl. Math. Mech., 37, 10, 2016, pp.1361-1374.

[4] Jędrysiak J., On stability of thin periodic plates, Eur. J. Mech.-A, 19, 2000, pp. 487-502.

[5] Marczak J., Jędrysiak J., Vibrations of plate strips with internal periodic structure, Eng. Trans., 63, 1, 2015, pp. 93-108.

[6] Domagalski Ł., Jędrysiak J., Geometrically nonlinear vibrations of slender meso-periodic beams. The tolerance modelling approach, Comp. Struct., 136, 2016, pp. 270-277.